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Factor Investing and Risk Allocation: From Traditional to Alternative Risk Premia Harvesting

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Foreword

The present publication was produced as part of the "Risk Allocation Solutions" research chair at EDHEC-Risk Institute, in partnership with Lyxor Asset Management. This chair is examining performance portfolios with improved hedging benefits, hedging portfolios with improved performance benefits, and inflation risk and asset allocation solutions.

This study, "Factor Investing and Risk Allocation: From Traditional to Alternative Risk Premia Harvesting", extends the analysis of factor investing beyond traditional factors and seeks to investigate what the best possible approach is for harvesting alternative long short-risk pemia.

There is a growing interest amongst sophisticated institutional investors in factor investing. It is now well accepted that the average long-term performance of active mutual fund managers can, to a large extent, be replicated through a static exposure to traditional factors, which implies that traditional long-only risk premia can be most efficiently harvested in a passive manner.

While the replication of hedge fund factor exposure appears to be a very attractive concept, the authors find that hedge fund replication strategies achieve in general a relatively low out-of-sample explanatory power, regardless of the set of factors and the methodologies used. Their results also suggest that risk parity strategies applied to alternative risk factors could be a better alternative than hedge fund replication for harvesting alternative risk premia in an efficient way.

A key challenge for the alternative investment industry remains the capacity to develop investable efficient low-cost proxies for harvesting alternative risk premia not only in equity markets but also in the fixed income, currencies and commodity markets.

I would like to thank Jean-Michel Maeso for his useful work on this research, and Laurent Ringelstein and Dami Coker for their efforts in producing the final publication. I would also like to extend particular thanks Nicolas Gaussel and Thierry Roncalli for their very useful comments and, more generally, to Lyxor Asset Management for their support of this research chair.

We wish you a useful and informative read.

Lionel Martellini
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It is now well accepted that the performance of active mutual fund managers can, to a large extent, be replicated through a static exposure to traditional factors (see for example Ang, Goetzmann, and Schaefer (2009) analysis of the Norwegian Government Pension Fund Global), which implies that traditional long-only risk premia can be most efficiently harvested in a passive manner. This paper extends the analysis of factor investing beyond traditional factors, and seeks to investigate what the best possible approach is for harvesting alternative long-short risk premia.

Hedge Fund Replication with Traditional and Alternative Factors

Benchmarking hedge fund performance is particularly challenging because of the presence of numerous biases in hedge fund return databases, the most important of which are the sample selection bias, the survivorship bias and the backfill bias. In what follows, we use EDHEC Alternative Indices, which aggregate monthly returns on competing hedge fund indices so as to improve the hedge fund indices' lack of representativeness and to mitigate the bias inherent to each database (see Amenc and Martellini (2003)). We consider the following thirteen categories: Convertible Arbitrage, CTA Global, Distressed Securities, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Global Macro, Long/Short Equity, Merger Arbitrage, Relative Value, Short Selling and Fund of Funds. We also define the set of relevant risk factors and suitable proxies that will be used in the empirical analysis. An overview of the 19 traditional and alternative risk factors considered in our empirical analysis is given in Table 1. We proxy traditional risk factors by returns of liquid and investable equity, bond, commodity and currency indices. For alternative risk factors, we inter alia consider long/short proxies for the two most popular factors, namely value and momentum, for various asset classes, using data from Asness, Moskowitz, and Pedersen (2013). A key difference between the traditional and alternative factors is that the latter cannot be regarded as directly investable, which implies that reported performance levels are likely to be overstated. Given the presence of performance biases in both hedge fund returns and alternative factor returns, we do not focus on differences in average performance between hedge fund indices and their replicating portfolios, and instead focus on the quality of replication measured by in-sample and out-of-sample (adjusted) R-squared and the annualised root mean squared error (RMSE).

As a first step, we perform an in-sample linear regression for each hedge fund strategy monthly returns against a set of K factors over the whole sample period ranging from January 1997 to October 2015. For each hedge fund strategy we have:

$$r_t^{HF} = \sum_{k=1}^{K} \beta_k F_{k,t} + \epsilon_t \qquad (0.1)$$

with r_t^{HF} being the monthly return of the hedge fund strategy at date t, β_k the exposure of the monthly return on hedge fund strategy to factor k (to be estimated), $F_{k,t}$ the monthly return at date t on factor k and ϵ_t the specific risk in the monthly return of hedge fund index at date t (to be estimated).

We estimate the explanatory power measured in terms of the linear regression adjusted R-squared on the sample period in three distinct cases.

Case 1: Linear regression on an exhaustive set of factors ("kitchen sink" regression),

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i.e. the set of 19 factors listed in Table 1.
Case 2: Linear regression on a subset of traditional factors only (5 factors: equity, bond, credit, commodity and currency).
Case 3: Linear regression on a bespoke subset of a maximum of 8 economically-motivated traditional and alternative factors for each hedge fund strategy (see Table 1 for the selection of factors for each hedge fund strategy).

The obtained adjusted R-squared values, reported in Table 2, suggest that we can explain a substantial fraction of hedge fund strategy return variability with traditional and alternative factors, validating that an important part of hedge fund performance can ex-post be explained by their systematic risk exposures. The kitchen sink regression (case 1) confirms that more dynamic and/or less directional strategies such as CTA Global, Equity Market Neutral, Fixed Income Arbitrage and Merger Arbitrage strategies, with respective adjusted R-squared of 31%, 32%, 50% and 39%, are harder to replicate than more static and/or more directional strategies such as long-short equity or short selling for which we obtain an adjusted R-squared of 81%.

The results we obtain also show the improvement in explanatory power when an economically motivated subset of factors that includes alternative factors is considered (case 3) compared to a situation where the same subset of traditional factors is used for all strategies (case 2). For example adjusted R-squared increases from 25% to 50% for the Global Macro strategy and from 52% to 80% for the Emerging Market strategy.

In a second step, we perform an out-of-sample hedge fund return replication exercise using for each strategy the bespoke subset of factors (case 3). The objective of this analysis is to assess whether one can capture the dynamic factor exposures of hedge fund strategies by explicitly allowing the betas to vary over time in a statistical model. The out-of-sample window considered is January 1999-October 2015, which allows us to build a "24-month rolling-window" linear clone for each strategy. For each hedge fund strategy we have:

$$r_t^{HF} = \sum_{k=1}^{K} \beta_{k,t} \ F_{k,t} + \epsilon_t \qquad (0.2)$$

with r_t^{HF} being the monthly return of the hedge fund strategy clone at date t, $\boldsymbol{\beta}_{k,t}$ the possibly time-varying exposure of the monthly return on hedge fund strategy to factor k on the rolling period [t - 24 months; t-1 month] (to estimate), $F_{k,t}$ the monthly return at date t on factor k and ϵ_t the specific risk in the monthly return of hedge fund index at date t (to estimate).

The hedge fund clone monthly return is:

$$r_t^{CL} = \sum_{k=1}^{K} \hat{\beta}_{k,t} \ F_{k,t} + (1 - \sum_{k=1}^{K} \hat{\beta}_{k,t}) \ r_t^f \qquad (0.3)$$

where $\hat{\beta}_{k,t}$ is the ordinary least squares (OLS) estimation of $\boldsymbol{\beta}_{k,t}$ from (0.2) on the rolling period [t - 24 months; t-1].

Since our focus is on hedge fund replication, we take into account the possible leverage of the strategy by adding a cash component r_t^f proxied by the US 3-month Treasury bill index monthly returns. A more sophisticated approach consists in explicitly modelling dynamic risk factor exposures through a linear state-space model and then solving it variables by Kalman filtering. Broadly speaking, a state-space model is defined by

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a transition equation and a measurement equation as follows:

$$\begin{cases} \beta_t = \beta_{t-1} + \eta_t \text{ (Transition Equation)} \\ r_t = \beta_t . F_t + \epsilon_t \text{ (Measurement Equation)} \end{cases} \quad (0.4)$$

where β_t is the vector of (unobservable) factor exposures at time t to the risk factors (to estimate via the Kalman filter), F_t the vector of factors monthly returns at time t. η_t and ϵ_t are assumed to be normally distributed with a variance assumed to be constant over time (to estimate). The hedge fund clone monthly return is:

$$r_t^{CL} = \sum_{k=1}^{K} \hat{\beta}_{k,t|t-1}\ F_{k,t} + (1 - \sum_{k=1}^{K} \hat{\beta}_{k,t|t-1})\ r_t^f \quad (0.5)$$

where $\hat{\beta}_{k,t|t-1} = \mathbb{E}_{t-1}[\beta_{k,t}]$ is the estimation of $\beta_{k,t}$ via the Kalman filter algorithm. The substantial decrease between in-sample (see Table 2) and out-of-sample (see Table 3) adjusted R-squared for all the strategies suggests that the actual replication power of the clones falls down sharply when taken out of the calibration sample. For example the Event Driven clones have an outof-sample adjusted R-squared below 50% whereas the Event Driven hedge fund strategy has a corresponding in-sample adjusted R-squared of 63%. The Equity Market Neutral clones have negative adjusted R-squared whereas the Equity Market Neutral hedge fund strategy has a corresponding in-sample adjusted R-squared of 16%. The CTA Global rolling-window clone has also a negative out-of-sample adjusted R-squared corroborating the lack of robustness of the clones.

To get a better sense of what the out-of-sample replication quality actually is, we compute the annualised root mean squared error (RMSE, see Table 3) which can be interpreted as the out-of-sample tracking error of the clone with respect to the corresponding hedge fund strategy. Our results suggest that the use of Kalman filter techniques does not systematically improve the quality of replication with respect to simple rolling-window approach: Kalman filter clones for the Distressed Securities, Emerging Markets, Event Driven, Global Macro, Short Selling and Fund of Funds have root mean squared errors greater than the root mean squared errors of their rolling-window counterparts. Overall, strategies such as CTA Global or Short Selling have clones with the poorest replication quality with root mean squared errors higher than 7.5%. Overall, these results do not support the belief that hedge fund returns can be satisfactorily replicated in a passive manner.

From Hedge Fund Replication to Hedge Fund Substitution

In this section we revisit the problem from a different perspective. Our focus is to move away from hedge fund replication, which anyway is not necessarily a meaningful goal for investors, and analyse whether naively diversified strategies based on systematic exposure to the same alternative risk factors perform better from a risk-adjusted perspective than the corresponding hedge fund clones. Since the same proxies for underlying alternative factor premia will be used in the clones and the diversified portfolios, we can perform a fair comparison in terms of risk-adjusted performance in spite of the presence of performance biases in both hedge fund return and factor proxies. We apply two popular robust heuristic portfolio construction methodologies, namely Equally-Weighted and Equal Risk Contribution, for each hedge fund strategy relative to its bespoke subset

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of economically identified risk factors for the period January 1999-October 2015. We use 24-month rolling windows to estimate the covariance matrix for the Equal Risk Contribution weighting scheme. We then compare the risk-adjusted performance of rolling-window and Kalman filter clones and the corresponding diversified portfolios of the same selected factors in terms of their Sharpe ratios. The first two rows of Table 4 show the Sharpe ratios of the rolling-window and Kalman filter clones and the last two rows show the Sharpe ratios of the corresponding Equal Risk Contribution and Equally-Weighted diversified portfolios. The clones for Distressed Securities, Event Driven, Global Macro, Relative Value and Fund of Funds have been built with the same 6 risk factors: Equity, Bond, Credit, Emerging Market, Multi-Class Value and Multi-Class Momentum. The corresponding Equal Risk Contribution and Equally-Weighted portfolios have respective Sharpe ratios of 0.74 and 0.63, which is higher than all of the previous clones' Sharpe ratios (see for example the Global Macro and Distressed Securities Kalman filter clones with respective Sharpe ratios of 0.53 and 0.17). Similarly, the Equity Market Neutral, Merger Arbitrage, Long Short Equity and Short Selling clones have been built with the same 6 risk factors: Equity, Equity Defensive, Equity Size, Equity Quality, Equity Value and Equity Momentum. All the clones' Sharpe ratios are lower (see for example the Equity Market Neutral Kalman filter clone with Sharpe ratio of 0.74) than those of the corresponding Equal Risk Contribution and Equally-Weighted portfolios (respectively 1.02 and 0.96), and sometimes substantially lower (see for example the Merger Arbitrage and Long/Short Equity Kalman filter clones with respective Sharpe ratios of 0.39 and 0.26).

Efficient Harvesting of Alternative Risk Premia

While the replication of hedge fund factor exposures appears to be a very attractive concept from a conceptual standpoint, our analysis confirms the previously documented intrinsic difficulty in achieving satisfactory out-of-sample replication power, regardless of the set of factors and the methodologies used. Our results also suggest that risk parity strategies applied to alternative risk factors could be a better alternative than hedge fund replication for harvesting alternative risk premia in an efficient way. In the end, the relevant question may not be "Is it feasible to design accurate hedge fund clones with similar returns and lower fees?", for which the answer appears to be a clear negative, but instead "Can suitably designed mechanical trading strategies in a number of investable factors provide a cost-efficient way for investors to harvest traditional but also alternative beta exposures?". With respect to the second question, there are reasons to believe that such low-cost alternatives to hedge funds may prove a fruitful area of investigation for asset managers and asset owners.

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Academic research (see Ang (2014) for a synthetic overview) has highlighted that risk and allocation decisions could be best expressed in terms of rewarded risk factors, as opposed to standard asset class decompositions, which can be somewhat arbitrary. For example, convertible bond returns are subject to equity risk, volatility risk, interest rate risk and credit risk. As a consequence, analysing the optimal allocation to such hybrid securities as part of a broad bond portfolio is not likely to lead to particularly useful insights. Conversely, a seemingly well-diversified allocation to many asset classes that essentially load on the same risk factor (e.g., equity risk) can eventually generate a portfolio with very concentrated risk exposure. More generally, given that security and asset class returns can be explained by their exposure to pervasive systematic risk factors, looking through the asset class decomposition level to focus on the underlying factor decomposition level appears to be a perfectly legitimate approach, which is also supported by standard asset pricing models relying on equilibrium arguments (the Intertemporal CAPM from Merton (1973)) or arbitrage arguments (the Arbitrage Pricing Theory from Ross (1976)).

In a recent paper, Martellini and Milhau (2015) provide further justification for the factor investing paradigm by formally showing that the most meaningful way for grouping individual securities is by forming replicating portfolios for asset pricing factors that can collectively be regarded as linear proxies for the unobservable stochastic discount factor, as opposed to forming arbitrary asset class indices. Building on this insight and a number of associated formal statistical tests, they provide a detailed empirical analysis of the relative efficiency of various forms of implementation of the factor investing paradigm and analyse the robustness of these findings with respect to a number of implementation choices, including the use of long-only versus long-short factor indices, the use of cap-weighted versus optimised factor indices, and the use of multi-asset factor indices versus asset class factor indices.

From a practical perspective, two main benefits can be expected from shifting to a representation expressed in terms of risk factors, as opposed to asset classes. On the one hand, allocating to risk factors may provide a cheaper, as well as more liquid and transparent, access to underlying sources of returns in markets where the value added by existing active investment vehicles has been put in question. For example, Ang, Goetzmann, and Schaefer (2009) argue in favour of replicating mutual fund returns with suitably designed portfolios of factor exposures such as the value, small cap and momentum factors. In the same vein, Hasanhodzic and Lo (2007) argue in favour of the passive replication of hedge fund vehicles, even though Amenc et al. (2008, 2010) found that the ability of linear factor models to replicate hedge fund performance is modest at best. On the other hand, allocating to risk factors should provide a better risk management mechanism, in that it allows investors to achieve an ex-ante control of the factor exposure of their portfolios, as opposed to merely relying on ex-post measures of such exposures.

Given the increasing interest in risk premia harvesting, and the desire to enhance the diversification of their portfolio, large sophisticated asset owners investors are turning their attention

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to so-called alternative risk premia, loosely defined as risk premia that can be earned above and beyond the reward obtained from standard long-only stock and bond exposure (see Section 2 for a tentative taxonomy of alternative risk premia). These alternative risk factors are empirically documented sources of return that can be systematically harvested typically through dynamic long/short strategies, which have been found to have explanatory power for some hedge fund strategies (see for example Fung and Hsieh (1997a,b, 2002, 2004, 2007) or Agarwal and Naik (2004, 2005)).

More precisely this paper aims at analysing what the best possible approach would be for harvesting alternative risk premia. To answer this question, we empirically analyse whether systematic rule-based strategies based on investable versions of alternative (and traditional) factors allow for the satisfactory in-sample and also out-of-sample replication of hedge fund performance, or whether it is instead the case that properly harvesting alternative risk premia, which are more complex to extract and trade compared to traditional risk premia, requires active managers' skills.

As such, our project is related to the stream of research on hedge fund replication (see Hasanhodzic and Lo (2007), Amenc et al. (2008, 2010), among many others), which we extend in the following two main directions. In a first step, in contrast to some of the previous research that has analysed the replication of global hedge fund indices, which are often dominated by long/short equity strategies that are arguably the easiest to replicate, our focus will be on replicating hedge fund strategy indices (see Asness et al. (2015) for a recent reference). It is in fact one of the goals of the research project to identify which strategies are easiest/hardest to replicate using alternative risk premia and possibly conditional models that may capture changes in hedge fund exposures by exploiting information from relatively high frequency conditioning variables (see Kazemi et al. (2008) for an analysis of conditional properties of hedge fund return distributions). Finally, we consider the possible improvement allowed for by the introduction of a specific set of factors for each strategy, as opposed to using a single set of systematic factors for all funds. Given the concern over data mining that would arise from a statistical search of the best factors, we have constrained ourselves to a purely economic selection of factors. In a second step, we shift the perspective from hedge fund replication to hedge fund substitution, and investigate whether suitably designed risk allocation strategies may provide a cost-efficient way for investors to get an attractive exposure to alternative factors, regardless of whether or not they can be regarded as proxies for any particular hedge fund strategy.

The rest of the paper is organised as follows. In Section 2, we first attempt to provide a definition for the rather loosely defined alternative risk factors as well as a list of the main alternative risk factors that have been analysed in the academic and practitioner literature. In Section 3, we analyse the explanatory power of various statistical model that can be used for the replication of hedge fund returns with (traditional and) alternative risk factors. In Section 4, we extend the analysis to the construction of investment strategies with attractive risk-adjusted performance based on these alternative risk premia. We present our conclusions

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and suggestions for further research in Section 5, while technical details are relegated to a dedicated appendix.

2. Taxonomy of Alternative Risk Premia

2. Taxonomy of Alternative Risk Premia

There is no well-accepted definition of *alternative* risk premia. These are in fact best defined by contrast to the so-called traditional risk premia, which essentially relate to long-term rewards earned from a long-only exposure to stocks and bonds. In other words, any factor premium, or documented anomaly, that is different from the long-only equity and bond risk premia can be regarded as an alternative factor premium. If the definition of alternative risk factors involves no a priori restrictions, we only consider in this paper alternative factors that have been documented to exhibit significant and persistent premia justified by academic research and economic intuition. Besides, we focus on those risk factors that can be harvested with relatively liquid instruments. These restrictions are most easily met in the equity universe, where the accepted list of alternative risk factors encompasses the standard long-short Fama and French (1992) value and size factors, but also the momentum factor (Carhart (1997)), the low volatility factor (Ang, Hodrick, Xing, and Zhang (2006, 2009)), as well as other factors such as the quality factors (Asness, Frazzini, and Pedersen (2013)) or liquidity factors (Idzorek et al. (2012)), among others. Overall and given that we wish to set the analysis in a multi-asset context that includes stocks, bonds but also commodities, credit and currencies, we choose to focus mainly on the following four risk factors (see Asness et al. (2015) for a similar choice of factors): value, momentum, carry and low risk. In what follows, we provide an overview of these risk factors, including a definition and a discussion of how to apply the definition in a multi-asset context.

2.1 The Value Risk Factor

The value risk factor is defined as a long exposure to assets that are "cheap" and a short exposure to those that are "expensive" according to a valuation measure. Common measures are the book-to-market ratio (Fama and French (1992, 1993)) and earnings yields for equities but more general measures applicable to other asset classes have been used in recent studies such as the past-five year return (Israel and Moskowitz (2013), Asness, Moskowitz, and Pedersen (2013)), which could perhaps be instead regarded as a return reversal factor. Asness et al. (2015) use the following value measures: real yield for bonds (10-year government yield minus consensus inflation forecast), purchasing power parity for currencies and the five-year reversal in price for commodities.

Value investing is an investment paradigm initially developed in equities and taught by Graham and Dodd (1934) at Columbia Business School in the 1920s. Since then numerous academics and practitioners published documents putting forward that value stocks outperform growth stocks on average in the United States (Graham and Dodd (1934), Basu (1977), Stattman (1980), Rosenberg et al. (1985), Fama and French (1992)) and around the world (Chan et al. (1991), Fama and French (1998), Liew and Vassalou (2000), Malkiel and Jun (2009)). Asness, Moskowitz, and Pedersen (2013) confirmed these results in different markets (US, UK, Europe and Japan) and asset classes (stocks, currencies, bonds, and commodities). Finally, Malkiel and Jun (2009) empirically showed the value effect in Chinese stocks with a nonparametric method of portfolio construction. The existence and persistence of the value effect has been empirically verified for many different markets and time periods, reducing the likelihood of a statistical fluke.

The economic explanation of these findings is a central question in academic finance.

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The value premium may be a compensation for forms of systematic risk other than market risk (Fama and French (1992)), such as recession risk (Jagannathan and Wang (1996)), cash-flow risk (Campbell and Vuolteenaho (2004); Campbell et al. (2010)), long-run consumption risk (Hansen et al. (2008)), or the costly reversibility of physical capital and countercyclical risk premia (Zhang (2005)). The underperformance of growth stocks relative to value stocks may also be evidence of the suboptimal behaviour of the typical investor (Lakonishok et al. (1994)), Daniel et al. (2001)). As another behavioural explanation to the value premium, Barberis and Huang (2001) give loss aversion and narrow framing as explanation of the cross-sectional return of value stocks.

Asness, Moskowitz, and Pedersen (2013) showed in their paper that the value risk factor can be extended to other asset classes. They define for each asset class a robust measure of cheapness of an asset relative to its asset class. The key idea is not to find the best predictors of returns for each asset class but rather to define a global and consistent approach of value across asset classes.

- For individual stocks, they choose the common ratio of the book value of equity to market value of equity. Book values are lagged of six months for data availability and market values are the most recent available.
- For country equity index futures, they choose the previous month ratio of the book value of equity to market value of equity for the MSCI index of the country considered.
- For government bonds, the measure is the 5-year change in the yields of 10-year bonds.
- For currencies, the measure is the 5-year change in purchasing power parity.
- For commodities futures, they define the negative of the spot return over the last 5 years.

For a security i within an asset class A made of n assets we denote $S_{i,t}$ the corresponding value measure at time t for the security i. We can write the value factor for the asset class A as follows:

$$r_t^A = c_t \sum_{i=1}^{n_t} \left[\left(\operatorname{rank}(S_{i,t}) - \frac{1}{n_t} \sum_{j=1}^{n_t} \operatorname{rank}(S_{j,t}) \right) r_{i,t} \right] \tag{2.1}$$

where $r_{i,t}$ is the monthly return of security i at time t and c_t a scaling factor for constraining the portfolio to be one dollar long and one dollar short.[1]

1 - For a given asset class A and $\alpha \in [\![1; n_t]\!]$, $\operatorname{rank}(S_{\alpha,t}) = n_t \iff \forall j \in [\![1; n_t]\!], S_{j,t} \leq S_{\alpha,t}$.

Then the authors construct a global average across eight global asset classes (country equity index futures, commodity futures, government bonds, currencies, US stocks, UK stocks, Europe stocks and Japan stocks). They define the global multi asset class value factor $VAL^{everywhere}$ as the inverse-volatility-weighted-across-asset-class value factor.

2.2 The Momentum Risk Factor

The momentum risk factor is designed to buy assets that performed well and sell assets that performed poorly over a certain historical time period. The premise of this investment style is that asset returns exhibit positive serial correlations. A common measure for all asset classes is the twelve-month cumulative total return (Jegadeesh and Titman (1993)). Some authors consider this measure omitting the last month for equities (Asness, Moskowitz, and Pedersen (2013)). The existence of such an effect first mentioned by Levy (1967) contradicts the hypothesis of efficient markets which states that past price returns alone cannot predict

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future performance. Jegadeesh and Titman (1993) in their pioneering work discovered the momentum effect anomaly in the US equity markets in their 1965-1989 sampling period. Fama and French (1996) underline in their academic paper that the "main embarrassment" of the three-factor model is its failure to capture the perpetuation of short-term momentum anomalies. Indeed, the first panel in Table VII of their paper shows that in the three-factor regressions, the intercepts are strongly negative for short-term-losers and strongly positive for short-term winners, which suggests the presence of an effect that is not explained by the three-factor model. In a later paper Carhart (1997) created a factor mimicking portfolio for the momentum effect like Fama and French (1996) did for the size and value factor. The three-factor model can be extended with the momentum factor resulting in the four-factor model for the expected excess return on a security.

Since Jegadeesh and Titman (1993) first reported momentum profits in the US equity markets, their findings have been corroborated and extended in a number of studies. For example Asness, Moskowitz, and Pedersen (2013) pointed out the momentum factor in international equities, government bonds, currencies and commodities (see also Menkhoff et al. (2012) for a focus on currencies). Daniel et al. (1998), Barberis et al. (1998), and Hong and Stein (1999) developed behavioural models to explain momentum profits. In contrast, some authors assess that the momentum factor can be at least partially explained by correlation with macro factor risks such as liquidity (Chordia and Shivakumar (2002), Pastor and Stambaugh (2003), Cooper et al. (2004)). Moskowitz et al. (2012) found "significant time series momentum" considering 58 diverse future and forward contracts across equities, bonds, currencies and commodities. Asness, Moskowitz, and Pedersen (2013) established the existence of a momentum risk premia across the major asset classes. They unified the momentum risk factor concept by proposing a unique robust measure for all asset classes. This measure is the return over the past 12 months skipping the most recent month, which again is justified by the desire to avoid 1-month reversal effect in stock returns.

The methodology used for the global momentum factor by Asness, Moskowitz, and Pedersen (2013) is the same as the one used for the global value factor. Nonetheless it is more straightforward due to the unique measure for all the asset classes considered. For a security i within an asset class A made of n_t assets we denote at time t $r_{i,[t-12M;t-1M]}$ the return of security i over the past 12 months skipping the most recent month. We can write the momentum factor for the asset class A as follows:

$$r_t^A = c_t \sum_{i=1}^{n_t} \left[\left(\operatorname{rank}\left(r_{i,[t-12M;t-1M]}\right) - \frac{1}{n_t} \sum_{j=1}^{n_t} \operatorname{rank}(r_{j,[t-12M;t-1M]}) \right) r_{i,t} \right] \tag{2.2}$$

where $r_{i,t}$ is the monthly return of security i at time t and c_t a scaling factor for constraining the portfolio to be one dollar long and one dollar short.[2]

Then the authors constructed a global average across eight global asset classes (country equity index futures, commodity futures, government bonds, currencies, US stocks, UK stocks, Europe stocks and Japan stocks). They define the global multi asset class momentum factor $MOM^{everywhere}$ as the inverse-volatility-weighted across-asset-class momentum factor.

2 - For a given asset class A and $\alpha \in [\![1;n_t]\!]$, $\operatorname{rank}(r_{\alpha,[t-12M;t-1M]}) - n_t \longleftrightarrow \forall j \in [\![1;n_t]\!], S_{j,[t-12M;t-1M]} \leq S_{\alpha,[t-12M;t-1M]}$.

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2.3 The Carry Risk Factor

The carry risk factor is designed to take advantage of the outperformance of higher yielding assets over lower yielding assets. Carry can be defined as the asset return when the underlying price does not move. The carry factor was historically most well-known in currencies where a common indicator is the three-month onshore cash rate (Koijen et al. (2015)).

Koijen et al. (2015) report evidence that carry provides investors with robust and risk-adjusted returns in every asset classes. For instance in fixed income, carry strategies can be defined as a differential of bond yields (buy the developed market government bonds with the highest yield and sells those with lowest yield). In commodities, investors can exploit the curve slide and be long the most backwardated commodity futures (downward sloping) and short contracts that are in contango (upward sloping). Carry strategies can also be defined in the equity asset class, even though the natural measure, which is the dividend yield, makes this factor highly correlated to, and somewhat redundant with, the value factor. Reported economic explanations for the risk premia on carry strategies are crash risk of carry strategies during liquidity dry-ups (Brunnermeier and Pedersen (2009)) as well as consumption growth risk (Lustig and Verdelhan (2007)).

Carry is defined by Koijen et al. (2015) as "the expected return on an asset assuming that market conditions, including its price, stay the same". They developed a unifying concept of carry as a directly observable quantity independent of any model:

$$\text{Return} = \underbrace{\text{Carry} + \text{Expected price appreciation}}_{\text{Expected return}} + \text{Unexpected price appreciation} \tag{2.3}$$

Koijen et al. (2015) in their paper extended the notion of carry to nine asset classes by considering (if need be synthetic) futures contracts. These asset classes are: currencies, equities, global bonds, commodities, US Treasuries, credit, slope of global yield curves, call index options and put index options.

They consider at time t a future contract that expires at time $t+1$ with a current futures price F_t, a spot price S_t of the underlying security, a risk-free rate noted r_t^f and an allocation of capital of X_t to finance the position and then write the return per allocated capital over one period as follows:

$$\begin{aligned} r_{t+1}^{total\ return} &= \frac{X_t(1+r_t^f) + F_{t+1} - F_t - X_t}{X_t} \\ &= \frac{F_{t+1} - F_t}{X_t} + r_t^f \end{aligned} \tag{2.4}$$

The return in excess of the risk free rate and the carry are respectively:

$$r_{t+1} = \frac{F_{t+1} - F_t}{X_t} \text{ and } C_t = \frac{S_t - F_t}{X_t} \tag{2.5}$$

The carry factor thus defined is directly observable from current market instruments. Finally we can rewrite the excess return as defined by Koijen et al. (2015):

$$\begin{aligned} r_{t+1} &= \frac{F_{t+1} - S_t - S_t - F_t}{X_t} \\ &= \underbrace{C_t + \mathbb{E}_t\left(\frac{S_{t+1} - S_t}{X_t}\right)}_{\mathbb{E}_t[r_{t+1}]:\ \text{Expected price appreciation}} \\ &+ \underbrace{\frac{S_{t+1} - \mathbb{E}_t(S_{t+1})}{X_t}}_{u_{t+1}:\ \text{Unexpected price appreciation}} \end{aligned} \tag{2.6}$$

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Considering the all assets future-based definition of Koijen et al. (2015) we can define the carry C_t for a large set of asset classes. In addition to the five main asset classes (equities, global bonds, currencies, credit and commodities), the authors treated the cases of US Treasuries of different maturities, the slope of global yield curves and options (see Koijen et al. (2015) for a detailed definition on carry relative to these other asset classes).

- For currencies,

$$F_t = S_t \frac{(1+r_t^f)}{(1+r_t^{f*})}, \quad C_t = \frac{S_t - F_t}{F_t} = \frac{r_t^{f*} - r_t^f}{1+r_t^f},$$

where S_t is the spot exchange rate, r_t^f the local interest rate and r_t^{f*} the foreign interest rate.

- For global equities,

$$F_t = S_t(1+r_t^f) - \mathbb{E}_t^Q(D_{t+1}),$$

$$C_t = \frac{S_t - F_t}{F_t} = \left(\frac{\mathbb{E}_t^Q(D_{t+1})}{S_t} + r_t^f\right)\frac{S_t}{F_t}'$$

where S_t is the current equity value, $\mathbb{E}_t^Q(D_{t+1})$ the expected future dividend payment under the risk-neutral measure and r_t^f the risk-free rate.

- For global bonds,

$$F_t = S_t \frac{(1+r_t^f)^{1/12}}{(1+y_t^{10Y})^{10}},$$

$$C_t = \frac{S_t - F_t}{F_t} \simeq \frac{1}{12}(y_t^{10Y} - r_t^f) - D^{mod}(y_t^{9Y11M} - y_t^{10Y})$$

where y_t^{10Y} is the current annualised yield of a 10-year zero-coupon bond, D^{mod} the modified duration of the bond and r_t^f the annualised short-term interest rate.

F_t corresponds to the current value of a 10-year zero-coupon bond futures contract with one month to expiration. We note that the definition of carry for global bonds is the same as for global equities and currencies.

- For credit, the same definition as for global bonds is used with a duration adjustment.
- For commodities,

$$F_t = S_t(1 + r_t^f - \delta_t),$$

$$C_t = \frac{S_t - F_t}{F_t} \simeq \frac{F_t^1 - F_t^2}{F_t^2(T_2 - T_1)} = (\delta_t - r_t^f)\frac{S_t}{F_t^2}$$

where F_t^1 is the nearest to maturity futures contract, F_t^2 the second nearest to maturity futures contract, T_i is expressed in months and δ_t is the convenience yield.

Note that unlike the previous asset classes, carry is defined for commodities as a difference between the slope of two futures prices of different maturities. This is due to the high illiquidity of the commodity spot markets.

Given that carry can be directly interpreted as a return, a global carry factor per asset class can be built as a weighted sum of the carry factors on the nt individual securities of the asset class available at time t:

$$C_t^{global} = \sum_{i=1}^{n} w_t^i C_t^i \tag{2.7}$$

where C_t^i denotes the carry at time t of asset i.

Koijen et al. (2015) propose the following weights:

$$w_t^i = z_t\left(\text{rank}(C_t^i) - \frac{n_t+1}{2}\right) \tag{2.8}$$

where n_t is the number of available securities at time t and z_t a scaling factor.[3] Likewise Asness, Moskowitz, and Pedersen (2013) they define the global multi asset class carry factor $GCR^{everywhere}$ as the inverse-volatility-weighted across single asset class carry factor.

3 - For a given asset class A and $\alpha \in [\![1;n_t]\!]$, $\text{rank}(C_{\alpha,t}) = n_t \iff \forall j \in [\![1;n_t]\!]$, $C_{j,t} \leq C_{\alpha,t}$

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2.4 The Low-Risk (or Low-Beta) Risk Factor

The low-risk factor is designed to take advantage of the reported outperformance of low-risk assets over the high-risk assets. Empirically, the relation between stock beta and returns has been proven to be flatter than predicted by the CAPM (Black et al. (1972), Haugen and Heins (1975)). Merton (1972) empirically showed in a number of different equity markets and extended time periods that stocks with low beta significantly outperformed high-beta stocks. Fama and French (1992) found in their study that beta did not explain significantly average returns. In a related effort, Ang, Hodrick, Xing, and Zhang (2006, 2009)) find that low idiosyncratic volatility stocks tend to outperform high idiosyncratic volatility stocks.

Building upon this body of evidence, Frazzini and Pedersen (2014) proposed to build a low-beta factor in several asset classes: they first consider the traditional definition of beta to define the low-beta risk factor for equities and then extend it to other asset classes. From an economic perspective, poor long-run performance of high-risk assets compared to low-risk assets may be due to leverage constraints (Black et al. (1972), Frazzini and Pedersen (2014)) or lottery preferences (Bali et al. (2011)).

Frazzini and Pedersen (2014) propose a model where investors are constrained by liquidity. They can invest in leveraged positions but have to sell these positions in bad times when the leverage is no longer sustainable. They extend the equity beta definition to several asset classes: equity indices, country bonds, currencies, US Treasury bonds, credit indices, credit and commodities. They use the following methodology to define their low-risk factor *BAB*: for a given asset class they first estimate the pre-ranking betas of its securities from rolling regressions of excess returns on market excess returns. Excess returns are above the US Treasury Bill rate.

For an asset class A composed of n_t securities at time t they estimate the beta of a security *i* as

$$\hat{\beta}_{i,t}^{ts} = \hat{\rho}_t \frac{\hat{\sigma}_{i,t}}{\hat{\sigma}_{m,t}} \qquad (2.9)$$

where $\hat{\sigma}_{i,t}$ and $\hat{\sigma}_{m,t}$ are the estimated volatilities at time t for the security i and the market and $\hat{\rho}_t$ the estimated correlation between the security and the market portfolio at time *t*. A 1-year rolling-window standard deviation is used for estimating volatilities and a 5-year time frame for estimating the correlation. Daily returns are preferred to monthly returns for estimations if available.

The market portfolio against which the pre-ranking betas are computed depends on the asset class considered:

- For US equities, the market portfolio is the CRSP value-weighted market index.
- For international equities, the market portfolio is the corresponding MSCI local market
index.
- For US Treasury bonds, the market portfolio is an aggregate Treasury Bond index.
- For equity indexes, country bonds and currencies, the market portfolio is a GDP-weighted
portfolio.
- For credit, the market portfolio is an equally-weighted portfolio of all the bonds in the database.
- For commodities, the market portfolio is an equal risk weight portfolio across commodities.

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Then a time series estimate of beta shrinkage for a given security *i* belonging to an asset class *A* is done as follows:

$$\hat{\beta}_{i,t} = 0.6 \times \hat{\beta}^{ts}_{i,t} + 0.4 \qquad (2.10)$$

They consider for a given asset class *A* composed by n_t securities at time *t* the $n_t \times 1$ vector $\hat{\beta}_t = (\hat{\beta}_{i,t})_{1 \leq i \leq n_t}$ and assign each security to the low-beta corresponding asset class portfolio if the security's beta is inferior to the asset class median or high-beta corresponding asset class portfolio if the security's beta is superior to the asset class median. Then they define the portfolio weights of the low-beta and high-beta portfolios relative to the n_t securities universe at time *t* as:

$$w_t^H = k_t(z_t - \overline{z}_t)^+$$

and

$$w_t^L = k_t(z_t - \overline{z}_t)^- \qquad (2.11)$$

where

$$z_t = (z_{i,t})_{1 \leq i \leq n_t} = (\text{rank}(\hat{\beta}_{i,t}))_{1 \leq i \leq n_t},$$

$$\bar{z}_t = \frac{1}{n_t}\sum_{i=1}^{n_t} z_{i,t}, \; k_t = \frac{2}{\sum_{i=1}^{n_t} |z_{i,t} - \overline{z}_t|}$$

is a normalising constant, $()^+$ and $()^-$ indicate respectively the positive and the negative elements of a vector.

The returns and ex-ante betas of low-beta and high-beta portfolios verify:

$$r_{t+1}^L =^t r_{t+1} w_t^L,$$

$$r_{t+1}^H =^t r_{t+1} w_t^H,$$

$$\hat{\beta}_t^L =^t \hat{\beta}_t w_t^L$$

and

$$\hat{\beta}_t^H =^t \hat{\beta}_t w_t^H \qquad (2.12)$$

Finally they define a global asset class betting against beta factor (*BAB*) which is market-neutral and goes long low-beta securities and short high-beta securities as:

$$r_{t+1}^{BAB} = \frac{1}{\hat{\beta}_t^L}(r_{t+1}^L - r_t^f) - \frac{1}{\hat{\beta}_t^H}(r_{t+1}^H - r_t^f) \qquad (2.13)$$

They proved that under the introduction of a funding constraint ψ_t, the expected excess return of this factor is positive and increasing in the funding tightness ψ_t and the ex-ante beta spread:

$$\mathbb{E}_t(r_{t+1}^{BAB}) = \frac{\hat{\beta}_t^H - \hat{\beta}_t^L}{\hat{\beta}_t^H \hat{\beta}_t^L} \geq 0 \qquad (2.14)$$

Frazzini and Pedersen (2014) also define a multi-asset global *BAB* factor (see Table 8 of their paper) by considering a portfolio with an equal risk contribution in each asset class global *BAB* factor with a 10% ex-ante volatility.

2.5 Other Factors

We now discuss two other factors that are often used in factor investing strategies, namely the size and quality factors, but which are only relevant for the equity asset class.

2.5.1 The Size Factor

The size factor is designed from a long portfolio of small cap stocks and a short portfolio of large cap stocks. The size effect was first discovered by Banz (1981) who studied stocks on the NYSE using a linear regression model, and found that very small stocks outperform medium and large stocks. A few years later Keim (1983) extended Banz's work by looking at a broader universe of stocks and showed that the "January effect" explains a significant part of the

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size effect. Fama and French (1992, 2012) test the size effect on both the U.S. and European market in several papers using cross-sectional regressions of stock returns. They found that the effect holds for both markets, but a large part of the size premium comes from micro cap stocks that are often non-investable. The robustness of the size effect has been questioned in the academic literature: several academics argued that the size effect did not persist after the mid-1980s (Eleswarapu and Reinganum (1993), Chan et al. (2000), Hirshleifer (2001)). Van Dijk (2011) provides an overview of empirical studies on the size premium on different markets and periods. His study shows the existence of the size effect in many markets (developed and emerging) and many time periods but does not explain the decline in the size premium after the mid-1980s. The standard explanations of the size effect are the following: a premium for illiquidity risk (Amihud (2002)), or a premium for idiosyncratic risk (Fu (2009), Hou and Moskowitz (2005)).

2.5.2 The Quality Factor

The quality factor is designed to invest in stocks with strong quality characteristics like low debt, stable earnings growth, management credibility. Sloan (1996) and Piotroski (2000) argue that quality stocks explain a significant part of the value strategy. Piotroski (2000) considers a three-dimensional score encompassing measures of profitability, leverage/liquidity/source of funds and operating efficiency. Asness, Moskowitz, and Pedersen (2013) built the quality-minus-junk factor (*QMJ*) by going long high quality stocks and short low quality stocks according to a four-dimension composite score. Novy-Marx (2013) also find that profitable stocks outperform unprofitable stocks. He proposed a quality score based on the ratio of a firm's gross profits to its assets (*GPA*) and assessed that: "Quality can even be viewed as an alternative implementation of value - buying high quality assets without paying premium prices is just as much value investing as buying average quality assets at a discount". In the same spirit, Fama and French (2014) extended their seminal three-factor model by adding two quality factors: investment and profitability. They showed that the value factor (*HML*) is then redundant for explaining cross-sectional differences in average returns.

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Hedge funds are by nature challenging to understand because of the diversity, opacity and complexity of their dynamic strategies with the possible use of leverage, short selling and derivatives in several asset classes. Traditionally hedge funds were thought to derive their returns from manager superior skills and capacity to take advantage of market inefficiencies (alpha). But the huge losses of quant funds (the quant meltdown) in the first week of August 2007, the poor performance of the hedge fund industry during the subprime crisis and the growing correlation between equities and hedge funds returns questioned the ability of managers to generate absolute return strategies and put the light on hedge funds systematic risk exposures (betas). Broadly speaking, hedge fund returns can be decomposed into alpha (manager's skill to exploit market inefficiencies), exposure to traditional risk factors (traditional betas) and exposure to alternative risk factors (alternative betas). While traditional betas and alternative betas both are the result of exposure to systematic risks in global capital markets, the factor exposures in hedge fund returns can be significantly more complex to analyse and track than the factor exposures in mutual fund returns. In what follows, we provide a brief review of the related academic literature.

3.1 Litterature Review

Three main approaches to hedge fund return replication have been analysed in the academic literature: (1) the mechanical duplication approach (used by Mitchell and Pulvino (2001) for merger arbitrage strategies, Fung and Hsieh (2002) for trend-following strategies and Agarwal and Naik (2005) for convertible arbitrage strategies), (2) the payoff distribution approach (Kat and Palaro (2005)) and (3) the factor replication approach (see for example Fung and Hsieh (1997b, 2001), Hasanhodzic and Lo (2007), Amenc et al. (2008, 2010), Asness et al. (2015)). In what follows, we only consider the third approach since our focus is precisely on characterising hedge fund exposure to alternative risk factors (see Table 5 for a synthetic overview of the literature).

3.1.1 Replication with Static Factor Models

The simplest approach to factor-based hedge fund replication is by using a linear regression to explain hedge fund index returns in terms of the return on a number of selected factors. Factors can be selected statistically on the basis of their explanatory power, or economically on the basis of their expected impact on a given hedge fund strategy. Each month, the regression analysis is performed over a fixed or rolling time frame, and weights for each of factor are selected. This backward-looking method has been used by Fung and Hsieh (1997a), Agarwal and Naik (2004), Hasanhodzic and Lo (2007) or Amenc et al. (2008, 2010) who found mixed in-sample results depending on the hedge fund strategy, and relatively disappointing out-of-sample results.

In the case of a fixed time frame, the hedge fund excess returns are modelled as follows:

$$r_t^{HF} - r_t^f = \sum_{k=1}^{K} \beta_k F_{k,t} + \epsilon_t \quad (3.1)$$

The hedge fund clone returns are:

$$r_t^{CL} = r_t^f + \sum_{k=1}^{K} \hat{\beta}_k F_{k,t} \quad (3.2)$$

where the exposure $\hat{\beta}_k$ on each factor $F_{k,t}$ is estimated via a standard OLS analysis.

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In his seminal analysis of US mutual fund returns, Sharpe (1992) uses a 12 asset class factor linear model, using a constrained OLS analysis where the fund exposures are forced to sum up to one, to explain the performance of a subset of open-end US mutual funds between 1985 and 1989. He empirically shows that it was possible to replicate the performance of a broad universe of US mutual funds only with a small number of asset classes. Fung and Hsieh (1997b) extend the analysis on a broader sample of US mutual funds with a 8 asset class factor model. They perform multilinear regressions against eight different asset class indices: US equities (MSCI), non-US equities (MSCI), emerging market equities (IFC), US government bonds (JP Morgan), non-US government bonds (JP Morgan), one-month Eurodollar deposit rate, gold, and the trade-weighted value of the US dollar (Fed). The results confirm the original Sharpe's work: on a sample of 3327 US mutual funds, they found that 75% of the mutual funds have R-squared above 0.75 and 92% above 0.50 according to their linear regression. When applying the same methodology to a subset of 409 hedge funds, they obtain disappointing results (48% of the hedge funds have R-squared below 0.25), confirming that hedge fund strategies are highly dynamic and could generate an option-like, nonlinear exposure to traditional underlying risk factors.

Agarwal and Naik (2004) consider a multifactor model with asset class factors (equities, bonds, currencies, credit and commodities), with the Fama-French and Carhart factors (size, value and momentum) and option-based factors in order to enhance the explanatory power of hedge fund return. They use a linear regression for eight hedge fund strategies according to these factors. They find that hedge funds have significant exposure to Fama-French and Carhart factors as well as the option-based factors. Fung and Hsieh (2004) propose a seven asset-based style factor model (two equity factors, two interest rate factors and three option factors) that explains up to 80% of monthly return variation in hedge funds. They emphasise time-varying beta loadings depending on the state of the market. The authors showed that the rolling-window model is more appropriate for replication than the fixed weight model. Diez de los Rios and Garcia (2007) consider options on an equity index as part of the factors and find statistical evidence of nonlinearities in some but not all hedge fund strategy returns. Jaeger and Wagner (2005), Hasanhodzic and Lo (2007) show that portfolios made up of common asset class risk factors can provide correct in-sample performance depending on hedge fund categories and have the benefit of being transparent and easily traded through liquid instruments. The six asset class factors used by Hasanhodzic and Lo (2007) are the following: US dollar, S&P 500 total return, spread between the Lehman Corporate Bond BAA Index and the Lehman Treasury Index, Lehman Corporate AA Intermediate Bond Index returns, Goldman Sachs Commodity Index total return and first difference of the end-of-month value of VIX index. These factors are used to run a constrained regression on hedge funds in each fund category to obtain portfolio weights of the risk factors in the clones. They obtain estimates with two methods: fixed-weight and rolling-window clones. More recently, Asness et al. (2015) use a multi-asset style factor linear regression on a subset of hedge fund indices for the

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1990-2013 period, where the factors are market, market lagged, value, momentum, carry and defensive. Overall, it appears that the non-linear and dynamic exposure of hedge fund returns must be taken into account in any attempt to improve out-of-sample performance of hedge fund replication models.

3.1.2 Replication with Dynamic Factor Models

The first dynamic approaches in the literature (Fung and Hsieh (2004), Hasanhodzic and Lo (2007)) are based on rolling-window OLS analysis. Improvements have been suggested through the use of a linear state-space model approach and its resolution with the Kalman filter algorithm. Amenc et al. (2010) examine new models designed to capture non-linearity and dynamics which characterise hedge fund strategies. They estimate hedge fund indices returns with a classic unconditional linear model, with an unconditional option-based model, with a conditional Markov regime-switching model (MRS) and a conditional Kalman filter model. In the first part of their study, they consider Hasanhodzic and Lo's factors except for the volatility factor. For the linear and the option-based models, the authors use a 24-month rolling-window and for the conditional models the calibration windows are readjusted as soon as new information arrives. According to their 7 years of out-of-sample empirical study, they conclude that even if conditional approaches gave better results from an in-sample perspective these conditional approaches deliver little, if any, improvement in replication performance over static linear clones. Otherwise the option-based model replication performance is poorer than the linear model performance.

On the second part of their study they emphasise on the importance of the factors used in the model relative to the quality of the replication process and found an improvement in the out-ofsample replication quality when factors are selected on an economic analysis basis. They adopt a bespoke factor selection for each hedge fund strategy based on economic criteria and then apply the four discussed models according to these factors. They obtain better out-of-sample results for more than half of the strategies from a tracking error perspective but mixed out-of-sample results from a performance perspective. Roncalli and Teiletche (2008) compare the rolling-window regression and the Kalman filter models in the framework of the hedge replication according to six underlying exposures. They conclude that the Kalman filter is a more efficient econometric method. Roncalli and Weisang (2009) applied more sophisticated Bayesian filters algorithms (particle filters with different numerical algorithms) in order to better take into consideration the non Gaussian aspect of hedge fund returns. They found that the matching of the hedge fund returns higher moments is done at the expense of a higher volatility of the tracking error of the clone. They confirmed the results of Diez de los Rios and Garcia (2007) by finding no clear evidence of nonlinearities relatively to hedge fund returns.

3.2 Data and Experimental Protocol

In what follows we describe the methodological protocol for our empirical analysis, after describing the data that we use.

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3.2.1 Data on Hedge Fund Returns

Benchmarking hedge fund performance is particularly challenging because of the presence of numerous biases, the most important of which are the sample selection bias, the survivorship bias and the backfill bias. Hedge funds also differ widely in terms of strategy and it is required to segregate them into a range of investment styles that can form a relatively homogenous set. A widely used database amongst the different academic studies concerning hedge funds is the Lipper TASS database. This database is composed of 11 categories. Amongst them are four non-directional strategies (Convertible Arbitrage, Equity Market Neutral, Event Driven, Fixed Income Arbitrage), five directional strategies (Global Macro, Long/Short Equity Hedge, Emerging Markets, Managed Futures, Dedicated Short Bias) and two categories which cannot be classified in directional or non-directional strategy (Multi-Strategy, Funds of Funds).

Amenc and Martellini (2003) compiled indices of indices (the so-called EDHEC hedge fund indices) in order to improve the hedge fund indices' lack of representativeness and to mitigate the bias inherent to each database. EDHEC-Risk Institute considers the following 13 categories which are coherent with the Lipper TASS database classification: Convertible Arbitrage, CTA Global, Distressed Securities, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Global Macro, Long/Short Equity, Merger Arbitrage, Relative Value, Short Selling and Fund of Funds. Each category has an index with data starting at January 1997. We will use the EDHEC indices in our empirical analysis.

Tables 6 and 7 present the descriptive statistics for each hedge fund strategy. Table 6 shows that apart from the CTA Global strategy, hedge fund strategies have high pairwise correlations: the absolute value of average pairwise correlations is 60%. The range of volatilities is rather wide, from 2.8% for Equity Market Neutral and 15.6% for Short Selling. Non directional strategies like Equity Market Neutral, Fixed Income Arbitrage, Merger Arbitrage and Relative Value display the lowest volatilities (respectively 2.8%, 3.8%, 3.1% and 4.2%). Sharpe ratios are all above 0.60 except for CTA Global (0.40) and Short Selling (-0.27). Merger Arbitrage is the strategy with the highest Sharpe ratio (1.43).

3.2.2 Data on Factor Returns

The first step consists in defining a set of relevant risk factors and then to find suitable proxies. An overview of the 19 traditional and alternative risk factors considered in our empirical analysis is given in Table 1. We proxy traditional risk factors by total returns of liquid and investable equity, bond, commodity and currency indices. For alternative risk factors, we inter alia consider long/short proxies for value, momentum and low beta, for various asset classes, using data from (Asness, Moskowitz, and Pedersen (2013) and Frazzini and Pedersen (2014)). Data for the carry risk factor were not available, so we decide to not consider it. All alternative factors considered (single and multi asset class) are global. We consider the following equity asset class alternative factors: value (Asness, Moskowitz, and Pedersen (2013)), momentum (Asness, Moskowitz, and Pedersen (2013)), size (Frazzini and Pedersen (2014)), defensive (Frazzini and Pedersen (2014)) and quality (Asness, Frazzini, and Pedersen (2013)). A key difference between the traditional

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and alternative factors is that the latter cannot be regarded as directly investable, which implies that reported performance levels are likely to be overstated. Given the presence of performance biases in both hedge fund returns and alternative factor returns, we shall not focus on differences in average performance between hedge fund indices and their replicating portfolios, and instead focus on the quality of replication measured by in-sample, out-of-sample adjusted R-squared and the out-of-sample root mean square error (RMSE). It is only in Section 4, where we compare different portfolios of alternative factors that a comparison in terms of performance becomes meaningful since the same biases will be found in all competing approaches. Tables 8 and 9 present the descriptive statistics for each traditional and alternative factors considered. The absolute value of factors' average pairwise correlations is 19% and suggest a low level of dependency amongst the factors. The range of volatilities vary from 2.4% for Fixed Income Global Value to 23.5% for Commodity and the Sharpe ratios vary from -0.64 for Fixed Income Global Momentum to 0.83 for Equity Global Defensive.

3.2.3 Experimental Protocol

Jaeger and Wagner (2005) and Amenc et al. (2010) adopt a bespoke factor selection based on economic rationale for each hedge fund strategy. They show that factor selection based on economic criteria combined with a classic linear regression approach leads to improved in-sample explanation and out-of-sample replication quality. We propose in our study to adopt a bottom-up approach according to a list of traditional and alternative risk factors defined above. We suggest to proceed in two steps: a first step where we consider an in-sample explanatory analysis approach with a basic static model (a linear regression on a fixed time window) distinguishing three cases for the subset of risk factors, and a second step where we consider an out-of-sample predictive analysis approach with two distinct approaches: a rolling-window linear regression and a Kalman filter.

In the following we perform linear regressions with no intercept and one should notice there is no consensus about the definition and interpretation (see Eisenhauer (2003) and Wooldridge (2012)) of the adjusted R-squared in this particular case (see Appendix A for more details). In our empirical study, we adopt the definition of Wooldridge (2012):

$$R^2_{adj} = 1 - \frac{\widehat{\sigma^2}}{\widehat{\sigma^2_{r^{HF}}}} \tag{3.3}$$

where the numerator $\widehat{\sigma^2} = \frac{\|\hat{\epsilon}\|^2}{T-K}$ is the unbiased estimation of the residual variance with $\|\hat{\epsilon}\|$ being the euclidian norm of the estimated specific risk, *T* the number of observations, *K* the number of regressors and the denominator $\widehat{\sigma^2_{r^{HF}}} = \frac{\|r^{HF} - \overline{r^{HF}}\mathbb{1}\|^2}{T-1}$ is the unbiased variance estimation relative to the observed hedge fund monthly returns.

3.2.3.1 Step 1: In-Sample Analysis

As a first step, we perform an in-sample linear regression for each hedge fund strategy monthly returns against a set of *K* factors over the whole sample period ranging from January 1997 to October 2015. For each hedge fund strategy we have:

$$r_t^{HF} = \sum_{k=1}^{K} \beta_k F_{k,t} + \epsilon_t \tag{3.4}$$

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with r_t^{HF} being the monthly return of the hedge fund strategy at date t, β_k the exposure of the monthly return on hedge fund strategy to factor k (to estimate), $F_{k,t}$ the monthly return at date t on factor k and ϵ_t the specific risk in the monthly return of hedge fund index at date t (to estimate).

The vector $\hat{\beta}$ is the OLS estimator which minimises the sum of squared errors:

$$\hat{\beta} = \arg\min_b \|r_t^{HF} - F_t b\|^2 \quad (3.5)$$

We estimate the explanatory power measured in terms of the regression adjusted R-squared on the sample period in three distinct cases.
Case 1: Regression on an exhaustive set of factors (kitchen sink regression), i.e. the 19-factors set listed in Table 1.
Case 2: Regression on a subset of traditional factors
Case 3: Regression on a bespoke subset of a maximum of 8 economically-motivated traditional and alternative factors for each hedge fund strategy (see Table 1 for the selection of factors for each hedge fund strategy).

The obtained adjusted R-squared values, reported in Table 2, suggest that we can explain a substantial fraction of hedge fund strategy return variability with traditional and alternative factors, validating that a substantial part of hedge fund performance can ex-post be explained by their systematic risk exposures. The kitchen sink regression (case 1) confirms that more dynamic and/or less directional strategies such as CTA Global, Equity Market Neutral, Fixed Income Arbitrage and Merger Arbitrage strategies, with respective adjusted R-squared of 31%, 32%, 50% and 39%, are harder to replicate than more static and more directional strategies such as long-short equity or short selling for which we both obtain an adjusted R-squared of 81%. The results we obtain also show the improvement of the explanatory power when an economically motivated subset of factors that includes alternative factors is considered (case 3) compared to a situation where the same subset of traditional factors is used for all strategies (case 2). For example adjusted R-squared increases from 25% to 50% for the Global Macro strategy and from 52% to 80% for the Emerging Market strategy.

We do not define hedge fund clones in this first step because the in-sample explanatory analysis presents a look-ahead bias that would drive the model out of the set of possible replication models. Instead, in what follows, we therefore rely on the 24-month rolling-window and Kalman filter approaches which generate truly out-of-sample results.

3.2.3.2 Step 2: Out-of-Sample Analysis

We propose to consider the same bespoke subset economic of factors for each strategy as done in case 3. The objective of this analysis is (i) to capture the dynamic allocation of hedge fund strategies by explicitly allowing the betas to vary over time in our models and (ii) to define hedge fund clones which make sense for an investor (i.e without look-ahead bias and with the sum of the clone's constituent weights equals to one). A classic approach in the literature (see Hasanhodzic and Lo (2007) and Amenc et al. (2010)) consists in regressing hedge fund excess returns against a subset risk factors under the constraint that the sum of the weights equals to one as follows:

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$$r_t^{HF} - r_t^f = \sum_{k=1}^{K} \beta_{k,t} \tilde{F}_{k,t} + \epsilon_t \text{ with } \tilde{F}_{k,t} = F_{k,t} - r_t^f$$

and

$$\sum_{k=1}^{K} \beta_{k,t} = 1 \tag{3.6}$$

with r_t^{HF} being the monthly return of the hedge fund strategy at date t, $\beta_{k,t}$ the (possibly time-varying) exposure of the monthly return on hedge fund strategy to factor k (to estimate), $F_{k,t}$ the monthly return at date t on factor k, ϵ_t the specific risk in the monthly return of hedge fund index at date t (to estimate), $\tilde{F}_{k,t}$ the factor k proxy excess return and $F_{k,t}$ the factor k proxy return.

When factors are traditional (i.e long-only) factors equation (3.6) is equivalent to:

$$r_t^{HF} = \sum_{k=1}^{K} \beta_{k,t} F_{k,t} + \epsilon_t$$

with

$$\sum_{k=1}^{K} \beta_{k,t} = 1 \tag{3.7}$$

Regressing hedge fund total returns against factor total returns is then equivalent to regress hedge fund excess returns against factors excess returns.

The hedge fund clone returns are then defined as:

$$r_t^{CL} = \sum_{k=1}^{K} \hat{\beta}_{k,t} F_{k,t}$$

with

$$\sum_{k=1}^{K} \hat{\beta}_{k,t} = 1 \tag{3.8}$$

where $F_{k,t}$ refers to investable support returns (asset class indices).

In our hedge fund replication framework where we also consider alternative (i.e., long/short) factors, we adopt the following different approach for the clone building: we regress the hedge fund total returns against factors total returns without constraint and take into account leverage in the clone's construction by adding the risk-free asset ex-post. This approach makes the assumption that all the factors have investable proxies and even alternative factors returns can be considered as total returns.

For each hedge fund strategy we define the hedge fund clones as follows:

$$r_t^{CL} = \sum_{k=1}^{K} \hat{\beta}_{k,t} F_{k,t} + \left(1 - \sum_{k=1}^{K} \hat{\beta}_{k,t}\right) r_t^f \tag{3.9}$$

with $\hat{\beta}_{k,t}$ the estimated time-varying exposure of the monthly return on hedge fund strategy to factor k, $F_{k,t}$ the monthly return at date t on factor k and r_t^f the risk-free asset return at date t.

<u>Rolling-Window Linear Regression</u>

In a second step, we perform an out-of-sample hedge fund return replication exercise using for each strategy the bespoke subset of factors (case 3). The objective of this analysis is to assess whether one can capture the dynamic allocation of hedge fund strategies by explicitly allowing the betas to vary over time in a statistical model. The out-of-sample time window considered is January 1999-October 2015, which allows us to build a "24-month rolling-window" linear clone for each strategy.

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Kalman Filter

A more sophisticated approach consists in explicitly modelling dynamic risk factor exposures through a linear state-space model and and then solving it variables by Kalman filtering. Broadly speaking, a state-space model is defined by a transition equation and a measurement equation (see Appendix B for more details). In our specific framework the state space model can be written as follows:

$$\begin{cases} \beta_t = \beta_{t-1} + \eta_t \text{ (Transition Equation)} \\ r_t^{HF} = \beta_t . F_t + \epsilon_t \text{ (Measurement Equation)} \end{cases} \tag{3.10}$$

where β_t is the unobservable vector of factor exposures which need to be estimated with all the available and suitable observations at time t, F_t the vector of factor returns at time $t-1$, r_t^{HF} the return of hedge strategy at time t, η_t and ϵ_t are supposed to be normally distributed with zero mean and constant diagonal covariance matrices. The out-of-sample period considered is January 1999-October 2015 and we fix the initial parameters $P_0 = \mathbb{O}_K$ and $E(\beta_0) = \hat{\beta}_{OLS}$ where $\hat{\beta}_{OLS}$ is the vector value stemming from an unconstrained least-squares estimation over the initial calibration period (January 1997-December 1998 in our case). Accordingly we obtain the Kalman clone for strategy as:

$$r_t^{CL} = \sum_{k=1}^{K} \mathbb{E}_{t-1}[\beta_{k,t}] F_{k,t} + (1 - \sum_{k=1}^{K} \mathbb{E}_{t-1}[\beta_{k,t}])\ r_t^f \tag{3.11}$$

where $\mathbb{E}_{t-1}[\beta_{k,t}] = \hat{\beta}_{k,t|t-1}$ is the best estimate of β_t given all available information up to time $t-1$.

An essential aspect of the Kalman filter implementation is the estimation of the K+1 coefficients of the covariance matrices η_t and ϵ_t. We fix these parameters heuristically in order to be consistent from an investor perspective. We also display an optimised version of the Kalman filter where we estimate the covariance matrix coefficients with maximum likelihood method on the whole sample period. This latter version presents a look-ahead bias and is useful to quantify the impact of the calibration of the Kalman filter. Details on technical aspects can be found in Appendix B.

Replication Power Analysis

To assess the robustness of the replication techniques it is necessary to analyse the out-of-sample replication power of the clones. The substantial decrease between in-sample (see Table 2) and out-of-sample (see Table 3) adjusted R-squared for most of the strategies suggests that the actual replication power of the clones falls down sharply when taken out of the calibration sample. For example the Event Driven clones have an out-of-sample adjusted R-squared below 50% whereas the Event Driven hedge fund strategy has a corresponding in-sample adjusted R-squared of 63%. The Equity Market Neutral clones have negative adjusted R-squared whereas the Equity Market Neutral hedge fund strategy has a corresponding in-sample adjusted R-squared of 16%. The CTA Global rolling-window clone also has a negative out-of-sample adjusted R-squared corroborating the lack of robustness of the clones. Table 11 shows that, in the kitchen sink case, the difference between the in-sample adjusted R-squared of the linear regression and the out-ofsample adjusted R-squared of the hedge fund clones is substantial. Apart from the CTA Global,

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Equity Market Neutral and Merger Arbitrage hedge fund strategies, the in-sample adjusted R-squared of the hedge fund strategies is greater than or equal to 50%, but out-of-sample the adjusted R-squared of almost all the clones are negative. For instance the Distressed Securities displays an in-sample adjusted R-squared of 71% when its corresponding clones display negative out-of-sample adjusted R-squared (-58% for the rolling-window clone and -16% for the Kalman clone).

To get a better sense of what the out-of-sample replication quality actually is, we compute the annualised root mean squared error (RMSE, see Table 3) which can be interpreted as the out-of-sample tracking error of the clone with respect to the corresponding hedge fund strategy. Our results suggest that the use of Kalman filter techniques does not systematically improve the quality of replication with respect to the simple rolling-window approach: the Kalman filter clones of the Distressed Securities, Emerging Markets, Event Driven, Global Macro, Short Selling and Fund of Funds have root mean squared errors above their rolling-window clones. Strategies like CTA Global or Short Selling have clones with the poorest replication quality with root mean squared errors superior to 7.5%.

Table 10 displays the adjusted R-squared and the annualised root mean squared error (RMSE) of the Kalman clones with two different calibration processes: the first process (used previously) consists in heuristically fixing the parameters to estimate (see Appendix B for more details), and the second process consists in estimating the parameters via the maximum likelihood method on the whole sample. As mentioned above, this second process has a look-ahead bias. The adjusted R-Squared and RMSE of all the clone strategies (except for Fixed Income Arbitrage) are substantially improved when the Kalman filter is calibrated with a look-ahead bias, underlying the importance and sensitivity of the calibration process: hedge fund strategy clones like Event Driven (48% vs. 20% adjusted R-squared, 4.0% vs. 5.0% RMSE) , Global Macro (31% vs. -23% adjusted R-squared, 3.9% vs. 5.2% RMSE) or Fund of Funds (58% vs. 37% adjusted R-squared, 3.5% vs.4.7% RMSE) show improved replication measures when look-ahead calibration is used. Overall, these results do not support the belief that hedge fund returns can be satisfactorily replicated.

4. Using Alternative Risk Premia to Generate Attractive Risk-Adjusted Performance

4. Using Alternative Risk Premia to Generate Attractive Risk- Adjusted Performance

The unifying idea of this section is to reconsider the hedge fund replication problem from a different perspective. Our focus is to move away from hedge fund replication, which is not per se a meaningful goal for investors, and to analyse whether (naively) diversified strategies based on systematic exposure to the same alternative risk factors do better from a risk-adjusted perspective than the corresponding hedge fund clones. The same proxies for underlying alternative factor premia will be used in both the clones and the diversified portfolios, which allows us to perform a fair comparison in terms of risk-adjusted performance in spite of the presence of performance biases in the factor proxies.

4.1 Diversified Portfolios of Alternative Risk Premia

In modern portfolio theory, starting with the seminal work of Markowitz (1952), all mean variance investors rationally seek to maximize the Sharpe ratio, subject to the constraint that the portfolio is fully invested in the K risky assets. Let $x = (x_1 \ldots, x_K)$ be the vector of weights in the portfolio, μ the corresponding vector of expected excess returns and $\mathbb{V}$ the covariance matrix. This program reads:

$$\max_x \frac{{}^t x\mu}{\sqrt{{}^t x\mathbb{V}x}} \text{ u.c} \sum_{i=1}^{K} x_i = 1 \quad (4.1)$$

In practice, the MSR (Maximum Sharpe Ratio) portfolio requires estimates for the vector of expected excess returns μ, and the covariance matrix $\mathbb{V}$, both of which are unobservable. A first approach consists in focusing on the Global Minimum Variance Portfolio (GMV), an efficient portfolio which does not require expected return estimates and only relies on the covariance matrix estimates. In implementation, weights constraints need to be introduced so as to avoid that this GMV portfolio become heavily concentrated on a few assets (see DeMiguel et al. (2009) for a thorough empirical analysis of the out-of-sample performance of GMV portfolios).

Moving away from scientific optimisation to naive optimisation procedures, we also consider risk parity portfolios. Formally, let consider a portfolio of K risky assets, let $x = (x_1 \ldots, x_K)$ be the vector of weights in the portfolio, $B = (B_1 \ldots, B_K)$ be the vector of risk budgets, $R = (x_1 \ldots, x_K)$ be a coherent convex risk measure and $RC_i = (x_1 \ldots, x_K)$ be the risk contribution of asset i to the portfolio risk. The Euler decomposition of the risk measure can be written as follows:

$$\begin{aligned} R(x_1,\ldots,x_K) &= \sum_{i=1}^{K} x_i \frac{\partial R(x_1,\ldots,x_K)}{\partial x_i} \\ &= \sum_{i=1}^{K} RC_i(x_1,\ldots,x_K) \end{aligned} \quad (4.2)$$

The risk budgeting portfolio is defined by the following constraints:

$$\begin{cases} RC_1(x_1,\ldots,x_K) = B_1 \times R(x_1,\ldots,x_K) \\ \vdots \\ RC_K(x_1,\ldots,x_K) = B_K \times R(x_1,\ldots,x_K) \end{cases} \quad (4.3)$$

with $\sum_{i=1}^{K} B_i = 1.$

Generally there is no analytical solution to the previous problem but we can always find a numerical solution. We can transform the previous non-linear system into an optimisation problem:

$$x^* = \min f(x; B) \text{ u.c} \sum_{i=1}^{K} x_i = 1$$

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and

$$f(x;B) = \sum_{i=1}^{K} \left(x_i \times \frac{\partial R(x_1, \ldots, x_K)}{\partial x_i} - B_i R(x_1, \ldots, x_K) \right)^2 \quad (4.4)$$

If we consider the volatility $\sigma = (x_1 \ldots, x_K)$ as a risk measure of the portfolio and let $\mathbb{V}$ be the covariance matrix of the K assets, the Euler decomposition of the risk measure can be written as follows:

$$\begin{aligned} R(x_1, \ldots, x_K) &= \sigma(x_1, \ldots, x_K) && (4.5) \\ &= \sum_{i=1}^{K} x_i \frac{\partial \sigma(x_1, \ldots, x_K)}{\partial x_i} && (4.6) \\ &= \sum_{i=1}^{K} x_i \times \frac{(\mathbb{V}x)_i}{\sqrt{^t x \mathbb{V} x}} && (4.7) \\ &= \frac{\sum_{i=1}^{K} x_i \left(x_i \sigma_i^2 + \sigma_i \sum_{j=1, j \neq i}^{K} \rho_{i,j} x_j \sigma_j \right)}{\sqrt{^t x \mathbb{V} x}} && (4.8) \\ &= \sum_{i=1}^{K} RC_i && (4.9) \end{aligned}$$

where $RC_i = x_i \times \frac{(\mathbb{V}x)_i}{\sqrt{^t x \mathbb{V} x}}$ is the total contribution of asset i to the portfolio volatility.

4.2 Reported Risk and Diversification Measures

In our analysis, we report commonly used risk measures such as volatility (a measure of average risk), Value-at-Risk (a measure of extreme risk) or tracking error (a measure of relative risk). These measures, however, are typically backward-looking risk indicators computed over one historical scenario. As a result, they provide very little information, if any, regarding the possible causes of the portfolio riskiness, the probability of a severe outcome in the future, or the reward that an investor can expect in exchange for bearing those risks. In this context, we propose to also report forward-looking risk indicators for the tested portfolios.

Common intuition and portfolio theory both suggest that the degree of diversification of a portfolio is a key indicator when assessing its ability to generate attractive risk-adjusted performance across various market conditions. The benefits of diversification are intuitively clear: efficient diversification generates a reduction of unrewarded risks that leads to an enhancement of the portfolio risk-adjusted performance. On the other hand, in the absence of a formal definition for diversification, it is not as straightforward a task as it might seem to provide a quantitative measure of how well or poorly diversified a portfolio is. The usual definition of diversification is that it is the practice of not "putting all your eggs in one basket". Having eggs (dollars) spread across many baskets is, however, a rather loose prescription in the absence of a formal definition for the true meaning of "many" and "baskets".

An initial approach to measuring portfolio diversification would consist of a simple count of the number of constituents the portfolio is invested in. One key problem with this approach is that what matters from a risk perspective is not the nominal number of constituents in a portfolio, but instead its effective number of constituents (ENC). To understand the nuance, let us consider the example of a fictitious equity portfolio that would allocate 99% of the wealth to one stock and spread the remaining 1% of the wealth to the 499 remaining stocks within the S&P 500 index universe. While the nominal number of stocks in that portfolio (defined as the number of stocks that receive some non zero allocation) is 500, it is clear

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that the effective number of stocks in the portfolio is hardly greater than one, and that this poorly diversified portfolio will behave essentially like a highly concentrated one-stock portfolio from a risk perspective. In this context, it appears that a natural and meaningful measure of the effective number of constituents (ENC) in a portfolio is given by the entropy of the portfolio weight distribution. This quantity, a dispersion measure for probability distributions commonly used in statistics and information theory, is indeed equal to the nominal number K for a well-balanced equally-weighted portfolio, but would converge to 1 if the allocation to all assets but one converges to zero as in the example above, thus confirming the extreme concentration in this portfolio. This dispersion measure of the dollar contributions could be defined as follows:

$$\mathrm{ENC}(x) = \frac{1}{\|x\|^2} \tag{4.10}$$

where ||·|| denotes the euclidian norm. An application of Cauchy-Schwarz inequality shows that $ENC \leq K$, and that equality holds if, and only if, all weights are equal.[4]

On the other hand, if one is indeed entitled to considering that a well-balanced allocation of dollars (eggs) to identical securities (baskets) may be regarded as a well-diversified allocation, the existence of differences in risks across securities would require some adjustment to the proposed measure of sound diversification. In other words, what needs to be well-balanced is not the number of eggs in each basket per se, but rather the risk contribution of each basket. In this context, a well-diversified portfolio would seek to have more eggs in more robust baskets, and fewer eggs in frailer baskets.

To try and identify a meaningful measure of the number of bets (baskets) to which investors' dollars (eggs) are allocated, one can first define the contribution of each constituent to the overall volatility of the portfolio σ_P (see Roncalli (2013) for further details) as :

$$\begin{aligned}\sigma_P(x_1,\ldots,x_K) &= \sum_{i=1}^{K} x_i \frac{\partial \sigma_P(x_1,\ldots,x_K)}{\partial x_i} \\ &= \sum_{i=1}^{K} x_i \times \frac{(\mathbb{V}x)_i}{\sqrt{{}^t x \mathbb{V} x}} \\ &= \sum_{i=1}^{K} RC_i\end{aligned} \tag{4.11}$$

This leads to the following scaled contributions:

$$\begin{aligned} RC_i^{scaled} &= \frac{RC_i}{\sigma_P} \\ &= \frac{\sum_{i=1}^{K} x_i \left(x_i \sigma_i^2 + \sigma_i \sum_{j=1, j\neq i}^{K} \rho_{i,j} x_j \sigma_j \right)}{\sigma_P}\end{aligned} \tag{4.12}$$

where $\sum_{i=1}^{K} RC_i^{scaled} = 1$.

To account for the presence of cross-sectional dispersion in the correlation matrix, one can apply the naive measure of concentration, ENC, introduced above to the scaled contributions to portfolio risk. This allows us to define *the effective number of correlated bets* in a portfolio as the dispersion of the volatility contributions of its constituents:

$$\mathrm{ENCB}(x) = \frac{1}{\|RC^{scaled}\|^2} \tag{4.13}$$

where ||·|| denotes the euclidian norm.[5]

4 - Cauchy-Schwarz inequality states that for any two vectors $(x_1, \ldots, x_K)$ and $(y_1, \ldots, x_K)$, we have $\left(\sum_i x_i y_i\right)^2 \leq \left(\sum_i x_i^2\right) \times \left(\sum_i y_i^2\right)$. Equality is achieved if, and only if, one of the two vectors is zero or the two vectors are parallel. Here, we take $y_i = 1$.

5 - An application of Cauchy-Schwarz inequality shows that $ENCB \leq K$, and that equality holds if, and only if, all risk contributions are equal.

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This measure takes into account not only the number of available assets but also the correlation properties between them. More specifically, a constituent that is highly positively correlated with all the other constituents will tend to have a higher contribution to the volatility (considering long-only portfolio for simplicity), leading to a lower effective number of correlated bets since most of the portfolio risk is concentrated in that constituent.

A shortcoming of the ENCB is that it can give a misleading picture for highly correlated assets. For instance, an equally-weighted portfolio of two highly correlated bonds with similar volatilities is well diversified in terms of dollars and volatility contributions, but portfolio risk is extremely concentrated in a single interest rate risk factor exposure. To better assess the contributions of underlying risk factors, Meucci (2009) and Deguest et al. (2013) propose to decompose the portfolio returns (which can be seen as combinations of correlated asset returns or correlated factor returns) as a combination of the contributions of K uncorrelated implicit factors. In other words, baskets should be interpreted as uncorrelated risk factors, as opposed to correlated asset classes, and it is only if the distribution of the contributions of various factors to the risk of the portfolio is well-balanced that the investor's portfolio can truly be regarded as well-diversified. Putting all these elements together, we propose using the effective number of uncorrelated bets (ENUB) in our empirical analysis, which would serve as a meaningful measure of diversification for investors' portfolios (see Meucci (2009) and Deguest et al. (2013) for more details). First we decompose the portfolio return r_P as the sum of K correlated asset returns $r_1, \dots, r_K$ and also the sum of K uncorrelated factor returns $r_{F1}, \dots, r_{FK}$ (see also Deguest et al. (2013) or Meucci et al. (2015)): $r_P = {}^t xr = {}^t x_F r_F$ where r denotes the vector of the original constituents' returns, r_F the vector of uncorrelated factors' returns, x the weight vector of correlated components and x_F the weight vector of uncorrelated factors. Note that to simplify we assess that the number of original correlated assets and the number of uncorrelated factors is the same. The main challenge with this approach is to turn correlated asset returns into uncorrelated factor returns.

The volatility of the portfolio is:

$$\sigma_P = \sqrt{{}^t x \mathbb{V} x} = \sqrt{{}^t x_F \mathbb{V}_F x_F}$$

where

$$\mathbb{V}_F = \begin{pmatrix} \sigma^2_{F_1} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma^2_{F_K} \end{pmatrix}$$

is the covariance matrix of the K uncorrelated factors.

The factor returns can typically be expressed as a linear transformation of the original returns: $r_F = {}^t Ar$ for some well chosen transformation A guaranteeing that the covariance matrix of the factors

$$\mathbb{V}_F = {}^t A \mathbb{V} A = \begin{pmatrix} \sigma^2_{F_1} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma^2_{F_K} \end{pmatrix}$$

is a diagonal matrix. A is a $K \times K$ transition matrix from the assets to the factors and

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it is therefore critical that it be invertible since $x_F = A^{-1}x$.

Then, we define the contribution of each factor to the overall variance of the portfolio σ_P^2 as:

$$\sigma_P^2 = {}^t x \mathbb{V} x = {}^t x_F \mathbb{V}_F x_F = \sum_{i=1}^{K} x_{F_i}^2 \sigma_{F_i}^2 \quad (4.14)$$

which leads to the following percentage contributions for each factor:

$$p_i = \frac{x_{F_i}^2 \sigma_{F_i}^2}{\sigma_P^2}$$

where $\sum_{i=1}^{K} p_i = 1, p_i \geq 0.$

The effective number of uncorrelated bets (ENUB) can be written as follows:

$$\text{ENUB}(x) = \exp\left(-\sum_{i=1}^{K} p_i \ln p_i\right) \quad (4.15)$$

ENUB reaches a minimum equal to 1 if the portfolio is loaded in a single risk factor, and a maximum equal to K, the nominal number of constituents, if the risk is evenly spread amongst factors. The portfolios named factor risk parity portfolios in Deguest et al. (2013) are built such that the contribution pi of each factor to the variance are all equal. This methodology relies on uncorrelated implicit factors which are not uniquely defined. In fact, it is easy to see that if A is a change of basis matrix from constituents to factors and Q is any orthogonal matrix, then the matrix $M = A\mathbb{V}_F^{1/2}Q$ is also a change of basis matrix such that ${}^t M\mathbb{V}M$ is diagonal.[6] Thus, one has to specify an orthogonalisation procedure. An option is to perform principal component analysis on the covariance matrix, so as to sequentially extract uncorrelated factors that have the maximum marginal explanatory power with respect to asset returns. This decomposition, however, has some undesirable properties: Carli et al. (2014) note that the principal factors lack interpretability and Meucci et al. (2015) show that if all assets have the same volatility and the same pairwise correlation, then an equally-weighted portfolio of the assets is fully invested in the first principal factor (i.e. the one with the largest variance) and that the other factors have zero weight. As a result, the portfolio risk is entirely explained by the first factor, regardless of the value of the common correlation. This property is counter-intuitive, as one would expect uncorrelated factors to have more balanced contributions when the correlation across assets shrinks to zero. To overcome these problems, Meucci et al. (2015) introduce an alternative method for extracting uncorrelated factors, known as "Minimum Linear Torsion". The MLT algorithm seeks to extract the matrix A such that the factor covariance matrix is diagonal while keeping the distance between the factors and the asset returns as small as possible, thus involving the smallest deformation of the original components. In detail, A is the solution to the following program:

$$\min_A \sum_{i=1}^{K} \text{Var}[r_{F_i} - r_i]$$

subject to $r_F = {}^t Ar$

and ${}^t A\mathbb{V}A = \text{diag}(\mathbb{V})$ (4.16)

where diag($\mathbb{V}$) denotes a diagonal matrix with diagonal elements equal to those of $\mathbb{V}$. Note that the second constraint implies two properties: first, the factor covariance

6 - A matrix Q is said to be orthogonal if it satisfies ${}^t QQ = Q^t Q = \mathbb{I}_K$.

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matrix is diagonal, and second, the factor variances are equal to the asset variances, which is a natural requirement since the factors should "resemble" asset returns as much as possible. In the previous example (uniform volatilities and correlations and equally-weighted portfolio), it can be shown that all uncorrelated factors have the same weight, which is more in line with the intuition. Fortunately, the solution to the optimisation problem is known in closed form, up to the singular value decompositions of and an auxiliary matrix: expressions for the change of basis matrix A can be found in Meucci et al. (2015) and Carli et al. (2014), and are recalled in Appendix C. Since efficient numerical algorithms exist for computing singular value decompositions, the solution is straightforward to obtain numerically. In what follows, we will use this "least intrusive" orthogonalisation procedure to extract uncorrelated factors starting from correlated factor indices.

4.3 Experimental Protocol

Overall, we propose to test the following heuristic weighting schemes with monthly rebalancing:

- Equally-Weighted (EW):

$$\forall i \in [\![1;K]\!] \;\; x_i = \frac{1}{K}$$

- Equal Risk Contribution (ERC):

$$\forall i \in [\![1;K]\!] \;\; RC_i^{scaled} = \frac{1}{K}$$

Equally-Weighted Portfolio (EW)

This heuristic portfolio simply involves attributing the same weight to each constituent in the investment universe. The EW portfolio is the most natural portfolio in the absence of any information on the covariance matrix $\mathbb{V}$ and on expected returns, and coincides with the tangency portfolio assuming that volatilities, correlations and expected returns are equal for all the constituents. The equally-weighted portfolio (EW) does not require any unobservable risk or return parameter and is thus completely free of estimation error. While maximising the ENC (Effective Number of Constituents), this approach can lead to an imbalanced portfolio in term of risk contributions of each constituent to the overall risk of the portfolio because some constituents have higher volatilities and there is no risk management in this approach. Because of this property, it often outperforms optimised portfolios constructed as proxies for efficient portfolios but plagued by estimation errors in covariances and expected returns (see Bloomfield et al. (1977), Jorion (1991), DeMiguel et al. (2009) and Martellini et al. (2014)).

Equal Risk Contribution Portfolio (ERC)

The underlying idea of the ERC portfolio is to equalise the contribution of each constituent to the overall risk. The ERC portfolio is the tangency portfolio if all constituents have the same Sharpe ratio and if all pairs of correlation are identical. The first formal analysis of the ERC portfolio was subsequently given by Maillard et al. (2010), who established its existence and uniqueness and proposed numerical algorithms to compute the portfolio. A drawback of the ERC portfolio is the disregard of the factors which have an impact on the portfolio: indeed large portfolios can be driven by a small number of factors. This limitation can be solved with the factor risk parity methodology (see Roncalli (2013) for further details). We estimate the covariance matrix considering the rolling past 24-month historical returns.

In this section we revisit the problem from a different perspective. Our focus

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is to move away from hedge fund replication, which anyway is not per se a meaningful goal for investors, and analyse whether diversified strategies based on systematic exposure to the same alternative risk factors perform better from a risk-adjusted perspective than the corresponding hedge fund clones. Since the same proxies for underlying alternative factor premia will be used in both the clones and the diversified portfolios, we can perform a fair comparison in terms of risk-adjusted performance in spite of the presence of performance biases in both hedge fund return and factor proxies. We apply two popular robust heuristic portfolio construction methodologies, namely Equally-Weighted and Equal Risk Contribution (using a 24-month rolling-window to estimate the covariance matrix) for each hedge fund strategy relative to its bespoke subset of economically identified risk factors for the period January 1999-October 2015. We then compare the risk-adjusted performance of rolling-window and Kalman filter clones with the corresponding diversified portfolio of the same selected factors by computing their Sharpe ratios.

The third row of Table 12 gives the Sharpe ratios of the rolling-window and Kalman filter clones, the Sharpe ratios of the corresponding Equal Risk Contribution and Equally-Weighted portfolios. The clones for Distressed Securities, Event Driven, Global Macro, Relative Value and Fund of Funds have been built with the same 6 risk factors: Equity, Bond, Credit, Emerging Market, Multi-Class Value and Multi-Class Momentum. The corresponding Equal Risk Contribution and Equally-Weighted portfolios have respective Sharpe ratios of 0.74 and 0.63 which is higher than all of the previous clones' Sharpe ratios (see for example the Global Macro and Distressed Securities Kalman filter clones with respective Sharpe ratios of 0.53 and 0.17). Similarly, the Equity Market Neutral, Merger Arbitrage, Long Short Equity and Short Selling clones have been built with the same 6 risk factors: Equity, Equity Defensive, Equity Size, Equity Quality, Equity Value and Equity Momentum. All the clones' Sharpe ratios are lower (see for example the Equity Market Neutral Kalman filter clone with Sharpe ratio of 0.74) than those of the corresponding Equal Risk Contribution and Equally-Weighted portfolios (respectively 1.02 and 0.96), and sometimes substantially lower (see for example the Merger Arbitrage and Long/Short Equity Kalman filter clones with respective Sharpe ratios of 0.39 and 0.26). Table 12 also displays extreme risks of clones and portfolios. We consider only non-directional strategies like Equity Market Neutral, Merger Arbitrage, Relative Value, Convertible Arbitrage and Fixed Income Arbitrage in order to compare clones and diversified portfolios with similar volatilities. The clones of Equity Market Neutral (12.4% maximum drawdown for the rolling-window clone and 7.7% for the Kalman clone) and Merger Arbitrage (20.7% maximum drawdown for the rolling-window clone and 17.2% for the Kalman clone), built with the same risk factors show higher extreme risk than the corresponding Equal Risk Contribution portfolio (2.6% maximum drawdown). It is also the case for the clones of Fixed Income Arbitrage (14.7% maximum drawdown for the rolling-window clone and 13.7% for the Kalman clone) which displays higher extreme risk than the corresponding Equal Risk Contribution portfolio (3.7% maximum drawdown).

In Table 13 we can compare for each strategy the diversification of its

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corresponding clones and diversified portfolios. For a given hedge fund strategy, the clones have by construction one more constituent (the risk-free asset) than the corresponding diversified portfolios due to the leverage. Consequently we consider relative values of the Average Effective Number of Constituents (ENC) and Average Effective Number of Uncorrelated Bets (ENUB) for the out-of-sample period ranging from January 1999 to October 2015. The values for these two statistics vary therefore from 0% to 100%. The Equal Risk Contribution portfolios have by construction an Average (*ENUB*) near from 100%, so we will compare the only the clones with the associated Equally-Weighted portfolios. All the clones for Equity Market Neutral, Merger Arbitrage, Long Short Equity and Short Selling built on the same factors display an inferior average ENUB (see for example the Long Short Equity strategy with 54% for the rolling-window clone and 45% for the Kalman clone; and the Merger Arbitrage strategy with 47% for the rolling-window clone and 46% for the Kalman clone) than the associated Equally-Weighted portfolio (66%). The assessment is still verified for more dynamic or non-directional strategies like CTA Global (57% for the rolling-window clone, 49% for the Kalman clone and 61% for the corresponding Equally-Weighted strategy) and Fixed Income Arbitrage (45% for the rolling-window clone, 42% for the Kalman clone and 71% for the corresponding Equally-Weighted strategy). On the contrary, all the clones for Distressed Securities, Event Driven, Global Macro, Relative Value and Fund of Funds built on the same factors have a superior (or equal) average ENUB (see for example the Relative Value strategy with 53% for the rolling-window clone and 55% for the Kalman clone; and the Fund of Funds strategy with 54% for the rolling-window clone and 65% for the Kalman clone) than the associated Equally-Weighted portfolio (53%). For every hedge fund strategy, the average ENC of clones is systematically inferior than those of the diversified portfolios: apart from the Equity Market Neutral and the Emerging Market strategies, the average ENC of the hedge fund clones is inferior or equal to 40%, indicating a high concentration of the clones in a few risk factors. Overall, diversified portfolios seem a better alternative to hedge fund clones from a diversification and extreme risk perspectives.

Table 14 contains the results for clones' and diversified portfolios' one-way annual turnover. At each rebalancing date t, the one-way turnover is computed as:

$$\theta_t = \frac{1}{2}\sum_{i=1}^{K} |x_{i,t} - x_{i,t_-}| \quad (4.17)$$

where $x_{i,t}$ is the weight of constituent i imposed on date t and $x_{i,t-}$ the effective weight just before the rebalancing. The annual turnover is the average of the $\boldsymbol{\theta}_t$ multiplied by 12 to convert it to an annual quantity.

We see that in all cases the turnover for the hedge fund clones is much higher than for the corresponding diversified portfolios. The clones' turnovers vary from 106 % for the Long Short Equity Kalman clone to 29935% for the Short Selling Kalman clone. Comparatively the diversified portfolios' turnovers vary from 4% for the Fixed Income Arbitrage Equally-Weighted portfolio to 79% for the CTA Global Equal Risk Contribution portfolio. On the other hand, except for the Short Selling strategy, the Kalman clones have lower turnover than the rolling-window clones.

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We can also assess that for all the hedge fund strategies, the Equally-Weighted portfolios display lower turnover than the Equal Risk Contribution portfolios. These results emphasise that the transaction costs inherent to dynamic trading strategies could be significant in the case of the hedge fund clones.

5. Conclusion

5. Conclusion

While the replication of hedge fund factor exposures appears to be a very attractive concept from a conceptual standpoint, our analysis confirms the previously documented intrinsic difficulty in achieving satisfactory out-of-sample replication power, regardless of the set of factors and the methodologies used. Our results also suggest that risk parity strategies applied to alternative risk factors could be a better alternative than hedge fund replication for harvesting alternative risk premia in an efficient way. In the end, the relevant question may not be "Is it feasible to design accurate hedge fund clones with similar returns and lower fees?", for which the answer appears to be a clear negative, but instead "Can suitably designed mechanical trading strategies in a number of investable factors provide a cost-efficient way for investors to harvest traditional but also alternative beta exposures?". With respect to the second question, there are reasons to believe that such low-cost alternatives to hedge funds may prove a fruitful area of investigation for asset managers and asset owners. A key challenge for the alternative investment industry remains the capacity to develop investable efficient low-cost proxies for harvesting alternative risk premia not only in the equity market but also in the fixed income, currencies and commodity markets. Our paper could also be extended in a number of useful directions. In particular, more factors, such as "carry everywhere" or "equity short volatility" for example, could be included in the analysis. Additionally, introducing regional single- and multi-asset factors would also be a worthwhile extension of our analysis.

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A. Linear Regression Without Intercept

We postulate that the observations are related by:

$$r^{HF} = F\beta + \epsilon \qquad \text{(A.1)}$$

where r^{HF} denotes the vector of hedge fund returns observations (endogenous $T \times 1$ variable), F denotes the matrix of factor returns observations (exogeneous $T \times K$ variable), β the unknown parameter of factors exposure ($K \times 1$ variable) to estimate and ϵ ($T \times 1$ variable) the residuals to estimate.

The three common assumptions of the standard linear model are:
(1) $\mathbb{E}[{}^t\epsilon F|F] = 0$: residuals are orthogonal to regressors,
(2) $\mathbb{V}[\epsilon|F] = \sigma^2 \mathbb{I}_T$ [7]: residuals are homoskedastic,
(3) $\text{rank}(F) = K$: F has the maximum possible rank, i.e there are no redundant regressors.

7 - $\mathbb{I}_T$ is the $T \times T$ identity matrix

The Ordinary Least Squares (OLS) estimator minimises the sum of squared errors:

$$\hat{\beta} = \arg\min_b \|r^{HF} - Fb\|^2.$$

This estimator is defined by:

$$\hat{\beta} = ({}^tFF)^{-1}\, {}^tF r^{HF} \qquad \text{(A.2)}$$

Assuming that the model includes a constant is equivalent to consider a $K \times 1$ vector of 1 as one of the column of F. In that general case, we have the following variance analysis:

$$\sigma^2_{r^{HF}} = \underbrace{\frac{1}{T}\|r^{HF} - \overline{r^{HF}}\mathbb{1}_T\|^2}_{\text{Total variation}}$$
$$= \underbrace{\frac{1}{T}\|\widehat{r^{HF}} - \overline{r^{HF}}\mathbb{1}_T\|^2}_{\text{Explained variation}}$$
$$+ \underbrace{\frac{1}{T}\|\widehat{r^{HF}} - r^{HF}\|^2}_{\text{Unexplained variation}} \qquad \text{(A.3)}$$

where $\overline{r^{HF}}$ is the average of vector r^{HF} components, T the number of observations on the sample, $\mathbb{1}_T$ is a $T \times 1$ vector of ones and $\widehat{r^{HF}} = F\hat{\beta}$.

We define the R-squared (R^2) as the percentage of variance explained by the regressors:

$$R^2 = 1 - \frac{\|\widehat{r^{HF}} - r^{HF}\|^2}{\|r^{HF} - \overline{r^{HF}}\mathbb{1}_T\|^2}$$
$$= 1 - \frac{\|\hat{\epsilon}\|^2}{\|r^{HF} - \overline{r^{HF}}\mathbb{1}_T\|^2}$$
$$= 1 - \frac{\widehat{\sigma^2}}{\widehat{\sigma^2_{r^{HF}}}}\,\frac{T-K-1}{T-1} \qquad \text{(A.4)}$$

where T is the number of observations on the sample, $K+1$ the number of regressors including the constant, $\widehat{\sigma^2_{r^{HF}}}$ the unbiased estimation relative to the observed hedge fund monthly returns defined as:

$$\widehat{\sigma^2_{r^{HF}}} = \frac{1}{T-1}\|r^{HF} - \overline{r^{HF}}\mathbb{1}_T\|^2$$

and $\widehat{\sigma^2}$ the estimation of residual variance defined as:

$$\widehat{\sigma^2} = \frac{\|\hat{\epsilon}\|^2}{T-K-1} = \frac{1}{T-K-1}\sum_{t=1}^{T}\hat{\epsilon}_t^2.$$

The R^2 lies between 0 and 1 (perfect fit) and increases with the number of regressors ($K+1$).

To take into account the number of regressors we define the adjusted R-squared

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as:
$$R^2_{adj} = 1 - \frac{\widehat{\sigma^2}}{\widehat{\sigma^2_{r^{HF}}}} \qquad \text{(A.5)}$$

We consider in our empirical studies linear regressions without intercept. In this case (see Eisenhauer (2003)), the equation (A.3) is not valid anymore. We have instead:

$$\underbrace{\frac{1}{T}\|r^{HF}\|^2}_{\text{Total variation}} = \underbrace{\frac{1}{T}\|\widehat{r^{HF}}\|^2}_{\text{Explained variation}} + \underbrace{\frac{1}{T}\|r^{HF} - \widehat{r^{HF}}\|^2}_{\text{Unexplained variation}} \qquad \text{(A.6)}$$

Then we define the R-squared and the adjusted R-squared as in Wooldridge (2012) :

$$R^2 = 1 - \frac{\|\widehat{r^{HF}} - r^{HF}\|^2}{\|r^{HF} - \overline{r^{HF}}\mathbb{1}_T\|^2} = 1 - \frac{\widehat{\sigma^2}}{\widehat{\sigma^2_{r^{HF}}}}\frac{T-K}{T-1},$$
$$R^2_{adj} = 1 - \frac{\widehat{\sigma^2}}{\widehat{\sigma^2_{r^{HF}}}} \qquad \text{(A.7)}$$

where $\widehat{\sigma^2} = \frac{\|\hat{\epsilon}\|^2}{T-K}$ is the unbiased estimation of residual variance and $\widehat{\sigma^2_{r^{HF}}} = \frac{1}{T-1}\|r^{HF} - \overline{r^{HF}}\mathbb{1}_T\|^2$ is the unbiased estimation of the hedge fund return variance.

B. Kalman filter

A state-space model (see Harvey (1991)) is defined by a measurement equation connecting present observations with unobservable variables. These variables could be interpreting as specifying a given state in which the observations are in. The state variables are supposed to be a first order Markov process. We denote this state vector by $\boldsymbol{\beta}_t$.

The transition equation is defined by:

$$\beta_t = A_t\beta_{t-1} + c_t + D_t\eta_t, \quad t = 1, ..., T \qquad \text{(B.1)}$$

where $\boldsymbol{\beta}_t$ is the $K \times 1$ dimensional state vector, A_t a $K \times K$ matrix, c_t a $K \times 1$ vector, D_t a $K \times l$ matrix and $\boldsymbol{\eta}_t$ a $l \times 1$ dimension white noise.

The measurement equation is defined by:

$$r_t^{HF} = F_t\beta_t + g_t + \epsilon_t, \ t = 1, ..., T \qquad \text{(B.2)}$$

where r_t^{HF} is a $N\times1$ vector, F_t is a $N\times K$ matrix, g_t a $N\times1$ vector and ϵ_t a $N\times1$ dimension white noise. Both $\boldsymbol{\eta}_t$ and ϵ_t are assumed to be normally distributed and independent (i.e uncorrelated).

We have:
$$\begin{cases} \mathbb{E}[\eta_t] = 0, \ \mathbb{V}[\eta_t] = Q_t \\ \mathbb{E}[\epsilon_t] = 0, \ \mathbb{V}[\epsilon_t] = H_t \end{cases} \qquad \text{(B.3)}$$

We also assume that initially the state space vector follows a gaussian distribution with $\mathbb{E}[\beta_0] = \hat{\beta}_0$ and $\mathbb{V}[\beta_0] = \hat{P}_0$.

We note $\hat{\beta}_t = \mathbb{E}_t[\beta_t]$, meaning that $\hat{\beta}_t$ is the best estimate of $\boldsymbol{\beta}_t$ given all available information up to time t. The covariance matrix associated to $\boldsymbol{\beta}_t$ is given by $P_t = \mathbb{E}_{t-1}[(\beta_t - \hat{\beta}_t)\ {}^t(\beta_t - \hat{\beta}_t)]$.

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In our particular time-varying beta framework we will consider the following simplified state-space model:

$$\begin{cases} \beta_t = \beta_{t-1} + \eta_t \text{ (Transition Equation)} \\ r_t^{HF} = \beta_t . F_t + \epsilon_t \text{ (Measurement Equation)} \end{cases} \quad \text{(B.4)}$$

where β_t is the K dimensional state vector (to estimate), r_t^{HF} is the observed value of the hedge fund strategy at time t, F_t is a K dimensional vector of factor returns.

We also assume that:

$$\begin{cases} \mathbb{E}[\eta_t] = 0, \ \mathbb{V}[\eta_t] = \hat{Q} \\ \mathbb{E}[\epsilon_t] = 0, \ \mathbb{V}[\epsilon_t] = \hat{H} = \widehat{\sigma_\epsilon^2} \end{cases} \quad \text{(B.5)}$$

where

$$\hat{Q} = \begin{pmatrix} \widehat{\sigma_1^2} & 0 & \cdots & \cdots & 0 \\ 0 & \widehat{\sigma_2^2} & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \widehat{\sigma_K^2} \end{pmatrix}$$

is supposed to be indepent of t.

The Kalman filter is then defined by the following recursive equations:

$$\begin{cases} \hat{\beta}_{t|t-1} = \hat{\beta}_{t-1} \\ \hat{P}_{t|t-1} = \hat{P}_{t-1} + \hat{Q} \\ \hat{r}_{t|t-1}^{HF} = F_t \hat{\beta}_{t|t-1} \\ v_t = r_t^{HF} - \hat{r}_{t|t-1}^{HF} \\ S_t = F_t \hat{P}_{t|t-1} \, {}^t F_t + \hat{H} \\ \hat{\beta}_t = \hat{\beta}_{t|t-1} + \hat{P}_{t|t-1} \, {}^t F_t S_t^{-1} v_t \\ \hat{P}_t = (I_m - \hat{P}_{t|t-1} \, {}^t F_t S_t^{-1} F_t) \hat{P}_{t|t-1} \end{cases} \quad \text{(B.6)}$$

where $\hat{\beta}_{t|t-1}$ and $\hat{P}_{t|t-1}$ are the best estimates of β_t and P_t conditionally on all the information available at time $t-1$. ϑ_t is the innovation process which calculates the difference between the current observation and its best predictor.

The parameters we have to estimate are: $(\beta_0, P_0, \sigma_\epsilon^2, \sigma_1^2, \ldots, \sigma_K^2)$.

We fix $\hat{P}_0 = \mathbb{O}_K$ and define the value of $\hat{\beta}_0$ as the parameter stemming from an unconstrained least-squares estimation over the initial calibration period (January 1997-December 1998). The other K + 1 parameters $\widehat{\sigma_\epsilon^2}, \widehat{\sigma_1^2}, \ldots, \widehat{\sigma_K^2}$ are heuristically fixed to 0.001.

The log-likelihood of the observation at time t corresponds to

$$l_t = -\tfrac{1}{2} ln(2\pi) - \tfrac{1}{2} ln(S_t) - \tfrac{1}{2S_t} v_t^2.$$

We consider it to calibrate the $\widehat{\sigma_\epsilon^2}, \widehat{\sigma_1^2}, \ldots, \widehat{\sigma_K^2}$ parameters and then compute the look-ahead biased version of the Kalman filter clones in our empirical study.

C. Minimum Linear Torsion Matrix

The solution to the optimisation problem is derived by Meucci et al. (2015) and Carli et al. (2014). In this appendix, we simply recall their result without proof.

Carli et al. (2014) show that the optimal matrix A can be written as:

$$A = PL^{-1} U^t W D$$

where notations are defined as follows:

- D is the diagonal matrix of standard deviations, i.e. the square root of the

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diagonal matrix of variances;

- L is the diagonal matrix of the square roots of the eigenvalues of the covariance matrix $\mathbb{V}$, ranked by decreasing order, and P is an orthogonal matrix such that $\mathbb{V} = PL^{2t}P$;
- U and W are two orthogonal matrices coming from the singular value decomposition of the auxiliary matrix $M = L^tPD$. This means that we have $M = US^tW$, where S is the diagonal matrix of singular values ranked by decreasing order.

Taking inverses of both sides of the equality $L^tPD = US^tW$, we obtain:

$$L^{-1} = {}^t PDV S^{-1t}U$$

Substituting this equality we get:

$$A = DWS^{-1t}WD$$

This expression shows in particular that A is symmetric.

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Table 1: List of risk factors

This table summarises in the first three columns the whole set of the traditional and alternative risk factors considered in our empirical analysis, their proxies and their sources. The other columns indicate the bespoke economic subset of factors used for each strategy in our empirical analysis. CA refers to Convertible Arbitrage, CTA to CTA Global, DS to Distressed Securities, EM to Emerging Markets, EMN to Equity Market Neutral, ED to Event Driven, FIA to Fixed Income Arbitrage, GM to Global Macro, LSE to Long Short Equity, MA to Merger Arbitrage, RV to Relative Value, SS to Short Selling and FoF to Fund of Funds.

	Risk Factors	Proxies	Source	CA	CTA	DS	EM	EMN	ED	FIA	GM	LSE	MA	RV	SS	FoF
"Traditional Factors"	Equity	S&P 500 TR	Bloomberg	X	X	X		X	X		X	X	X	X	X	X
	Currency	US Dollar Index	Bloomberg				X									
	Commodity	S&P GSCI TR	Bloomberg		X											
	Emerging market	MSCI EM TR	MSCI Website			X	X		X		X			X		X
	Bond	"Barclays US TreasuryBond Index"	Datastream	X	X	X			X	X	X			X		X
	Credit	"Barclays US CorporateInv Grade Index"	Datastream	X		X			X	X	X			X		X
"Alternative Factors"	Multi-Class Momentum	Multi-Class Global MOM	AQR website			X			X		X			X		X
	Multi-Class Value	Multi-Class Global VAL	AQR website			X			X		X			X		X
	Equity Momentum	Eq Global MOM	AQR website	X	X			X				X	X		X	
	FX Momentum	FX Global MOM	AQR website		X											
	FI Momentum	FI Global MOM	AQR website		X					X						
	Commo Momentum	COM Global MOM	AQR website		X											
	Equity Value	Eq Global VAL	AQR website	X				X				X	X		X	
	FX Value	FX Global VAL	AQR website													
	FI Value	FI Global VAL	AQR website							X						
	Commo Value	COM Global VAL	AQR website													
	Equity Defensive	Eq Global BAB	AQR website	X				X				X	X		X	
	Equity Quality	Eq Global QMJ	AQR website	X				X				X	X		X	
	Equity Size	Eq Global SMB	AQR website	X				X				X	X		X	

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Table 2: In-Sample Ajusted R-squared for empirical data
This table reports for each hedge fund strategy the linear regression adjusted R-squared of its monthly returns against different set of factors (3 cases) over the whole sample period ranging from January 1997 to October 2015.

	CA	CTA	DS	EM	EMN	ED	FIA	GM	LSE	MA	RV	SS	FoF
Case 1: 19 Factors	56	31	71	85	32	77	50	58	81	39	70	81	77
Case 2: Traditional Factors (except Emerging Market)	49	12	42	52	-2	55	35	25	64	22	55	60	46
Case 3: Economic Factors (see Table 1)	54	28	52	80	16	63	28	50	71	31	56	73	68

Table 3: Out-of-Sample Adjusted R-squared and Annualised Root Mean Squared Error for empirical data
This table reports for each hedge fund strategy the out-of-sample adjusted R-squared and the root mean squared error of the corresponding rolling-window and Kalman filter clones relative to its bespoke relative subset of economically identified factors in table 1 over the period ranging from January 1999 to October 2015.

	HF clone Rolling-Window Out-of-Sample Adjusted R-Squared (%)	HF clone Kalman Filter Out-of-Sample Adjusted R-Squared (%)	HF Clone Rolling Window RMSE (%)	HF Clone Kalman Filter RMSE (%)
CA	38	47	4.8	4.4
CTA	-13	8	8.5	7.6
DS	29	-1	4.9	5.8
EM	81	77	4.6	5.0
EMN	-4	-8	2.8	2.8
ED	48	20	4.0	5.0
FIA	21	31	3.3	3.1
GM	26	-23	4.0	5.2
LSE	57	58	4.6	4.5
MA	-4	20	3.2	2.8
RV	46	49	3.0	2.9
SS	74	71	7.8	8.3
FoF	60	37	3.4	4.7

Table 4: Sharpe ratios for empirical data
This table shows for each hedge fund strategy the annualised Sharpe ratios (annualised return in excess of the risk-free rate divided by the annualised volatility of monthly excess returns) of the corresponding rolling-window and Kalman filter clones and of the corresponding equal risk contribution and equally-weighted portfolios relative to its bespoke subset of economically identified risk factors in table 1. The period considered is the out-of-sample period ranging from January 1999 to October 2015.

	HF clone Rolling Window	HF clone Kalman Filter	Equal Risk Risk Contribution Portfolio	Equal Weight Portfolio
CA	0.56	0.48	1.21	1.13
CTA	0.42	0.57	0.55	0.37
DS	0.16	0.17	0.74	0.63
EM	0.42	0.33	0.25	0.40
EMN	0.47	0.74	1.02	0.96
ED	0.27	0.18	0.74	0.63
FIA	0.05	0.22	-0.25	0.37
GM	0.32	0.53	0.74	0.63
LSE	0.09	0.26	1.02	0.96
MA	0.32	0.39	1.02	0.96
RV	0.38	0.35	0.74	0.63
SS	-0.01	0.03	1.02	0.96
FoF	0.19	0.20	0.74	0.63

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Table 5: Overview of the focus and findings of the literature on hedge fund performance replication
This table represents a non-exhaustive summary of the main findings of the academic literature concerning hedge fund performance replication.It focuses on the methods, the risk factors used and the conclusions of the authors.

Authors	Year	Data	Method	Factors (proxies)	Conclusion
Fung and Hsieh	2004	HFR FoF index, from Jan-94 to Dec-02, monthly returns	OLS linear regression	**7 factors:** 2 equity factors (S&P, SC-LC): the stock market (S&P 500) and the difference, the SMB Fama-French factor (difference between returns on Wilshire 1750 Small Cap and Wilshire 750 Large Cap) 2 fixed income factors (10Y, Cred Spr): change in the 10Y US T-Bond and spread between Moody's Baa bonds ans US T-Bond 3 option factors (Bd Opt, FX Opt, Com Opt): lookback straddles on bonds, on currencies and on commodities.	The adjusted R-squared for the model is 0,69 between January 1994 to September 1998 and 0,80 between April 2000 and December 2002. Significant variables on 1% significance level are S&P, SC-LC and Cred Spr for the first period; S&P, SC-LC and 10Y for the second period. The model has an adjusted R-squared of 69%.
Hasanhodzic and Lo	2007	1610 hedge funds from TASS Hedge Fund Live database, from Feb-86 to Sep-05, monthly returns	OLS linear regression	**6 factors:** 1 equity factor (SP500): the S&P 500 total return 1 bond factor (BOND): the return on the Lehman Corporate AA Intermediate Bond Index 1 credit factor (CREDIT): the spread between the Lehman BAA Corporate Bond Index and the Lehman Treasury Index 1 currency factor (USD): the US Dollar Index return 1 commodity factor (GSCI): the GSCI Index total return 1 volatility factor (DVIX): the first difference of the end-of-month value of CBOE VIX.	Matching of the hedge fund returns' higher moments is done at the expense of a higher volatility of the tracking error of the clone. No clear evidence of nonlinearities relative to hedge fund returns.
Roncalli and Weisang	2008	HFRI Fund Weighted Composite, from Jan-94 to Sep-08	Bayesian approaches: Kalman filter (Gaussian case) and particle filter (Non Gaussian case). Linear and non-linear assets.	**6 factors:** 1 equity factor (SP500): the S&P 500 total return 3 long/short factors (RTY/SPX, SX5E/SPX, TPX/SPX): between Russell 2000 and S&P500, between DJ Eurostoxx 50 and S&P500 and between Topix and S&P500 1 bond factor (UST): a bond position in the US 10Y-Treasury 1 currency factor (USD): an FX position in the EUR/USD	Matching of the hedge fund returns' higher moments is done at the expense of a higher volatility of the tracking error of the clone. No clear evidence of nonlinearities relative to hedge fund returns.
Amenc, Martellini, Meyfredi and Ziemann	2010	1610 hedge funds from TASS Hedge Fund Live database, from Jan-99 to Dec-06, monthly returns	OLS linear regression; conditional linear and option models, Markov-Switching models and Kalman filter	**5 factors (Halihodzic and Lo's factor without the volatility factor):** 1 equity factor (SP500): the S&P 500 total return 1 bond factor (BOND): the return on the Lehman Corporate AA Intermediate Bond Index 1 credit factor (CREDIT): the spread between the Lehman BAA Corporate Bond Index the Lehman Treasury Index 1 currency factor (USD): the US Dollar Index return 1 commodity factor (GSCI): the GSCI Index total return **+ 6 Economic factors:** the Small/Large spread (proxied by the return differential between the S&P 600 Small Cap index and the S&P 500 Composite index) the FC Emerging Markets index, the Merrill Lynch 300 Global Convertible Bond index, the Default spread (proxied by the return differential between Lehman US Aggregate Intermediate Credit BAA and the Lehman US Aggregate Intermediate AAA indices), the Mortgage spread (modeled by the excess return of the GNMA index over the Lehman US Treasury Bill index)	Evidences of non linear risk exposures o f hedge funds to different risk factors, better out of sample results than with a standard linear regression.
Asness, Ilmanen, Israel and Moskowitz	2015	"HFRI and HFR sub-indices, 1 value factor from 1990 to 2013	OLS linear regression	**6 multi-assets/multi-styles factors:** 1 market factor 1 market lagged factor 1 momentum factor 1 carry factor 1 defensive factor	Innovative approach with style regressors. for the sub-indices. R-squared of 65% for HFRI and between 23% and 63%

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Table 6: Hedge fund strategy correlations
Statistics are estimated from monthly returns over the period Jan. 1999-Oct. 2015. CA refers to Convertible Arbitrage, CTA to CTA Global, DS to Distressed Securities, EM to Emerging Markets, EMN to Equity Market Neutral, ED to Event Driven, FIA to Fixed Income Arbitrage, GM to Global Macro, LSE to Long/Short Equity, MA to Merger Arbitrage, RV to Relative Value, SS to Short Selling and FoF to Fund of Funds.

	CA	CTA	DS	EM	EMN	ED	FIA	GM	LSE	MA	RV	SS	FoF
CA	100	1	73	62	50	74	84	40	60	59	85	-37	64
CTA		100	5	11	19	7	6	60	11	8	5	7	23
DS			100	80	60	93	73	56	79	63	85	-61	81
EM				100	55	84	63	73	87	61	83	-69	88
EMN					100	64	48	52	64	53	62	-35	67
ED						100	69	63	90	81	92	-69	88
FIA							100	43	55	50	80	-33	63
GM								100	74	50	59	-49	80
LSE									100	73	85	-80	92
MA										100	76	-50	71
RV											100	-61	83
SS												100 -	69
FoF													100

Table 7: Summary statistics on hedge fund strategies
Statistics are estimated from monthly returns over the period Jan. 1999-Oct. 2015. Average excess return is the average of the difference between the monthly returns of the index and that of the risk-free asset. Average excess returns and volatilities are annualised. The Sharpe ratio is the ratio of the annualised average excess return over the annualised volatility of the excess returns.

	Average excess return (%)	Volatility (%)	Sharpe ratio	Maximum Drawdown (%)
CA	5.1	6.2	0.82	28
CTA	3.2	8.1	0.40	12
DS	7.1	5.9	1.22	22
EM	7.5	10.6	0.71	35
EMN	3.5	2.8	1.33	11
ED	6.0	5.7	1.05	20
FIA	4.1	3.8	1.07	17
GM	4.6	4.8	0.97	8
LSE	5.3	7.1	0.75	21
MA	4.3	3.1	1.43	6
RV	5.3	4.2	1.28	16
SS	-4.2	15.6	-0.27	61
FoF	3.3	5.4	0.62	20

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Table 8: Factors and risk-free asset correlations

Correlations (%) are estimated from monthly returns over the period Jan. 1999-Oct. 2015. In line with Table 1, EQ stands for Equity, FX for Currency, CO for Commodity, EM for Emerging Market, FI for Bond, CRE for Credit, GM for Multi-Class Global Momentum, GV for Multi-Class Global Value, EQM for Equity Global Momentum, FXM for Currency Global Momentum, FIM for Fixed Income Global Momentum, COM for Commodity Global Momentum, EQV for Equity Global Value, FXV for Currency Global Value, FIV for Fixed Income Global Value, COV for Commodity Global Value, EQD for Equity Global Defensive, EQQ for Equity Global Quality and EQS for Equity Global Size. RF stands for the Risk-Free Asset, proxied by the US 3-month Treasury bill index.

	EQ	FX	CO	EM	FI	CRE	GMO	GVA	EQM	FXM	FIM	COM	EQV	FXV	FIV	COV	EQD	EQQ	EQS	RF
EQ	100	-32	29	77	-31	19	-32	6	-31	-2	-10	-2	22	-20	5	6	-37	-70	13	-7
FX		100	-44	-43	-14	-30	8	-6	9	4	-4	-1	-36	6	5	15	-22	29	-14	0
CO			100	44	-13	8	4	-12	2	-3	4	20	26	-15	13	-49	14	-32	18	14
EM				100	-25	28	-21	-2	-19	1	0	-5	42	-29	3	-2	-24	-75	35	3
FI					100	56	18	-9	23	-4	32	4	-11	1	-41	-6	15	30	-15	7
CRE						100	-3	-2	-3	8	20	4	20	-23	-27	-5	11	-23	14	-4
GMO							100	-70	68	45	30	46	-27	-28	-14	-25	26	27	-7	5
GVA								100	-44	-19	-8	-32	39	30	12	34	11	6	-10	3
EQM									100	28	15	27	-34	-17	-23	-18	23	31	0	2
FXM										100	15	8	-12	-49	-10	5	3	5	-4	0
FIM											100	17	-2	-21	-23	-9	15	11	-9	4
COM												100	-13	-13	0	-61	12	3	-2	14
EQV													100	-6	9	0	13	-36	18	8
FXV														100	9	7	-13	11	0	-8
FIV															100	-6	-3	-15	19	8
COV																100	-21	5	-14	-15
EQD																	100	41	-7	-2
EQQ																		100	-57	3
EQS																			100	-5
RF																				100

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Table 9: Summary statistics on factors
Statistics are estimated from monthly returns over the period Jan. 1999-Oct. 2015. Average excess return is the average of the difference between the monthly returns of the index and that of the risk-free asset. Average excess return and returns volatility are annualised. The Sharpe ratio is the ratio of the annualised average excess return over the annualised volatility of the excess returns.

	Average excess return (%)	Volatility (%)	Sharpe ratio	Maximum Drawdown (%)
EQ	4.3	15.1	0.28	49%
FX	-1.4	8.3	-0.17	39%
CO	2.6	23.5	0.11	69%
EM	9.6	22.9	0.42	57%
FI	2.8	4.5	0.63	5%
CRE	3.8	5.5	0.68	15%
GM	1.9	8.4	0.22	22%
GV	0.1	6.8	0.01	29%
EQM	0.7	9.3	0.07	15%
FXM	0.1	7.9	0.02	21%
FIM	-1.8	2.8	-0.64	10%
COM	6.9	19.3	0.36	40%
EQV	-0.4	8.0	-0.05	37%
FXV	-0.7	7.4	-0.09	22%
FIV	-1.4	2.4	-0.57	8%
COV	-2.9	21.2	-0.14	64%
EQD	9.3	11.1	0.83	31%
EQQ	3.7	8.9	0.41	23%
EQS	0.5	6.8	0.07	25%

Table 10: Kalman filter clones out-of-sample adjusted R-squared and RMSE
This table reports for each hedge fund strategy the out-of-sample adjusted R-squared and the root mean squared error of the corresponding Kalman filter clones (with heuristic and look-ahead calibrations) over the period ranging from January 1999 to October 2015.

	HF clone Kalman Filter Out-of-Sample Adjusted R-Squared (%) (heuristic calibration)	HF clone Kalman Filter Out-of-Sample Adjusted R-Squared (%) (look-ahead calibration)	HF Clone Kalman Filter RMSE (%) (heuristic calibration)	HF Clone Kalman Filter RMSE (%) (look-ahead calibration)
CA	47	53	4.4	4.2
CTA	8	15	7.6	7.3
DS	-1	33	5.8	4.7
EM	77	82	5.0	4.5
EMN	-8	5	2.8	2.7
ED	20	48	5.0	4.0
FIA	31	22	3.1	3.3
GM	-23	31	5.2	3.9
LSE	58	68	4.5	4.0
MA	20	24	2.8	2.7
RV	49	54	2.9	2.8
SS	71	79	8.3	7.2
FoF	37	58	4.7	3.5

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Table 11: In-sample and out-of-sample adjusted R-squared for the kitchen sink case
This table reports for each hedge fund strategy the in-sample adjusted R-squared of the kitchen sink regression over the period ranging from January 1997 to October 2015 and the out-of-sample adjusted R-squared of the corresponding rolling-window and Kalman filter clones over the period ranging from January 1999 to October 2015.

	HF In-Sample Linear Regression Adjusted R-Squared (%)	HF clone Rolling-Window Out-of-Sample Adjusted R-Squared (%)	HF clone Kalman Filter Out-of-Sample Adjusted R-Squared (%)
CA	56	-141	18
CTA	31	-423	-391
DS	71	-58	-16
EM	85	-23	-94
EMN	32	-595	-165
ED	77	-113	14
FIA	50	-293	-185
GM	58	-123	-185
LSE	81	-2	3
MA	39	-555	-55
RV	70	-49	42
SS	81	-33	-264
FoF	77	-31	-38

Table 12: Performance statistics
This table reports for each hedge fund strategy corresponding clones and weighting schemes the annualised average excess return over the risk-free asset(%), the annualised volatility of the returns(%), the Sharpe ratio defined as the annualised average excess return over the annualised volatility of the excess returns, the maximum drawdown(%) over the period and the Value at Risk 99% with a 1-month horizon over the out-of-sample period ranging from January 1999 to October 2015.

		Average excess return (%)	Volatility (%)	Sharpe ratio	Maximum Drawdown (%)	VaR 1% 1-month (%)
EMN	Rolling-Window	1.1	2.4	0.47	12.4	-3.0
	Kalman filter	1.6	2.1	0.74	7.7	-2.8
	Equal Risk	2.2	2.2	1.02	2.6	-1.8
	Equal Weight	3.0	3.1	0.96	11.5	-3.8
LSE	Rolling-Window	0.6	7.0	0.09	34.7	-10.0
	Kalman filter	1.7	6.4	0.26	32.2	-10.4
	Equal Risk	2.2	2.2	1.02	2.6	-1.8
	Equal Weight	3.0	3.1	0.96	11.5	-3.8
MA	Rolling-Window	1.1	3.4	0.32	20.7	-5.5
	Kalman filter	1.2	3.0	0.39	17.2	-5.0
	Equal Risk	2.2	2.2	1.02	2.6	-1.8
	Equal Weight	3.0	3.1	0.96	11.5	-3.8
SS	Rolling-Window	-0.2	17.5	-0.01	57.2	-17.8
	Kalman filter	0.6	18.7	0.03	51.1	-17.9
	Equal Risk	2.2	2.2	1.02	2.6	-1.8
	Equal Weight	3.0	3.1	0.96	11.5	-3.8
DS	Rolling-Window	0.8	5.1	0.16	19.7	-5.3
	Kalman filter	1.1	6.7	0.17	17.9	-9.4
	Equal Risk	1.8	2.4	0.74	5.0	-2.1
	Equal Weight	3.7	5.9	0.63	19.4	-7.7

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Table 12 (continued)

		Average excess return (%)	Volatility (%)	Sharpe ratio	Maximum Drawdown (%)	VaR 1% 1-month (%)
ED	Rolling-Window	1.4	5.2	0.27	19.9	-6.2
	Kalman filter	1.1	6.3	0.18	20.3	-8.8
	Equal Risk	1.8	2.4	0.74	5.0	-2.1
	Equal Weight	3.7	5.9	0.63	19.4	-7.7
GM	Rolling-Window	1.5	4.6	0.32	10.8	-4.1
	Kalman filter	3.0	5.6	0.53	10.6	-5.7
	Equal Risk	1.8	2.4	0.74	5.0	-2.1
	Equal Weight	3.7	5.9	0.63	19.4	-7.7
RV	Rolling-Window	1.5	3.9	0.38	14.1	-5.2
	Kalman filter	1.5	4.2	0.35	15.9	-6.7
	Equal Risk	1.8	2.4	0.74	5.0	-2.1
	Equal Weight	3.7	5.9	0.63	19.4	-7.7
FoF	Rolling-Window	0.9	4.9	0.19	17.1	-6.0
	Kalman filter	1.1	5.3	0.20	17.2	-8.0
	Equal Risk	1.8	2.4	0.74	5.0	-2.1
	Equal Weight	3.7	5.9	0.63	19.4	-7.7
CA	Rolling-Window	3.0	5.4	0.56	23.8	-7.4
	Kalman filter	2.3	4.8	0.48	21.2	-8.9
	Equal Risk	2.4	2.0	1.21	1.9	-1.1
	Equal Weight	3.1	2.7	1.13	7.9	-3.6
CTA	Rolling-Window	3.1	7.4	0.42	12.0	-7.5
	Kalman filter	3.6	6.3	0.57	10.8	-5.1
	Equal Risk	1.9	3.4	0.55	6.9	-2.5
	Equal Weight	2.2	6.0	0.37	20.8	-5.6
EM	Rolling-Window	4.0	10.4	0.39	29.6	-12.4
	Kalman filter	3.1	10.4	0.30	29.7	-12.1
	Equal Risk	1.7	6.9	0.25	17.7	-5.7
	Equal Weight	4.1	10.4	0.40	29.6	-9.8
FIA	Rolling-Window	0.2	3.5	0.05	14.7	-7.5
	Kalman filter	0.8	3.5	0.22	13.7	-7.1
	Equal Risk	-0.4	1.6	-0.25	3.7	-2.1
	Equal Weight	0.9	2.3	0.37	4.4	-2.3

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Table 13: Diversification statistics
This table reports for each hedge fund strategy corresponding clones and weighting schemes the Average Effective Number of Constituents (%) and the Average Effective Number of Uncorrelated Bets over the out-of-sample period ranging from January 1999 to October 2015.

		Average Effective Number of Constituents (%)	Average Effective Number of Bets (%)
EMN	Rolling-Window	36	58
	Kalman filter	63	52
	Equal Risk	78	92
	Equal Weight	100	66
LSE	Rolling-Window	28	54
	Kalman filter	38	45
	Equal Risk	78	92
	Equal Weight	100	66
MA	Rolling-Window	40	47
	Kalman filter	38	46
	Equal Risk	78	92
	Equal Weight	100	66
SS	Rolling-Window	3	49
	Kalman filter	6	52
	Equal Risk	78	92
	Equal Weight	100	66
DS	Rolling-Window	13	61
	Kalman filter	6	66
	Equal Risk	64	97
	Equal Weight	100	53
ED	Rolling-Window	17	58
	Kalman filter	9	63
	Equal Risk	64	97
	Equal Weight	100	53
GM	Rolling-Window	23	53
	Kalman filter	16	55
	Equal Risk	64	97
	Equal Weight	100	53
RV	Rolling-Window	25	56
	Kalman filter	14	60
	Equal Risk	64	97
	Equal Weight	100	53
FoF	Rolling-Window	18	54
	Kalman filter	13	65
	Equal Risk	64	97
	Equal Weight	100	53
CA	Rolling-Window	12	55
	Kalman filter	22	66
	Equal Risk	78	91
	Equal Weight	100	68

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Table 13 (continued)

		Average Effective Number of Constituents (%)	Average Effective Number of Bets (%)
CTA	Rolling-Window	10	57
	Kalman filter	11	49
	Equal Risk	60	97
	Equal Weight	100	61
EM	Rolling-Window	64	47
	Kalman filter	64	58
	Equal Risk	92	100
	Equal Weight	100	79
FIA	Rolling-Window	25	45
	Kalman filter	12	42
	Equal Risk	75	99
	Equal Weight	100	71

Table 14: Turnover
This table reports for each hedge fund strategy corresponding clones and weighting schemes the annualised 1-way turnover(%) over the out-of-sample period ranging from January 1999 to October 2015.

		Annualised 1-way Turnover (%)
EMN	Rolling-Window	1576
	Kalman filter	195
	Equal Risk	55
	Equal Weight	12
LSE	Rolling-Window	1460
	Kalman filter	106
	Equal Risk	55
	Equal Weight	12
MA	Rolling-Window	830
	Kalman filter	435
	Equal Risk	55
	Equal Weight	12
SS	Rolling-Window	6973
	Kalman filter	29935
	Equal Risk	55
	Equal Weight	12
DS	Rolling-Window	1488
	Kalman filter	501
	Equal Risk	60
	Equal Weight	13
ED	Rolling-Window	6322
	Kalman filter	379
	Equal Risk	60
	Equal Weight	13

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Table 14 (continued)

		Annualised 1-way Turnover (%)
GM	Rolling-Window	2337
	Kalman filter	466
	Equal Risk	60
	Equal Weight	13
RV	Rolling-Window	1744
	Kalman filter	1114
	Equal Risk	60
	Equal Weight	13
FoF	Rolling-Window	2309
	Kalman filter	421
	Equal Risk	60
	Equal Weight	13
CA	Rolling-Window	7149
	Kalman filter	732
	Equal Risk	62
	Equal Weight	11
CTA	Rolling-Window	230
	Kalman filter	230
	Equal Risk	79
	Equal Weight	15
EM	Rolling-Window	634
	Kalman filter	444
	Equal Risk	29
	Equal Weight	18
FIA	Rolling-Window	9941
	Kalman filter	4054
	Equal Risk	36
	Equal Weight	4

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- A tireless quest for tailor-made, flexible solutions

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- Extensive experience in managing risk and volatility, options and hedge funds

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- Constant monitoring of pricing, valuations, investment limits and market risks

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239 ETFs listed on 13 regulated exchanges
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About EDHEC-Risk Institute

About EDHEC-Risk Institute

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This institute now boasts a team of close to 50 permanent professors, engineers and support staff, as well as 38 research associates from the financial industry and affiliate professors.

The Choice of Asset Allocation and Risk Management and the Need for Investment Solutions

EDHEC-Risk has structured all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds. EDHEC-Risk Institute has also developed an ambitious portfolio of research and educational initiatives in the domain of investment solutions for institutional and individual investors.

Academic Excellence and Industry Relevance

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute's advisory board. This board is made up of internationally recognised researchers, the Institute's business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:

- Asset allocation and alternative diversification
- Performance and risk reporting
- Indices and benchmarking
- Non-financial risks, regulation and innovations
- Asset allocation and derivative instruments
- ALM and asset allocation solutions

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in Singapore, which was established at the invitation of the Monetary Authority of Singapore (MAS); the City of London in the United Kingdom; Nice and Paris in France.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- ETF and Passive Investment Strategies, *in partnership with Amundi ETF*
- Regulation and Institutional Investment, *in partnership with AXA Investment Managers*
- Asset-Liability Management and Institutional Investment Management, *in partnership with BNP Paribas Investment Partners*
- New Frontiers in Risk Assessment and Performance Reporting, *in partnership with CACEIS*
- Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, *in partnership with CME Group*

About EDHEC-Risk Institute

• Asset-Liability Management Techniques for Sovereign Wealth Fund Management, *in partnership with Deutsche Bank*
• The Benefits of Volatility Derivatives in Equity Portfolio Management, *in partnership with Eurex*
• Structured Products and Derivative Instruments, *sponsored by the French Banking Federation (FBF)*
• Optimising Bond Portfolios, *in partnership with the French Central Bank (BDF Gestion)*
• Risk Allocation Solutions, *in partnership with Lyxor Asset Management*
• Infrastructure Equity Investment Management and Benchmarking, *in partnership with Meridiam and Campbell Lutyens*
• Risk Allocation Framework for Goal-Driven Investing Strategies, *in partnership with Merrill Lynch Wealth Management*
• Investment and Governance Characteristics of Infrastructure Debt Investments, *in partnership with Natixis*
• Advanced Modelling for Alternative Investments, *in partnership with Société Générale Prime Services (Newedge)*
• Advanced Investment Solutions for Liability Hedging for Inflation Risk, *in partnership with Ontario Teachers' Pension Plan*
• Active Allocation to Smart Factor Indices, *in partnership with Rothschild & Cie*
• Solvency II, *in partnership with Russell Investments*
• Structured Equity Investment Strategies for Long-Term Asian Investors, *in partnership with Société Générale Corporate & Investment Banking*

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and global conferences.

To ensure the distribution of its research to the industry, EDHEC-Risk also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international risk and asset management research. The website, which has more than 70,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC-Risk's analysis and expertise in the area of applied portfolio management research. Its quarterly newsletter is distributed to more than 200,000 readers.

EDHEC-Risk Institute: Key Figures, 2014-2015

Number of permanent staff	48
Number of research associates & affiliate professors	36
Overall budget	€6,500,000
External financing	€7,025,695
Nbr of conference delegates	1,087
Nbr of participants at research seminars and executive education seminars	1,465

About EDHEC-Risk Institute

Research for Business

The Institute's activities have also given rise to executive education and research service offshoots. EDHEC-Risk's executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of asset-liability management for institutions and individuals, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of risk and investment management.

As part of its policy of transferring know-how to the industry, in 2013 EDHEC-Risk Institute also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.

EDHEC-Risk Institute Publications and Position Papers (2013-2016)

EDHEC-Risk Institute Publications (2013-2016)

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2014

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