
An EDHEC-Risk Institute Publication

Factor Investing in Fixed-Income – Defining and Exploiting Value in Sovereign Bond Markets

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Foreword

This paper has been produced as part of the "ETF, Indexing and Smart Beta Investment Strategies" Research Chair at EDHEC-Risk Institute, in partnership with Amundi.

Value has been recognised as one of the most important factors for equities since the pioneering work by Fama and MacBeth (1973). In equities, the book-to-market value ratio has traditionally been used as a proxy for the value factor. Natural as this choice is for this asset class, it is difficult to translate the concept of value to the fixed-income domain.

In this paper, "Factor Investing in Fixed-Income – Defining and Exploiting Value in Sovereign Bond Markets", we propose a definition of value in Treasury bonds that, we believe, is more satisfactory than definitions found in the recent literature, and that allows statistically significant and economically relevant predictions of cross-sectional excess returns. Our value pricing factor exploits the differences between the market and the theoretical values of Treasury bonds, where the theoretical value is assessed using an economically-justifiable Gaussian dynamic term structure model.

We show that the profitability of the strategy we build using our value signal is closely linked to Treasury market volatility, and we provide an explanation for this strong link using arguments similar to those that can be found in the recent literature on liquidity in Treasuries.

In a companion paper, we undertake a systematic, security-level analysis of momentum and reversal strategies in US Treasuries covering more than 40 years of data. We find that, after adjusting for duration, the long/short (zero-cost) reversal cross-sectional strategy is profitable over a wider range of look-back and investment periods. This strategy can be adapted to a portfolio (long/only) context.

I would like to thank Riccardo Rebonato and Jean-Michel Maeso for their leadership in this research effort, and Laurent Ringelstein and Dami Coker for their efforts in producing the final publication.

I would also like to extend particular thanks to Amundi for their support of this research chair.

We wish you a useful and informative read.



Lionel Martellini
Professor of Finance,
Director of EDHEC-Risk Institute

ABOUT THE AUTHORS



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Riccardo Rebonato was previously Global Head of Rates and FX Research at PIMCO. He also served as Head of Front Office Risk Management and Head of Clients Analytics, Global Head of Market Risk and Global Head of Quantitative Research at Royal Bank of Scotland (RBS). Prior to joining RBS, he was Head of Complex IR Derivatives Trading and Head of Derivatives Research at Barclays Capital. Riccardo Rebonato has served on the Board of ISDA (2002-2011), and has been on the Board of GARP since 2001. He was a visiting lecturer in Mathematical Finance at Oxford University (2001-2015). He is the author of several books, in particular having published extensively on interest rate modelling, risk management, and most notably books on SABR/LIBOR Market Model pricing of interest rate derivatives, as well as on the use of Bayesian nets for stress testing and asset allocation. He has published articles in international academic journals such as *Quantitative Finance*, the *Journal of Derivatives* and the *Journal of Investment Management*, and has made frequent presentations at academic and practitioner conferences.

1. Introduction and Motivation

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Value has been recognised as one of the most important factors for equities since the pioneering work by Fama and MacBeth (1973). In equities, the book-to-market value ratio has traditionally been used as a proxy for the value factor. Natural as this choice is for this asset class, it is difficult to translate the concept of value to the fixed-income domain, and, for this reason, Fama and French (1993) have argued that value does not apply to fixed-income instruments in general, and to Treasury bonds in particular¹.

This seems to be at odds with recent literature, which claims to have found value (and momentum) "everywhere". For instance Asness, Moskowitz and Pedersen (2013) have defined value for bonds as the (negative of the) 5-year bond returns – a choice motivated by the observation that, in equities, this difference in returns is found to be positively correlated with the book-to-market ratio². The factor thus defined may well predict future bond returns, but its interpretation as "value" seems at least stretched, and one, if not two, steps removed from the true latent underlying factor. At best, it plays the role of a proxy of a proxy, and, as a result, the labelling of the chosen measure as value becomes rather arbitrary.

In this paper, we provide what we believe is a more intuitive and satisfactory definition of value in US Treasury bonds, and we show that the value quantity we define has very strong predictive power of future cross-sectional Treasury returns. More precisely, we identify "cheap" ("valuable") and "expensive" bonds using a dynamic Gaussian term structure model, and show that a systematic, no-peek-ahead strategy of investing in the cheap

and shorting the expensive bonds has a strongly positive Sharpe ratio. Our results are so robust that, before and after adjusting for duration exposure, the strategy we propose has positive Sharpe ratios in 14 out of the 15 3-year periods from 1975 to 2017, a Sharpe ratio which is statistically significantly different from zero at the 99.9% confidence level in 13 of the three-year sub-periods out of 15, and an average Sharpe ratio (before transaction costs) above 1.

1.1 In Which Ways Is Our Approach Different?

As mentioned, we make our assessment of cheapness or expensiveness with respect to an economically motivated affine dynamic term structure model. We justify our choice of model in Section 3, but for the moment we stress that our strategy is different from those often encountered in the industry. In market practice, highly flexible fitting models (such as the popular Nelson-Siegel, 1987, model), devoid of economic content or justification, are used to obtain very tight fits to market prices, and their pricing errors are interpreted as indicators of cheapness or expensiveness of individual bonds (or, as in the case of Hu, Pan and Wang, 2013, of market liquidity). Since the industry fitting models are devoid of economic interpretability, they just implicitly enforce a criterion of statistical smoothness for the par-coupon curve, and do not convey information about the economic value of any individual bond. What we do is different: we estimate what the value of a bond "should" be (given a simple but reasonable arbitrage-free model) and exploit differences from this economically-motivated value. By constraining

1 - "... explanatory variables like size and book-to-market equity have no obvious meaning for government and corporate bonds. . . " – Fama and French, 1993

2 - "...[we] show that individual stock portfolios formed from the negative of past 5-year returns are highly correlated with those formed on BE/ME ratios in our sample. [. . .] Hence, using past 5-year returns to measure value seems reasonable. . . ." This being the case, their "value" factor can arguably be better understood as a long-term reversal factor.

1. Introduction and Motivation

the possible values of the model parameters and state variables to ranges consistent with the economic interpretation of our model we obtain a worse fit than the "flexible" fitting models can achieve, but we can endow the pricing errors with the meaning of indications of where the price should have been (given our model). So, obtaining a tight fit to market prices is not our goal; obtaining possibly large, but informative, pricing errors is – and, indeed, we show in Section 7 that the strategy *makes more money when the fit is poor, not when it is tight*.

We focus instead on the pricing information embedded in these pricing errors...

What is the economic origin of the profitability of our strategy? The most natural explanation points to prices of individual bonds straying away from fundamentals, and reverting to where they should be with a characteristic reversion speed. In this respect, our results are in line with the findings in Hu, Pan and Wang (2012), who carry out an analysis similar to ours using the Nelson-Siegel (1987) model, and find that the largest pricing errors occur in periods of low market liquidity. They justify this finding by arguing that, in periods of market turmoil, arbitrage capital, which would normally iron away price deviations from fundamentals, is less forthcoming, and this allows the price discrepancies to appear in the first place, and to persist until liquidity is restored. The focus of the work by Hu, Pan and Wang is on liquidity (indeed, the title of their paper is "*Noise as*

Information for Illiquidity"), and therefore they do not explore the return predictability implications of their findings. We focus instead on the pricing information embedded in these pricing errors, but the conceptual framework is otherwise quite similar.

The procedure we employ to capture the "value of value" in Treasury bonds appears *prima facie* very similar to the procedure usually employed in the traditional equity factor studies: a zero-cost portfolio is set up by buying securities that load³ positively on (a proxy for) the chosen factor and selling those securities that load negatively on the factor, and the excess returns of securities in different percentiles are compared. (In the traditional studies, the investment universe is usually split in percentiles – we adopt a continuous rather than discrete partitioning of the bonds, but the approach is otherwise very similar). Also in common with traditional studies, we test a joint hypothesis, ie, i) that the chosen proxy correctly captures the underlying factor, and ii) that the factor is indeed priced.

Despite these formal similarities, our approach is, conceptually, rather different. In the usual factor studies, the underlying working assumption is that the long portfolios attract extra return over the short ones because they have exposure to a rewarded factor, for which the chosen market-mimicking portfolio is a proxy. In our study, we do not assume that investors seek compensation (in terms of a risk premium) for bearing the risk associated with a bond being more valuable than another. Rather, we posit that individual bonds temporarily move out of line with their fundamental value, but revert to it with an

3 - The use of the word "load" might be a bit misleading here since our factor is not defined exogenously as a time-series factors with respect to which we estimate a beta for each security, but rather through an observable (or at least computable) characteristic. This slight abuse of language is common to the factor investing literature, and we use the term "load" to conform to the common usage.

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exploitable reversion speed. So, in our universe, level risk may well be rewarded via the (slope-dependent) market price of risk (as in the early studies by Fama and Bliss, 1987, Campbell and Shiller, 1991). However, in our model, an extra source of predictability comes from temporary and reversible deviations from fundamentals. Cieslak and Povala (2010) and Radwansky (2010) indeed give this interpretation to the additional predictability afforded by the Cochrane-Piazzesi and Cieslak-Povala factors.

If our interpretation is correct, the value attribute that we uncover is therefore not truly a proxy for a priced factor, in the sense that our long-short portfolio does not load positively and negatively, respectively, on bonds that have a positive or negative exposure to this latent priced factor: our strategy is not profitable because it exploits the attending market price of risk, but because it exploits temporary market inefficiencies.

We mentioned that the recent literature on predictability of excess returns in Treasuries (such as the work by Cieslak and Povala, 2010) invokes a similar explanation in terms of mean-reverting deviations from fundamentals to explain the higher predictive power of the new-generation return-predicting factors – see, in this respect, also the work by Rebonato and Hatano (2018). We stress, however, that the deviations from fundamentals identified by a return-predictive factor such as the one by Cieslak and Povala by construction refer to deviations of the overall level of the yield curve from where it should be. In our study, also by construction, we cannot identify deviations in the overall level of rates from its fundamental value, but only deviations

of higher principal components from where they should be. We discuss this point at greater length in Section 7.

The rest of the paper is organised as follows: in Section 2 we describe the data used for the analysis; in Section 3 we introduce the affine dynamic term structure model used, we explain how we have calibrated it (Section 4), and we discuss the reasonableness of the derived time series of the model degrees of freedom (Section 5). In Section 6 we explain how the trading signal is generated, and we present and discuss our results in Section 7. In Section 8 we look at the profitability of a long-only version of the strategy and our conclusions are presented in Section 9.

2. The Data

2. The Data

The data used for the study is made up of the daily close-of-business-day prices of 1,562 US Treasury coupon bonds over the period 27 December 1973 to 29 June 2018.⁴ All these bonds are non-callable, non-puttable and non-inflation-linked. We also excluded from the data-set prices of individual bonds that were deemed to be erroneous. The exclusion was determined by setting a threshold in standard deviations for the price changes, and then excluding those bonds with price moves that exceeded the threshold *while the other bonds in the universe for that day did not show a similar move*. We stress that this culling procedure is conservative because spurious spikes would generate fictitious profits: we therefore prefer to miss a true sharp price deviation / reversal than to include a fake one.

⁴ - We thank ICE for providing us with the data-set used for our empirical analysis.

3. The Model

3. The Model

In this section, we provide a description and interpretation of the model starting from the real-world \mathbb{P} measure (Section 3.1); we discuss which constraints its parameters should satisfy if this interpretation is to hold (Section 3.2); we generalise the model formulation for ease of treatment (Section 3.3); and finally we provide in Section 3.4 a link between the model formulation in the \mathbb{P} and the \mathbb{Q} measures; this link will become essential for the calibration phase, discussed in Section 4.

3.1 Description and Interpretation of the Model

The affine model we employ can be defined in the physical and risk-neutral measures. Starting from the \mathbb{P} measure, it can be written as

$$dr_t = \kappa_r^{\mathbb{P}} (\theta_t^{\mathbb{P}} - r_t) dt + \sigma_r dw_r \quad (1)$$

$$d\theta_t = \kappa_\theta^{\mathbb{P}} (\theta_\theta^{\mathbb{P}} - \theta_t) dt + \sigma_\theta dw_\theta \quad (2)$$

$$\mathbb{E} [dw_r dw_\theta] = \rho dt \quad (3)$$

As written, the model can be interpreted as describing the actions of the monetary authorities who respond to deviations of the inflation and / or output gap from their desired target levels by adjusting the Fed Funds rate (in our model, the "short rate") towards the long-term NAIU-compatible⁵ nominal rate (the ultimate reversion level $\theta_\theta^{\mathbb{P}}$); they do so, however, with a degree of urgency (of "aggressiveness") that depends on the economic conditions of the moment; the adjustment is therefore achieved by letting the short rate revert to a time-dependent reversion level, which in turn reverts towards

the unchanging NAIU-compatible long-term nominal rate.

In moving from the physical to the risk-neutral measure, we assume that investors only seek compensation for level risk (see, in this respect, Cochrane and Piazzesi, 2005, Adrian, Crump and Moench, 2013), and therefore modify the \mathbb{P} -measure dynamics in Equations (1-3) to

$$dr_t = \kappa_r^{\mathbb{P}} (\theta_t^{\mathbb{P}} - r_t) dt + \sigma_r dw_r \quad (4)$$

$$d\theta_t = \kappa_\theta^{\mathbb{P}} (\theta_\theta^{\mathbb{P}} - \theta_t) dt + \lambda (r_t, \theta_t) \sigma_\theta dt + \sigma_\theta dw_\theta \quad (5)$$

$$\mathbb{E} [dw_r dw_\theta] = \rho dt \quad (6)$$

In general, the market price of risk could depend on both state variables. Given the discussion in the Introduction, we make the assumption that the slope of the yield curve accounts for the degree of predictability associated with the business-cycle variation of risk aversion. However, we assume (see, eg, Rebonato and Hatano (2018), Cieslak and Povala (2010)) that the additional predictability afforded by the new-generation return-predicting factors is due to deviations from fundamentals, and not to non-level rewarded risk factors. Since our approach tries to precisely capture these deviations from fundamentals, we do not add other contributions to the market price of risk other than its business-cycle/slope-related component. In addition, we show in what follows that the slope of the yield curve is strongly correlated with $\theta_t - r_t$. Therefore we posit for the market price of risk the affine form discussed in detail in Section 3.4.

⁵ - The NAIU is defined as the non-accelerating inflation rate of unemployment, ie, the unemployment rate that produces neither inflationary nor deflationary pressures.

3. The Model

3.2 Constraints on the Model

For the model to describe, under \mathbb{P} , the stylised behaviour of the monetary authorities sketched earlier, several constraints have to be satisfied. These are discussed in detail in Rebonato (2108) and Ehrekinsky (2013), but can be summarised as follows:

1. the reversion speed of the short rate to the instantaneous reversion level should be higher than the reversion speed of the instantaneous reversion level to θ_{inf} ;
2. the volatility of the short rate should be lower than the volatility of the reversion level;
3. the correlation coefficient should be positive and between, approximately, 0 and 0.4 (see, in this respect, Ehrekinski, 2013);
4. the initial value of the short rate should be very close to the Fed Funds rate;
5. the short rate volatility should be very close to the volatility of a proxy rate such as the yield of the 3-month Treasury yield.

We impose conditions 1 to 3 in the calibration phase, and we check after the calibration that conditions 4 and 5 are indeed satisfied. We describe, in the next section, how we estimate the reversion speeds in the physical and risk-neutral measures.

3.3 Generalising the Formulation of the Model

In the calibration of the model, discussed in detail in Section 4, we are going to make use of information about the covariance matrix among yields. This source of information will determine, among other quantities, the reversion speeds in the physical measure, \mathbb{P} , $\kappa_{\theta}^{\mathbb{P}}$ and $\kappa_r^{\mathbb{P}}$. However,

if the market price of risk depends on the state variables – as in our model it is reasonable to impose – the reversion speeds will change in moving across measures, and therefore we cannot assume $\kappa_r^{\mathbb{P}} = \kappa_r^{\mathbb{Q}}$. So, the yield covariance matrix observed under \mathbb{P} cannot be directly compared with the covariance matrix obtained using the reversion speed matrices, $\kappa_{\theta}^{\mathbb{Q}}$ and $\kappa_r^{\mathbb{Q}}$. We explain, in this section, how this subtle but important point is handled. To do so we slightly generalise the setting we have considered so far as follows.

Consider n state variables, $x_t^1, x_t^2, \dots, x_t^n$, which follow under \mathbb{P} a mean-reverting process of the form

$$d\vec{x}_t = \mathcal{K}^{\mathbb{P}} \left(\vec{\theta}^{\mathbb{P}} - \vec{x}_t \right) dt + S d\vec{z}_t, \quad (7)$$

under the additional affine constraint

$$r_t = u_r + \vec{g}^T \vec{x}_t. \quad (8)$$

The doubly-mean-reverting Vasicek model under \mathbb{P} has the form

$$dr_t = \kappa_r^{\mathbb{P}} (\theta_t^{\mathbb{P}} - r_t) dt + \sigma_r dw_r \quad (9)$$

$$d\theta_t = \kappa_{\theta}^{\mathbb{P}} (\theta_t^{\mathbb{P}} - \theta_t) dt + \sigma_{\theta} dw_{\theta} \quad (10)$$

$$\mathbb{E} [dw_r dw_{\theta}] = \rho dt \quad (11)$$

In order to cast it into the mold of the general affine formulation in Equation (7), we set

$$\vec{x}_t = \begin{bmatrix} \theta_t \\ r_t \end{bmatrix} \quad (12)$$

Then we have

$$\mathcal{K}^{\mathbb{P}} = \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} \quad (13)$$

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$$\vec{\theta}^{\mathbb{P}} = \begin{bmatrix} \theta_{\theta}^{\mathbb{P}} \\ \theta_r^{\mathbb{P}} \end{bmatrix} \quad (14)$$

$$S = \begin{bmatrix} s_{11} & 0 \\ s_{21} & s_{22} \end{bmatrix} \quad (15)$$

with

$$s_{11} = \sigma_{\theta} \quad (16)$$

$$s_{21}^2 + s_{22}^2 = \sigma_r^2 \quad (17)$$

$$\rho = \frac{s_{11}s_{21}}{\sigma_{\theta}\sigma_r} = \frac{\sigma_{\theta}s_{21}}{\sigma_{\theta}\sigma_r} \implies s_{21} = \sigma_r\rho \quad (18)$$

and therefore

$$\begin{aligned} s_{22}^2 &= \sigma_r^2 - \sigma_r^2\rho^2 = \sigma_r^2(1 - \rho^2) \\ \implies s_{22} &= \sigma_r\sqrt{(1 - \rho^2)}. \end{aligned} \quad (19)$$

As for the vector \vec{g}^T and the scalar u_r , we clearly have

$$\vec{g}^T = [0, 1] \quad (20)$$

$$u_r = 0. \quad (21)$$

3.4 Moving from the Real-World to the Pricing Measure

So far, the model has been introduced purely in the real-world measure. We now establish the connection between the \mathbb{P} - and the \mathbb{Q} -measure formulations of the model.

If we want to retain the essentially affine formulation, the market price of risk must display at most an affine dependence on the state variables, ie, it must have the form

$$\lambda_t = \lambda_0 + \Lambda x_t. \quad (22)$$

We assume $\lambda_0 = 0$ – see Duee (2002) for a justification of this choice. Cochrane and Piazzesi (2005), Adrian, Crump and Moench (2013) document that investors only seek compensation for bearing level risk. Given the high reversion speed of the short rate, we therefore impose that only the uncertainty about the reversion level, θ_t , should attract a risk premium. This implies that the process for the short rate should be the same under \mathbb{P} and under \mathbb{Q} . Finally, we require that the market price of risk should depend on the slope of the yield curve (Fama and Bliss (1987), Campbell and Shiller(1991)). We show in Appendix II that, after enforcing these assumptions, the reversion speeds matrices under \mathbb{P} and under \mathbb{Q} must be related by the expression

$$\begin{aligned} \mathcal{K}^{\mathbb{Q}} &= \mathcal{K}^{\mathbb{P}} - S\Lambda = \\ &= \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} - \begin{bmatrix} \sigma_{\theta}a & -\sigma_{\theta}a \\ \sigma_r\rho a & -\sigma_r\rho a \end{bmatrix} = \\ &= \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} - \sigma_{\theta}a & \sigma_{\theta}a \\ -\kappa_r^{\mathbb{P}} - \sigma_r\rho a & \kappa_r^{\mathbb{P}} + \sigma_r\rho a \end{bmatrix} \end{aligned} \quad (23)$$

for some positive constant, a . So, when we fit to coupon-bond prices, we use a reversion speed matrix, $\mathcal{K}^{\mathbb{Q}}$, that has a different form from $\mathcal{K}^{\mathbb{P}}$, namely:

$$\mathcal{K}^{\mathbb{Q}} = \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} - \sigma_{\theta}a & \sigma_{\theta}a \\ -\kappa_r^{\mathbb{P}} - \sigma_r\rho a & \kappa_r^{\mathbb{P}} + \sigma_r\rho a \end{bmatrix} \quad (24)$$

As the extra parameter, a , goes to zero, the risk-neutral and real-world reversion-speed matrices clearly coincide. As an added bonus of this approach we can also get an estimate of the risk premium, which we can compare with the

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usual econometric estimates of the same quantity. If there is a good match, this will strengthen the claim that we are using an economically justifiable model.

Establishing this link between the reversion speed matrices in the two measures is important, because, as explained in Section 4, from the statistically estimated yield covariance matrix we obtain information about the reversion speeds in the real-world measure, but in order to price bonds we need the same quantities in the risk-neutral measure. We explain the procedure in detail in Section 4, but, in short, we proceed as follows.

After estimating the reversion speed matrix $\mathcal{K}^{\mathbb{P}}$ using real-world information about the yield covariance matrix, we keep the estimated reversion speed matrix, $\mathcal{K}^{\mathbb{P}}$, fixed and we move to the reversion speed matrix $\mathcal{K}^{\mathbb{Q}}$ needed for pricing using the relationship in Equation (54). Having obtained the remaining model parameters in the pricing measure by best fit to market prices of coupon-bearing bonds, closed-form solutions for discount bond prices are given by the expressions reported in Appendix I. The model-implied prices of coupon-bearing bonds can be obtained by multiplying, for each bond, its contractual cash flows (coupons and principal at maturity) by the model discount bond price. Section 4 explains how the model parameters are determined.

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4. Model Calibration

4. Model Calibration

By 'calibration' we mean the process by means of which the parameters and the initial values of the state variables are determined. This process is repeated for every trading day for which we have price information. If the model were perfectly specified, the fitted parameters would, of course, have to be the same on every single day. In practice, some features of the model, such as the constant volatility for the short rate and the reversion speeds it assumes, are clearly unrealistic. We therefore allow the least-square fit procedure to determine the optimal parameters on each trading day, and simply check that the variations in the fitted parameters are smooth, and congruent with their macro-financial interpretation (see Section 3.2).

For each trading day the calibration is carried out in two distinct phases: with the first we fit the reversion speeds under \mathbb{P} , the volatilities, σ_r , and σ_θ , and the correlation, ρ , to the covariance matrix of the changes in the yields of discount bonds described in Section 2. The co-variance matrix is estimated using a 5-year rolling window of daily yield changes with equal weights.

Once these quantities have been estimated, they fully specify the \mathbb{P} -measure reversion speed matrix, $\mathcal{K}^{\mathbb{P}}$. To this matrix we apply the transformation in Equation (54), to obtain the reversion speed matrix under $\mathcal{K}^{\mathbb{Q}}$ needed for pricing. After the calibration to the covariance matrix has been carried out, the associated parameters are kept fixed, and the market price information from real coupon bonds is brought into play. The model prices of the coupon bonds are calculated as

$$CP_mod_t^{TN} = \sum_{i=1, N} cash_flow_i P_t^{T_i} \quad (25)$$

where $CP_mod_t^{TN}$ denotes the time- t price of a

T -maturity coupon-bearing bond with N coupons still to pay, $P_t^{T_i}$ signifies the time- t price of a discount bond of maturity T_i , and the cash flows include both the coupons and the final repayment at maturity. On every trading day, we add the squares of the differences between the model prices, $CP_mod_t^{TN}$, and the market prices, $CP_mkt_t^{TN}$, of all the Treasury bonds active on that day, and we vary the initial value of the short rate r_0 , the initial value of the reversion speed, θ_0 , and the reversion level of the reversion level, θ_{inf} , until the sum of the squared errors is minimised. The parameters are allowed to vary within (wide) bands, chosen to ensure that the optimised values are consistent with their macro-financial interpretation.

The reason for splitting the calibration in two phases (first to the covariance matrix, and then to the market price), is to avoid the "tug of war" between reversion level and volatilities that often occurs in the model calibration: in order to get a closer fit to the yield curve, the optimisation often increases the reversion levels to very high values, and simultaneously increases the volatilities, and hence the convexity. The high reversion level "pulls" the yield curve higher, and the high volatilities, via convexity, have the opposite effect. As a consequence, the result of a joint optimisation sometimes is a set of extremely and implausibly high volatilities, and a similarly high and implausible reversion level. The procedure we employ ensures that this cannot happen because, by requiring the optimised yield volatilities to be approximately correct, we also ensure that the curvature of the yield curve due to convexity should be broadly correctly recovered. See Rebonato and Putiatyn (2017) on this point.

5. Checking the Financial Plausibility of the Calibrated Model

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As explained in the Introduction, we want to endow the model with a macro-financial interpretation. Such an interpretation will only be warranted if the parameters estimated in the course of the calibration phase bear a strong resemblance to the correspondent \mathbb{P} -measure quantities. We explore, in this section, whether this turns out to be indeed the case.

The "entry-level" check is whether the time series of the state variable, r_t , matches closely the time series of any reasonable proxy for the unobservable short rate. We choose the US 3-month Treasury Bill rate for such a proxy, and show the result in Fig 1. The match is so close that, at the level of resolution of the figure, it is hardly possible to detect the existence of two separate curves, and the resulting correlation is over 99%.

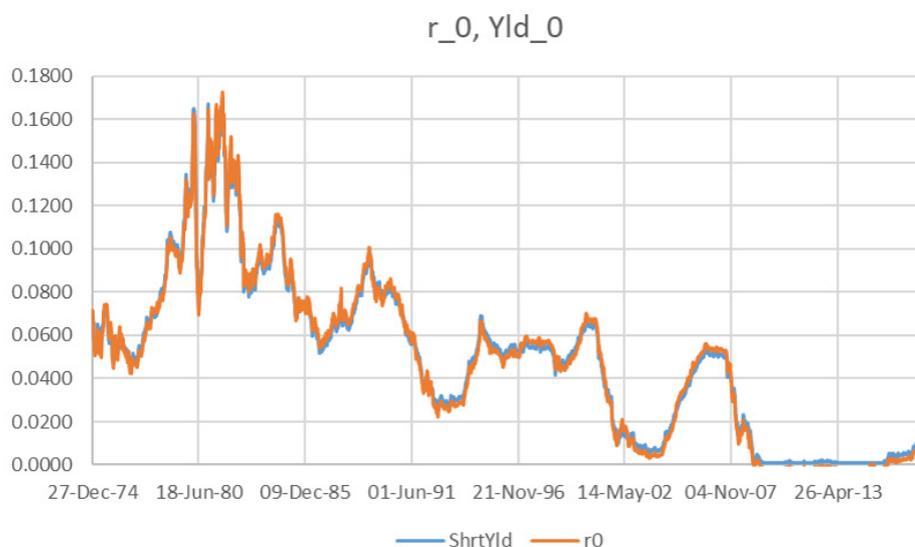
A more stringent test is afforded by the comparison between the volatility of the short rate, σ_r , as calibrated within the model, and the volatility

of the same proxy for the short rate discussed above. The results are shown in Fig 2, which displays the high degree of congruence between the two curves. To quantify the similarity between the two curves, when we regress the fitted short rate volatility against the volatility of the Treasury Bill, we find an intercept not statistically significantly different from zero, a regression slope of 0.8, and a correlation coefficient of 0.57.

As mentioned in Section 3.2, and discussed at greater length in Rebonato (2018) and Ehrekinsky (2013), a financially grounded interpretation of the model requires $\mathcal{K}_r > \mathcal{K}_\theta$ and $\sigma_r < \sigma_\theta$. Figs 3 and 4 show that these conditions were indeed satisfied by the calibration procedure.

We also argued that it is reasonable to use as a proxy for the slope of the yield curve the difference $\theta_t - r_t$. We define the slope of the yield curve as the difference between the 1- and the 10-year yields, and show the time series of the slope thus defined and of our proxy (the quantity $\theta_t - r_t$) in

Figure 1: The time series for the short rate (labelled 'r0') and the 3-month Treasury Bill yield (labelled 'ShrtYld')



5. Checking the Financial Plausibility of the Calibrated Model

Figure 2: The time series of the volatility of the short rate and of the 3-month Treasury Bill yield.

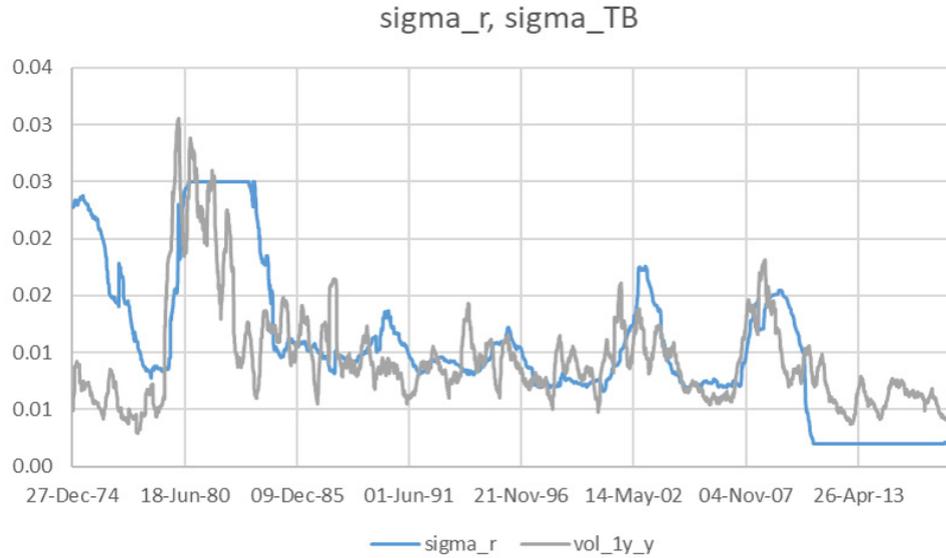


Figure 3: Time series of the fitted values of the short rate reversion speed, and of the reversion speed of the reversion level.

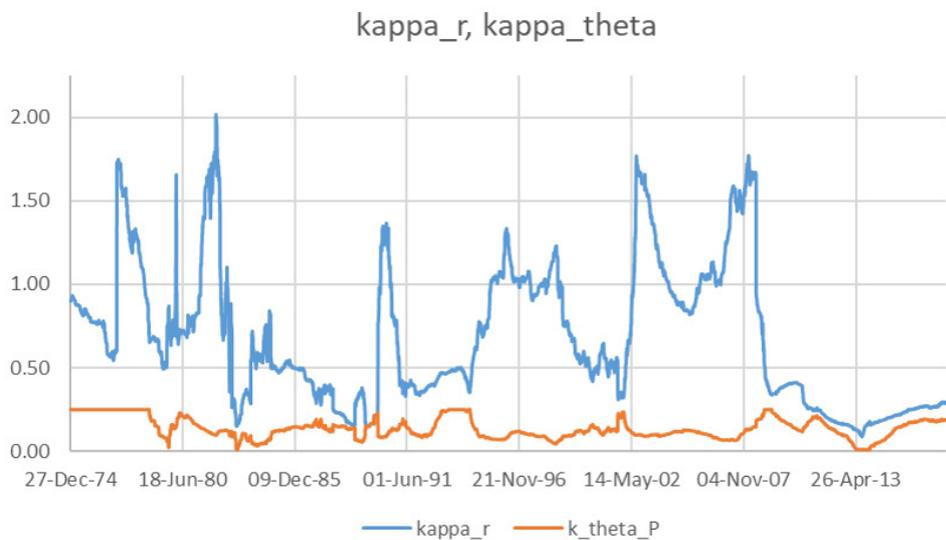


Figure 5. As one can see the match is very close, with the intercept in the regression of the slope against the difference $\theta_t - r_t$ not statistically significant from zero, a regression slope of 1.3 and a correlation coefficient of 0.76.

Finally, we comment on the closeness of the estimate of the slope return-predicting factor estimated statistically and as implied by the model. Here, the correlation between the two quantities is poor (0.21), indicating that the magnitude of the market price of risk is poorly

5. Checking the Financial Plausibility of the Calibrated Model

captured by the fitting procedure of our model. From our perspective, this poor recovery of the market price of risk (in our model the parameter α in Equation (23)) translates into a poor estimate of the difference between the real-world and risk

neutral reversion-speed matrix. Since we are not interested in the recovery of the slope-dependent market price of level risk, our results are mildly affected by this shortcoming of the model.

Figure 4: The time series of the volatility of the short rate and of reversion level.

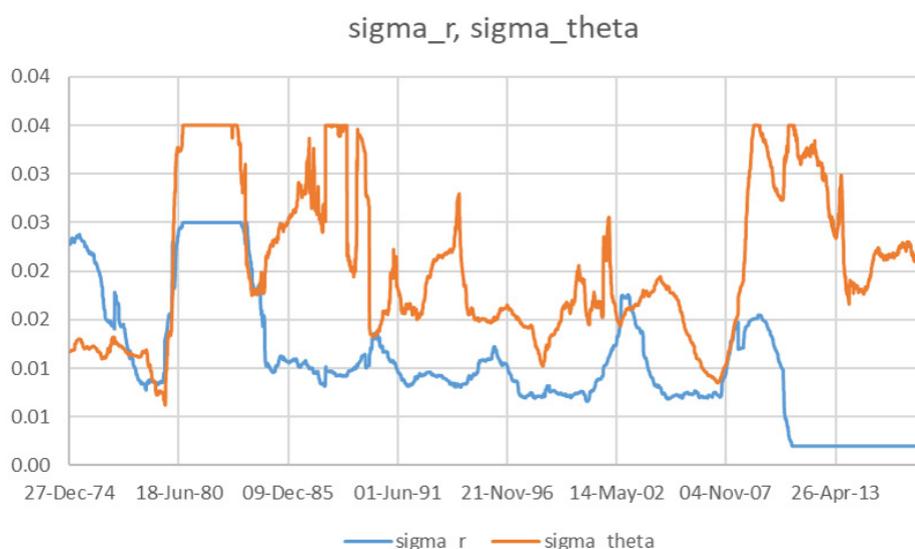
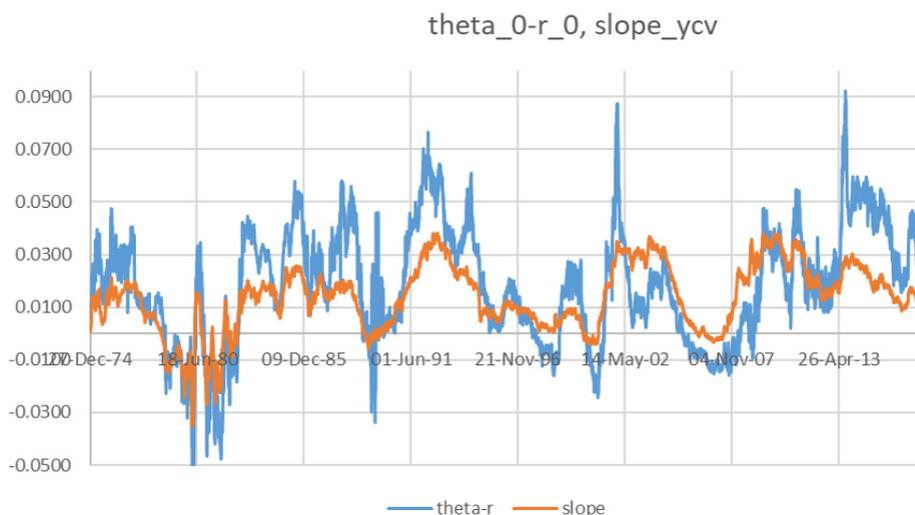


Figure 5: The time series of the slope of the yield curve and of the difference between the instantaneous reversion level and the short rate, $\theta_t - r_t$.



6. Creating the Strategy Signal

6. Creating the Strategy Signal

After the calibration procedure has been carried out, for each bond we have a time series of pricing errors. One such time series for a particular Treasury bonds is shown in Fig 6. In order to establish a trading strategy, we create a trading signal, by setting the notional of the position in each bond to be proportional to the strength of the signal for that bond on that day.

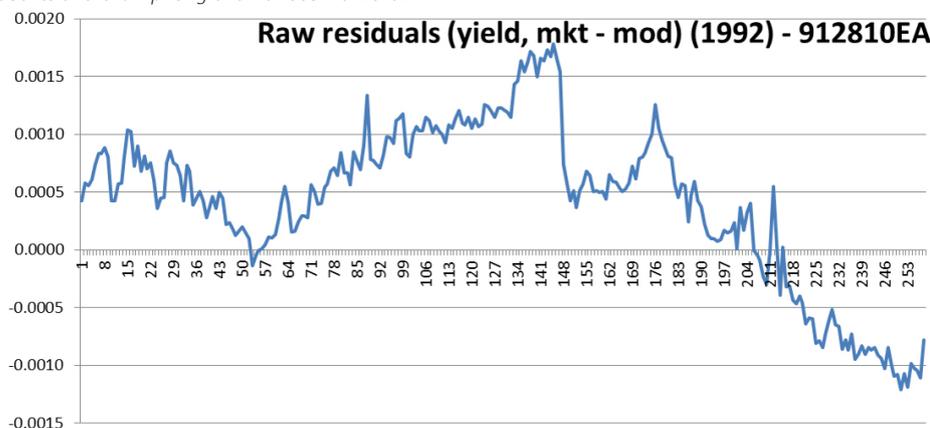
For each bond, the trading signal is formed by taking the difference between a slow moving average and an adjusted fast moving average of price errors. The adjusted fast moving average is obtained by summing the last n_{short} price errors, and dividing the sum by n_{long} rather than n_{short} , where n_{long} and n_{short} are the number of price errors in the long and short sum, respectively. We use a slow moving average rather than the zero level for the pricing errors because some bonds (perhaps for liquidity or other reasons) may have an unconditional average price error different from zero. The reason for using an adjusted fast moving average, ie, for dividing the short sum by n_{long} rather than n_{short} is to make the signal more stable and to filter out high-frequency (quickly reversed) price errors, clearly visible in

the time series displayed in Fig 6, that can lead to over-trading. The differences in signal using a "proper" and an adjusted moving average are shown in Fig 7, which was obtained using a random walk to obtain the price errors, $n_{long} = 20$ and $n_{short} = 5$. It is clear how the adjusted signal retains the salient trends, but removes the high-frequency fluctuations, which is exactly what we wanted to achieve.

As Fig 8 shows, the trading signals tend to display mean-reverting behaviour...

We took the number of days in the slow moving average equal to 22 business days (corresponding to roughly one month), and the number of days in the fast moving average ranging from 1 to 5 business days (with the last choice corresponding to roughly one week). We analysed the robustness of our results using several values for the number of days in the slow and fast moving averages, and we found the results to be largely insensitive to

Figure 6: The time series of the raw pricing error for CUSIP 912810EA.



6. Creating the Strategy Signal

sensible variations around the chosen values. We stress that the results we report in Section 7 were not obtained for any "optimised" combination of days in the fast and slow moving averages: as the round numbers (22 and 1 or 5) and the simple interpretation (one month and one day / one week) indicate, we did not engage in a data-mining exercise of optimisation. The same applies to the cut-off maturities (2 and 15 years).

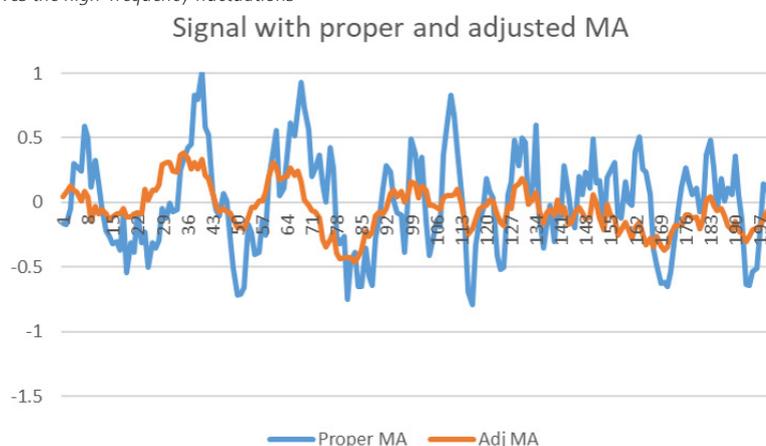
Typical patterns for the two moving averages and the resulting signal are shown in Fig 8.

As Fig 8 shows, the trading signals tend to display mean-reverting behaviour, with reversion speeds implying half-lives of several weeks to a few months. This observation is important, because it suggests that the signal is practically exploitable, in that it neither requires excessively long strategies, nor does it require overly frequent rebalancing.

On any given day, that strategy will consist of long positions in "cheap" bonds, and short positions

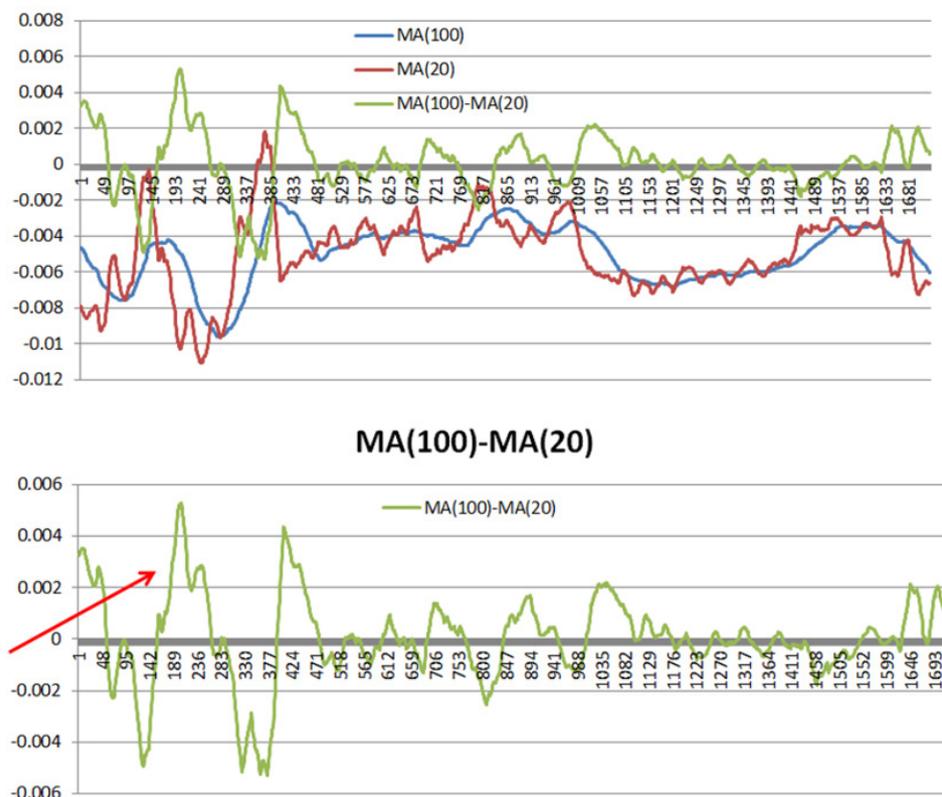
in "expensive" bonds. The resulting portfolio will not have a systematic long or short bias but, on any given day, it will not have exactly zero cost, nor will it be exactly duration neutral. Since yields have fallen considerably over the period under study, and even a small residual duration exposure may vitiate the results, we control for a possible residual duration exposure in our portfolio by calculating the net portfolio duration, and by subtracting the hypothetical profit (or loss) that a portfolio with that residual duration would make given the change in average yield from one day to the next. We note that subtracting the duration exposure this way would flatter the results from long positions, and penalise short positions, because achieving "physical" (as opposed to "virtual") immunisation requires selling an actual bond. Over the period under study, Treasuries have commanded an unconditional positive risk premium, and therefore physical hedging requires paying rather than receiving this premium. (To give an idea of the magnitude of the effect, the magnitude of the unconditional risk premium for the 10-year point is over 200 basis points

Figure 7: The time series of the signals obtained using a 'proper' and an adjusted moving average. For the purpose of this test, the pricing errors were obtained using a random walk, and the number of terms in the short and long sum were chose to be $n_{long} = 20$ and $n_{short} = 5$. The two signals were the difference between the proper-fast moving average and the slow moving average (curve labelled "Proper MA", and the difference between the adjusted-fast moving average and the slow moving average (curve labelled "Adj MA"). Note how the adjusted signal retains the salient trends of the "proper" signal, but removes the high-frequency fluctuations



6. Creating the Strategy Signal

Figure 8: The 20-day and 5-day moving averages for CUSIP 912810CU and their difference (top panel), and the associated trading signal (bottom panel). Note the clear mean-reverting behaviour of the trading signal. The amplitude of the signal declines over time as the bond in question approaches maturity, and its price therefore becomes closer and closer to par.



per annum.) In order to compensate for this, we increase the funding cost by an amount required to ensure zero realised return in each three-year period for a virtually-duration-neutralised equal-weight long bond portfolio. (The construction of the equal-weight portfolio is described in greater detail in Section 8.)

We funded the difference between the proceeds from the short sales and the cost of the long positions by borrowing or lending at the Treasury Bill rate. Finally, we reinvested all coupons received in the same bond they originated from.

7. Profitability of the Strategy

7. Profitability of the Strategy

We carried out our analysis of the results by splitting the data into 15 blocks of three years (the last block is slightly shorter than three years). We have no return results for the first few days of each three-year block because of the need to build the moving average needed for the signal. On any given day, the overall strategy will in general consist of long and short positions in different bonds.

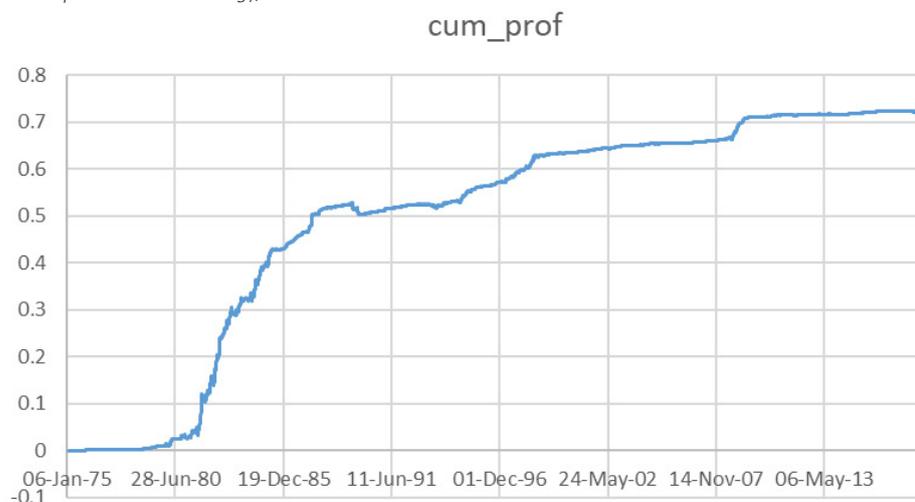
Fig 9 shows the cumulative profits for the duration-corrected strategy⁶. Fig 10 shows the rolling two-year return and rolling standard deviation from the strategy.⁷ The two curves show that both return and their volatility are highly variable, and, above, all, that they are very strongly correlated, with the highest returns and the highest return volatility observed in the early 1980s, in the mid 1990s and in the immediate aftermath of the 2008 financial crisis. We discuss this feature in detail in what follows.

The ratio of the strategy returns and volatility, ie, the Sharpe ratio of the funded, duration-

neutralised strategy, is much more stable, and is shown in tabular form in Tab 1, and graphically in Fig 11. In particular, Tab 1 shows the Sharpe ratios for each of the 15 three-years blocks into which the full data set was subdivided. We stress that the Sharpe ratio is positive in 14 out of 15 of the three-year blocks, is often very high, is never significantly negative, and is significantly greater than zero at the 99.9% confidence level in 12 out of 15 blocks.

From Table 1 it is clear that the Sharpe ratio of the strategy is very high, but also that it has tended to decline over time. See also Fig 11 in this respect. By far the most interesting observation, however, is the high correlation (75%) between the short-rate volatility (either as obtained from the fitting of the model, or as estimated statistically as the volatility of the 3-month Treasury Bill rate), and the profitability of the strategy, displayed in Fig 12. We also note that the strategy tends to produce high returns (but not necessarily high Sharpe ratios!) when the market volatility is high; in these periods the volatility of the strategy is

Figure 9: The cumulative profit from the strategy, added across all the individual bonds



6 - In what follows we omit the "duration-corrected" qualifier unless required for clarity.

7 - In order to focus on the profitability of the long - short strategy the results shown in Fig 10) refer to returns without funding. The Sharpe ratios reported in Tab 1 and in Fig 11 include funding costs.

7. Profitability of the Strategy

also high, and therefore the Sharpe ratios do not display this link with the market volatility.

This finding is significant because it suggests a clear indication of the origin of the profitability of the strategy. Our results can in fact be reconciled with the findings by Hu, Pan and Wang (2013),

who establish a link between "price errors" ("noise" in their terminology) for Treasury bonds and a general decrease in market liquidity. The explanation they offer is that, the greater the decrease in liquidity, the greater the difficulty encountered by pseudo-arbitrageurs in carrying out the trades that should bring Treasury prices

Figure 10: The rolling returns (left-hand y axis), and standard deviation (right-hand x axis) from the strategy



Table 1: The Sharpe ratios for the strategy in the 3-year block in the left column for 2 and 20 days in the short and long moving averages.

Date	Sharpe Ratio
1975-1977	0.565
1978-1980	0.573
1981-1983	1.348
1984-1986	1.820
1987-1989	1.081
1990-1992	1.235
1993-1995	-0.014
1996-1998	0.110
1999-2001	1.282
2002-2004	0.691
2005-2007	0.326
2008-2010	2.716
2011-2013	0.876
2014-2016	1.812
2017-2018	1.121

7. Profitability of the Strategy

Figure 11: The Sharpe ratios for each of the three-year periods for the full strategy and the long-only strategy, as described in Sec 8.

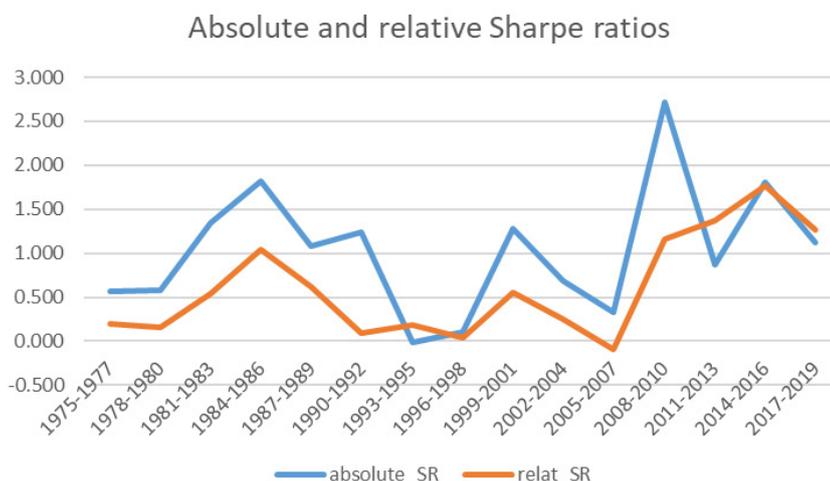


Figure 12: The rolling returns and standard deviation of the strategy (left-hand y-axis) and the volatility of the 3-month Treasury Bill (right-hand y-axis)



in line with fundamentals. To the extent that an increase in volatility can be associated with a decrease in market liquidity (see, in this respect, Foucault, Pagano and Roell (2014)), the findings of our study are consistent with the interpretation in Hu, Pan and Wang (2013), and provide a rationale for the source of profitability of our strategy. And if, indeed, high returns are reaped in periods of high market volatility, it is not surprising that in these periods also the volatility of the strategy should

be high, as the deviations from fundamentals may well increase (giving rise to temporary losses) before eventually reverting towards their reversion level.

This explanation can be further corroborated by running a regression of the running returns from the strategy and the root-mean-squared error of the yield curve fitting. When we do so, we find an even higher correlation of 78% than what we found between the strategy profitability

7. Profitability of the Strategy

and the short-rate volatility: the strategy makes most money when the fit is poor, not when it is good. This is also fully consistent with the findings by Hu, Pan and Wang (2013), who, as explained, associate periods of market turbulence with periods when pseudo-arbitrageurs find it difficult to exploit and correct price deviations from fundamentals. And, indeed, the correlation between the rolling root-mean-squared fitting error and the market volatility (proxied by the 1-year yield volatility) is as high as 57%.

7. Profitability of the Strategy

8. Long-Only Analysis

8. Long-Only Analysis

We also explored a long-only version of our strategy. More precisely, we only invested in those bonds that, according to the model, were under-priced ("cheap"), and we invested an equal amount in all the bonds in the universe (we call this the "equal-weight portfolio"). The market and strategy portfolios were sized as to require the same outlay of cash, and both versions of the strategy were funded and duration neutralised as explained in the previous section. We report the results in Tab 2. As mentioned above, the funding rate was adjusted in each three-year block so as to give a zero Sharpe ratio for the long-only equal-weight portfolio.

The long-only strategy outperforms in terms of Sharpe ratio the market portfolio in 14 out of the 15 three-year periods. The average Sharpe ratio for the strategy is significantly higher than the Sharpe ratio of the long-always strategy at the 99% confidence level. While from the theoretical point of view these results do not add much to the results shown in Section 7, they are very important for the practical applicability of the strategy for many institutional investors, who often have long-only constraints.

9. Conclusions

9. Conclusions

In this paper, we have proposed a definition of value in Treasury bonds that, we believe, displays more clearly the features intuitively associated with the term "value" than what has recently been offered in the literature. In our definition, value is the difference between the market price of a Treasury bond and its theoretical price, with the latter determined by a financially motivated dynamic Gaussian term structure model. We show that the calibration of the model to yield volatilities and market yields produces parameters that are in good agreement with their macro-financial interpretation, and therefore justify the association of value with the difference between the market and the theoretical prices.

Using this definition of value, we construct long/short self-financing portfolios that load positively/negatively on our value factor. After controlling for residual duration exposure, we show that the

portfolios thus constructed consistently earn a very attractive Sharpe ratio (average Sharpe ratio of 1.03, with a positive Sharpe ratio in 14 of the 15 three-year periods in our data set). The Sharpe ratio of a long-only version of the strategy outperforms the Sharpe ratio of an equally weighted long portfolio by 0.822.

We have shown that the profitability of the strategy is closely linked to the volatility of the 3-month Treasury Bill. We can explain this finding if we establish a link between higher market volatility and poorer market liquidity. In this account of our finding, in periods of market turmoil (of high volatility), less arbitrage capital is forthcoming to bring prices back to fundamentals, and pricing "errors" temporarily appear. As market conditions revert to normal, the pricing errors are arbitrated away towards zero. This interpretation is closely linked to the view of "liquidity as noise" in Hu,

Table 2: The difference in Sharpe Ratio between the long-only strategy and the equal-weight portfolio for the periods shown in the left-hand column.

Date	Difference in Sharpe ratio
1975-1977	0.197
1978-1980	0.157
1981-1983	0.536
1984-1986	1.044
1987-1989	0.625
1990-1992	0.097
1993-1995	0.178
1996-1998	0.037
1999-2001	0.0551
2002-2004	0.256
2005-2007	-0.099
2008-2010	1.159
2011-2013	1.376
2014-2016	1.768
2017-2018	1.262

9. Conclusions

Pan and Wang (2013), and also explains the close link between market volatility and the volatility of returns from our strategy.

Our study did not try to account for trading costs, however, given the size of the Sharpe ratio, it appears unlikely that trading costs in the extremely liquid Treasury market could wipe out, or significantly reduce, the profitability of the strategy.

Finally, it would be interesting to undertake a systematic study of the timing of the profitability of our strategy compared with the returns from a diversified US equity index, or from the various equity factors that have been identified in the literature. We leave this as a possible future development.

9. Conclusions

Appendices

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Appendix 1

Take a process for a vector, \vec{x}_t , of generic state variables

$$d\vec{x}_t = \mathcal{K} \left(\vec{\theta} - \vec{x}_t \right) dt + S d\vec{z}_t \quad (26)$$

such that

$$r_t = u_r + \vec{g}^T \vec{x}_t \quad (27)$$

Let also the reversion-speed matrix, \mathcal{K} , be invertible, and let it admit orthogonalisation:

$$\mathcal{K} = L\mathcal{L}L^{-1} \quad (28)$$

with its eigenvalues, l_{ii} , on the diagonal of the matrix \mathcal{L} ,

$$\mathcal{L} = \text{diag}[l_{ii}] \quad (29)$$

and its eigenvectors in the matrix \mathcal{L} .

Then it is shown in Rebonato (2018) that the discount bond price, $P_t^T = P(\tau)$ ($\tau = T - t$), is given by

$$P_t^T = e^{A_t^T + (\vec{B}_t^T)^T \vec{x}_t} \quad (30)$$

with A_t^T and $(\vec{B}_t^T)^T$ given by

$$\vec{B}(\tau) = \left(e^{-\mathcal{K}^T \tau} - I_n \right) (\mathcal{K}^T)^{-1} \vec{g} \quad (31)$$

$$(\vec{B}_t^T)^T = \vec{g}^T \mathcal{K}^{-1} [e^{-\mathcal{K} \tau} - I_n] \quad (32)$$

$$A(\tau) = \text{Int}_1 + \text{Int}_2 + \text{Int}_3 \quad (33)$$

and with

$$\text{Int}_1 = -u_r T \quad (34)$$

$$\text{Int}_2 = \vec{g}^T \left(L\mathcal{L}^{-1} D(T) \mathcal{L}L^{-1} \vec{\theta} - \vec{\theta} T \right) \quad (35)$$

$$\text{Int}_3 = \text{Int}_3^a + \text{Int}_3^b + \text{Int}_3^c + \text{Int}_3^d \quad (36)$$

$$\text{Int}_3^a = \frac{1}{2} \vec{g}^T \mathcal{K}^{-1} L F(T) \mathcal{L}^{-1} L^T \vec{g} \quad (37)$$

$$\text{Int}_3^b = -\frac{1}{2} \vec{g}^T \mathcal{K}^{-1} C (L^{-1})^T D(T) \mathcal{L}^{-1} L^T \vec{g} \quad (38)$$

$$\text{Int}_3^c = -\frac{1}{2} \vec{g}^T \mathcal{K}^{-1} L D(T) L^{-1} C (\mathcal{K}^T)^{-1} \vec{g} \quad (39)$$

$$\text{Int}_3^d = \frac{1}{2} \vec{g}^T \mathcal{K}^{-1} C (\mathcal{K}^T)^{-1} \vec{g} T \quad (40)$$

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with $M = L^{-1}C(L^{-1})^T$, $F = [f_{ij}]_n$, and

$$f_{ij} = m_{ij} \frac{1 - e^{(l_{ii} + l_{jj})T}}{l_{ii} + l_{jj}} \quad (41)$$

$$D(T) = \text{diag} \left[\frac{1 - e^{l_{ii}T}}{l_{ii}} \right]_n \quad (42)$$

$$C = SS^T. \quad (43)$$

Appendix 2

In this Appendix we derive the link between the reversion speed matrices under \mathbb{P} and under \mathbb{Q} , if the assumptions discussed in Section 3.4, and reported again below, are satisfied.

First, we impose that an essentially affine formulation for the model should be retained; if this is the case, the market price of risk must have the form

$$\lambda_t = \lambda_0 + \Lambda x_t. \quad (44)$$

We then assume that $\lambda_0 = 0$ – See Duffee (2002), and that only the reversion level should attract a risk premium. Finally, we require that the market price of risk should depend on the slope of the yield curve. This means that we have

$$\begin{aligned} \text{drift}^{\mathbb{Q}}[d\vec{x}_t] &= \mathcal{K}^{\mathbb{P}} \left(\vec{\theta}^{\mathbb{P}} - \vec{x}_t \right) + \underline{S}\Lambda x_t = \\ & \begin{bmatrix} \mu_{\theta}^{\mathbb{Q}} \\ \mu_r^{\mathbb{Q}} \end{bmatrix} = \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \theta_{\theta}^{\mathbb{P}} - \theta_t \\ \theta_{\theta}^{\mathbb{P}} - r_t \end{bmatrix} + \\ & \begin{bmatrix} \sigma_{\theta} & 0 \\ \sigma_r \rho & \sigma_r \sqrt{(1 - \rho^2)} \end{bmatrix} \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \begin{bmatrix} \theta_t \\ r_t \end{bmatrix} = \\ & \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \theta_{\theta}^{\mathbb{P}} - \theta_t \\ \theta_{\theta}^{\mathbb{P}} - r_t \end{bmatrix} + \begin{bmatrix} \sigma_{\theta} & 0 \\ \sigma_r \rho & \sigma_r \sqrt{(1 - \rho^2)} \end{bmatrix} \begin{bmatrix} \lambda_{11}\theta_t + \lambda_{12}r_t \\ \lambda_{21}\theta_t + \lambda_{22}r_t \end{bmatrix} \quad (45) \end{aligned}$$

It is at this point that we impose the condition (Cochrane and Piazzesi, 2005; Adrian et al, 2014) that only the reversion level should attract a risk premium, ie, that the process for the short rate should be the same under \mathbb{P} and under \mathbb{Q} .

This implies

$$(\lambda_{21}\theta_t + \lambda_{22}r_t) = 0 \quad (46)$$

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For this to be true for any value of r_t and θ_t we must have separately

$$\lambda_{21} = 0 \quad (47)$$

and

$$\lambda_{22} = 0 . \quad (48)$$

Now we impose that the market price of risk should depend positively on the slope of the yield curve. This implies

$$\begin{aligned} \lambda_{11} &= a \\ \lambda_{12} &= -a \end{aligned} \quad (49)$$

for some *positive* constant, a .

Therefore we have for the drifts of the state variables under \mathbb{Q}

$$\begin{aligned} & \begin{bmatrix} \mu_{\theta}^{\mathbb{Q}} \\ \mu_r^{\mathbb{Q}} \end{bmatrix} = \\ & \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \theta_{\theta}^{\mathbb{P}} - \theta_t \\ \theta_{\theta}^{\mathbb{P}} - r_t \end{bmatrix} + \begin{bmatrix} \sigma_{\theta} & 0 \\ \sigma_r \rho & \sigma_r \sqrt{(1-\rho^2)} \end{bmatrix} \begin{bmatrix} a & -a \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_t \\ r_t \end{bmatrix} \\ & \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \theta_{\theta}^{\mathbb{P}} - \theta_t \\ \theta_{\theta}^{\mathbb{P}} - r_t \end{bmatrix} + \begin{bmatrix} \sigma_{\theta} & 0 \\ \sigma_r \rho & \sigma_r \sqrt{(1-\rho^2)} \end{bmatrix} \begin{bmatrix} a(\theta_t - r_t) \\ 0 \end{bmatrix} \\ & = \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \theta_{\theta}^{\mathbb{P}} - \theta_t \\ \theta_{\theta}^{\mathbb{P}} - r_t \end{bmatrix} + \begin{bmatrix} \sigma_{\theta} a (\theta_t - r_t) \\ \sigma_r \rho a (\theta_t - r_t) \end{bmatrix} \end{aligned}$$

This means that the relationship between the reversion speeds in the two measures, $\mathcal{K}^{\mathbb{P}}$ and $\mathcal{K}^{\mathbb{Q}}$, can be obtained as follows:

$$\begin{aligned} \text{drift}^{\mathbb{Q}} [d\vec{x}_t] &= \mathcal{K}^{\mathbb{P}} (\theta^{\mathbb{P}} - x_t) + S\Lambda x_t = \\ & \mathcal{K}^{\mathbb{P}} \theta^{\mathbb{P}} - \mathcal{K}^{\mathbb{P}} x_t + S\Lambda x_t = \\ & \mathcal{K}^{\mathbb{P}} \theta^{\mathbb{P}} - (\mathcal{K}^{\mathbb{P}} - S\Lambda) x_t = \\ & (\mathcal{K}^{\mathbb{P}} - S\Lambda) \left[(\mathcal{K}^{\mathbb{P}} - S\Lambda)^{-1} \mathcal{K}^{\mathbb{P}} \theta^{\mathbb{P}} - x_t \right] \end{aligned} \quad (50)$$

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and therefore

$$\mathcal{K}^{\mathbb{Q}} = \mathcal{K}^{\mathbb{P}} - S\Lambda \quad (51)$$

$$\theta^{\mathbb{Q}} = (\mathcal{K}^{\mathbb{P}} - S\Lambda)^{-1} \mathcal{K}^{\mathbb{P}} \theta^{\mathbb{P}} \quad (52)$$

Given the results above we have

$$S\Lambda = \begin{bmatrix} \sigma_{\theta} & 0 \\ \sigma_r \rho & \sigma_r \sqrt{(1-\rho^2)} \end{bmatrix} \begin{bmatrix} a & -a \\ 0 & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_{\theta} a & -\sigma_{\theta} a \\ \sigma_r \rho a & -\sigma_r \rho a \end{bmatrix}}_{S\Lambda} \quad (53)$$

and therefore

$$\begin{aligned} \mathcal{K}^{\mathbb{Q}} &= \mathcal{K}^{\mathbb{P}} - S\Lambda = \\ & \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} & 0 \\ -\kappa_r^{\mathbb{P}} & \kappa_r^{\mathbb{P}} \end{bmatrix} - \begin{bmatrix} \sigma_{\theta} a & -\sigma_{\theta} a \\ \sigma_r \rho a & -\sigma_r \rho a \end{bmatrix} = \\ & \begin{bmatrix} \kappa_{\theta}^{\mathbb{P}} - \sigma_{\theta} a & \sigma_{\theta} a \\ -\kappa_r^{\mathbb{P}} - \sigma_r \rho a & \kappa_r^{\mathbb{P}} + \sigma_r \rho a \end{bmatrix} \end{aligned} \quad (54)$$

Appendices

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About Amundi ETF,
Indexing and
Smart Beta

About Amundi ETF, Indexing and Smart Beta

With more than €112 billion¹ in assets under management, Amundi ETF, Indexing and Smart Beta is one of Amundi's strategic business areas and is a key growth driver for the Group.

Amundi ETF, Indexing and Smart Beta business line provides investors - whether institutionals or distributors - with robust, innovative, and cost-efficient solutions, leveraging Amundi Group's scale and large resources. The platform also offers investors fully customized solutions (ESG, Low Carbon, specific exclusions, risk constraints, etc.).

With over 30 years of benchmark construction and replication expertise, Amundi is a trusted name in ETF & Index management among the world's largest institutions. The team is also recognized for its ability to develop Smart Beta & Factor Investing solutions, with more than 10-year track-record.

1- All figures and data are provided by Amundi ETF, Indexing & Smart Beta at end March 2019

About EDHEC-Risk Institute

About EDHEC-Risk Institute

Founded in 1906, EDHEC is one of the foremost international business schools. Operating from campuses in Lille, Nice, Paris, London and Singapore, EDHEC is one of the top 15 European business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This Institute boasts a team of permanent professors, engineers and support staff, and counts a large number of affiliate professors and research associates from the financial industry among its ranks.

The Need for Investment Solutions and Risk Management

Investment management is justified as an industry only to the extent that it can demonstrate a capacity to add value through the design of dedicated and meaningful investor-centric investment solutions, as opposed to one-size-fits-all manager-centric investment products. After several decades of relative inertia, the much needed move towards investment solutions has been greatly facilitated by a true industrial revolution triggered by profound paradigm changes in terms of (1) mass production of cost- and risk-efficient smart factor indices; (2) mass customisation of liability-driven investing and goal-based investing strategies; and (3) mass distribution, with robo-advisor technologies. In parallel, the investment industry is strongly impacted by two other major external revolutions, namely the digital revolution and the environmental revolution.

In this fast-moving environment, EDHEC-Risk Institute positions itself as the leading academic think-tank in the area of investment solutions, which gives true significance to the investment management practice. Through our multi-faceted programme of research, outreach, education and industry partnership initiatives, our ambition is to support industry players, both asset owners and asset managers, in their efforts to transition towards a novel, welfare-improving, investment management paradigm.

EDHEC-Risk New Initiatives

In addition to the EDHEC Alternative Indexes, which are used as performance benchmarks for risk analysis by investors in hedge funds, and the EDHEC-IEIF Monthly Commercial Property index, which tracks the performance of the French commercial property market through SCPIs, EDHEC-Risk has recently launched a series of new initiatives.

- The EDHEC-Princeton Retirement Goal-Based Investing Index Series, launched in May 2018, which represent asset allocation benchmarks for innovative mass-customised target-date solutions for individuals preparing for retirement;
- The EDHEC Bond Risk Premium Monitor, the purpose of which is to offer to investment and academic communities a tool to quantify and analyse the risk premium associated with Government bonds;
- The EDHEC-Risk Investment Solutions (Serious) Game, which is meant to facilitate engagement with graduate students or investment professionals enrolled on one of EDHEC-Risk's various campus-based, blended or fully-digital educational programmes.

About EDHEC-Risk Institute

Academic Excellence and Industry Relevance

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute's advisory board. This board is made up of internationally recognised researchers, the Institute's business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Seven research programmes have been conducted by the centre to date:

- Investment Solutions in Institutional and Individual Money Management;
- Equity Risk Premia in Investment Solutions;
- Fixed-Income Risk Premia in Investment Solutions;
- Alternative Risk Premia in Investment Solutions;
- Multi-Asset Multi-Factor Investment Solutions;
- Reporting and Regulation for Investment Solutions;
- Technology, Big Data and Artificial Intelligence for Investment Solutions.

EDHEC-Risk Institute's seven research programmes explore interrelated aspects of investment solutions to advance the frontiers of knowledge and foster industry innovation. They receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in the City of London (United Kingdom) and Nice, (France).

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- **Financial Risk Management as a Source of Performance,**
in partnership with the *French Asset Management Association (Association Française de la Gestion financière – AFG)*;
- **ETF, Indexing and Smart Beta Investment Strategies,**
in partnership with *Amundi*;
- **Regulation and Institutional Investment,**
in partnership with *AXA Investment Managers*;
- **Optimising Bond Portfolios,**
in partnership with *BDF Gestion*;
- **Asset-Liability Management and Institutional Investment Management,**
in partnership with *BNP Paribas Investment Partners*;
- **New Frontiers in Risk Assessment and Performance Reporting,**
in partnership with *CACEIS*;
- **Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation,**
in partnership with *CME Group*;

About EDHEC-Risk Institute

- **Asset-Liability Management Techniques for Sovereign Wealth Fund Management**, in partnership with *Deutsche Bank*;
- **The Benefits of Volatility Derivatives in Equity Portfolio Management**, in partnership with *Eurex*;
- **Innovations and Regulations in Investment Banking**, in partnership with the *French Banking Federation (FBF)*;
- **Dynamic Allocation Models and New Forms of Target-Date Funds for Private and Institutional Clients**, in partnership with *La Française AM*;
- **Risk Allocation Solutions**, in partnership with *Lyxor Asset Management*;
- **Infrastructure Equity Investment Management and Benchmarking**, in partnership with *Meridiam and Campbell Lutyens*;
- **Risk Allocation Framework for Goal-Driven Investing Strategies**, in partnership with *Merrill Lynch Wealth Management*;
- **Financial Engineering and Global Alternative Portfolios for Institutional Investors**, in partnership with *Morgan Stanley Investment Management*;
- **Investment and Governance Characteristics of Infrastructure Debt Investments**, in partnership with *Natixis*;
- **Advanced Investment Solutions for Liability Hedging for Inflation Risk**, in partnership with *Ontario Teachers' Pension Plan*;
- **Cross-Sectional and Time-Series Estimates of Risk Premia in Bond Markets**, in partnership with *PIMCO*;
- **Active Allocation to Smart Factor Indices**, in partnership with *Rothschild & Cie*;
- **Solvency II**, in partnership with *Russell Investments*;
- **Advanced Modelling for Alternative Investments**, in partnership with *Société Générale Prime Services (Newedge)*;
- **Structured Equity Investment Strategies for Long-Term Asian Investors**, in partnership with *Société Générale Corporate & Investment Banking*.

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies and global conferences.

To ensure the distribution of its research to the industry, EDHEC-Risk also provides professionals with access to its website, <https://risk.edhec.edu>, which is devoted to international risk and investment management research for the industry. The website is aimed at professionals who wish to benefit from EDHEC-Risk's analysis and expertise in the area of investment solutions. Its quarterly newsletter is distributed to more than 150,000 readers.

About EDHEC-Risk Institute

Research for Business

EDHEC-Risk Institute also has highly significant executive education activities for professionals, in partnership with prestigious academic partners. EDHEC-Risk's executive education programmes help investment professionals upgrade their skills with advanced asset allocation and risk management training across traditional and alternative classes.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements. The first was with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of investment solutions for institutions and individuals. The second was with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of risk and investment management.

As part of its policy of transferring know-how to the industry, in 2013 EDHEC-Risk Institute also set up Scientific Beta, which is an original initiative that aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.

EDHEC-Risk Institute also contributed to the 2016 launch of EDHEC Infrastructure Institute (EDHEC*infra*), a spin-off dedicated to benchmarking private infrastructure investments. EDHEC*infra* was created to address the profound knowledge gap faced by infrastructure investors by collecting and standardising private investment and cash flow data and running state-of-the-art asset pricing and risk models to create the performance benchmarks that are needed for asset allocation, prudential regulation and the design of infrastructure investment solutions.

About EDHEC-Risk Institute

EDHEC-Risk Institute
Publications and Position
Papers (2016-2019)

EDHEC-Risk Institute

Publications and Position Papers (2016–2019)

2019

- Maeso, J.M., Martellini, L. and R. Rebonato. Cross-Sectional and Time-Series Momentum in Sovereign Bond Markets (May).
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