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SCIENTIFIC BETA SPECIAL

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Introduction
Noël Amenc

It is my pleasure to introduce this latest issue of the Research Insights supplement to IPE. In this issue, a 'Scientific Beta' special, we present research that has been developed by ERI Scientific Beta, an EDHEC-Risk Institute entity that aims to help investors understand and invest in advanced beta equity strategies.

One of the key points in the choice of investors to call cap-weighted indices into question as an investment benchmark is their poor diversification. It therefore seems logical for this first edition of the Scientific Beta supplement devoted to smart beta to begin with smart beta diversification indices.

Indeed, diversification strategy indices address the limitations of cap-weighted indices, such as their high concentration levels (in weight or risk contributions) or inefficient return-to-risk profiles. In our first article we examine five such diversification strategy indices (maximum deconcentration, diversified risk parity, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe) and draw the relevant conclusions for investors.

Since the performance of any investment cannot be dissociated from the risks taken, we then address the question of the risks of smart beta indices and the customisation of those risks. Clearly, alternative weighting schemes carry significant risks, in absolute terms and relative to their cap-weighted reference index. Since departing from the traditional cap-weighting portfolio construction will lead to different risk return profiles, as well as a specific set of risk exposures, investors need to be aware of the risks they bear when making the smart beta choice.

We subsequently turn to the subject of smart beta allocation. Although the asset management industry has traditionally been divided into passive and active management, this distinction has been fading away recently and smart beta indices can be applied in both areas. We particularly focus on the benefits to be gained from a diversified allocation to a variety of different smart beta benchmarks. This diversification can be a genuine added-value contribution from asset managers, who too often see smart beta as a threat and not as an opportunity for active management.

We then analyse the conditional performance of smart beta strategies. Apart from showing that portfolios that perform best in bull markets are riskier and have lower Sharpe ratios than portfolios that perform best in bear markets, our study demonstrates that the contrasting characteristics of portfolios that perform best in bull/bear markets could work in favour of the investors if they are mixed equally into a single portfolio.

An analysis of the specific risks of diversification strategies looks specifically at the strategies that were presented in the article on diversification strategy indices. Again we see that a combination of these different strategies will allow the risks that are specific to each strategy to be diversified by exploiting the imperfect correlation between the different strategies' parameter estimation errors and the differences in their underlying optimality assumptions.

Finally, we examine the robustness of smart beta strategies. Since these strategies are for the most part recent, there is little in the way of live historical track records. Informing investors of the risk and return drivers of the strategies, ie, the risk factors they are exposed to, will allow them to control for risk exposures that drive returns in order to extract the substance of the chosen weighting scheme instead of having inconsistent results through time.

We hope that the 'Scientific Beta' articles in the supplement will prove useful and informative. We wish you an enjoyable read and extend our thanks as ever to our friends at IPE for their collaboration on the supplement.

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Contents

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Overview of diversification strategies

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Modern Portfolio Theory prescribes that every rational investor should split the investment process into two steps: (i) construct a portfolio of risky assets with the maximum Sharpe ratio (the MSR portfolio); and (ii) allocate wealth between the MSR portfolio and a riskless asset in a proportion that matches the investor's risk appetite. Therefore, the only portfolio of risky assets that should be of interest to a rational investor is the MSR portfolio.

Implementing this objective of Sharpe ratio maximisation, however, is a complex task because of the presence of estimation risk for the required expected returns and risk parameters. The costs of parameter estimation error may in some cases entirely offset the benefits of optimal portfolio diversification (see, eg, De Miguel et al [2009b]). Therefore some methodologies for constructing diversification strategy indices do not explicitly aim to obtain a portfolio with an optimal risk/reward ratio, but instead adopt heuristic approaches to diversification by trying to have fewer parameters to estimate or parameters whose estimation would be easier.

Heuristic or ad-hoc strategies, which have objectives different from Sharpe ratio maximisation, can be further categorised into *deconcentration* and *decorrelation*-based approaches. Deconcentration-based strategies simply focus on reducing the weight and risk concentration of portfolios by spreading out the constituents' weights or their risk contributions equally.[1] This can be seen as a response to concerns about weight or risk concentration which may arise in cap-weighted equity indices.[2] Decorrelation strategies focus on risk reduction that stems from the fact that assets are imperfectly correlated.

In contrast to these heuristic approaches, scientific or *efficient diversification* methodologies are based on the theoretical framework of Modern Portfolio Theory and aim at obtaining efficient frontier portfolios – ie, portfolios that obtain the lowest level of volatility for a given level of expected return (and thus the highest risk-adjusted return).[3] We will now briefly describe three *heuristic diversification* weighting schemes (Maximum Deconcentration, Diversified Risk Parity and Maximum Decorrelation) and then two *efficient diversification* strategies, Efficient Minimum Volatility and Efficient Maximum Sharpe. Additionally, it should be noted that ERI Scientific Beta applies turnover control and liquidity rules to all its indices to ensure that they take into account practical investment constraints.

Diversification strategies

Equal weighting (also known as the '1/N' weighting scheme) is a simple way of 'deconcentrating' a portfolio in terms of stock weights or maximising the effective number of stocks.[4] This strategy has been shown to deliver attractive performance even in comparison with sophisticated portfolio optimisation strategies (De Miguel et al [2009b]). Depending on the size of the stock universe, equal weighting can lead to relatively high turnover and liquidity problems.[5] Maximum Deconcentration addresses this drawback and minimises the distance of weights from the equal weights subject to constraints on turnover and liquidity.

Extending the notion of weight deconcentration to risk deconcentration, the general risk parity approach aims to equalise the risk contributions of constituent stocks to the total portfolio risk:

$$w_i \frac{\partial \sigma_p}{\partial w_i} = w_j \frac{\partial \sigma_p}{\partial w_j}$$

where w_i is the (positive) portfolio weight of stock i and σ_p the portfolio volatility (see Maillard, Roncalli and Teïletche [2010] for a detailed discussion). It should be noted that in the general case no analytical solution is available to this problem; it therefore needs to be solved numerically. Diversified Risk Parity, which is based on a specific case of the general risk parity problem, is a weighting scheme that attempts to equalise the risk contributions of individual stocks to the total risk of the index, assuming uniform correlations across stocks. This assumption has the advantage that the optimal weights can be derived analytically, without relying on any numerical resolution. Indeed, in the absence of any constraints, such as tracking error or sector neutrality constraints, Diversified Risk Parity boils down to inverse volatility weighting. Furthermore, the use of identical pairwise correlations allows a high level of robustness to be achieved. Indeed, Elton and Gruber (1973) show that the assumption of identical correlations leads to surprisingly reliable estimates of realised correlations. In theory the constant correlation model may appear very unrealistic but in practice, setting all pair-wise correlations between stocks to their overall average may be a reasonable approach. This is the case because estimates of the entire set of correlation coefficients tend to be very noisy when the number of constituents is large, and hence it may be better in some cases to use a simplifying assumption than to use potentially very noisy correlation estimates.

In fact, Maximum Deconcentration and Diversified Risk Parity overlook the fact that exploiting the imperfect interactions between the underlying assets is at the heart of diversification. A large body of literature has assessed diversification benefits, notably in the area of international equity portfolio management, by focusing on a measure of how well the portfolio exploits correlation effects.[6] For instance, Goetzmann, Li and Rouwenhorst (2001) measure the diversification benefits of an international investment as the ratio of the total variance of an equally-weighted portfolio to the average variance of its constituents. We refer to this measure as the GLR measure.[7] In

1 The risk contribution of a constituent is defined as the product of the constituent's weight with the marginal contribution of this constituent to the total portfolio volatility.

2 Some of the known shortcomings of cap-weighted equity indices arise from the issue of: (i) their high concentration in the larger capitalisation stocks – Malevergne, Santa-Clara and Sornette (2009) show that cap-weighted indices hold a very low effective number of stocks (as measured by the reciprocal of the Herfindahl index) relative to their nominal number of constituents – or (ii) their lack of risk/return efficiency (see for instance Ferson, Kandel and Stambaugh [1987], as well as Goltz and Le Sourd [2011], and the references therein).

3 It should be noted however that the heuristic and scientific approaches to diversification are not mutually exclusive – for instance, the motivation for the addition of weight constraints to a scientific diversification methodology can be to bring it closer to a heuristic methodology in order to gain robustness.

4 The effective number of stocks is defined as the reciprocal of the Herfindahl Index, which is a commonly used measure of portfolio concentration:

$$\text{Effective number of stocks} = \frac{1}{\sum_{i=1}^{N} w_i^2}$$

where N is the number of constituent stocks in the index and w_i is the weight of stock i in the index. In brief, the effective number of stocks in a portfolio indicates how many stocks would be needed in an equal-weighted portfolio to obtain the same level of concentration (as measured by the Herfindahl Index). Equal-weighting stocks in a portfolio will lead to the maximum effective number of stocks.

5 In particular, in very broad universes that contain stocks with little liquidity, the rebalancing back to equal weights may be difficult to implement (see Blitz [2013]). Plyakha, Uppal and Vilkov (2012), Demey, Maillard and Roncalli (2010) and Leote de Carvalho, Xu and Moulin (2012) show equal-weighted strategies have moderately higher levels of turnover compared to market capitalisation weighted portfolios. Dash and Loggie (2008) point out that transaction costs can become important for equal-weighting when the universe includes less liquid stocks. However, the intensity of the liquidity problems depends on the universe being chosen. Intuitively, the liquidity risk will be lower if one were to apply the equal-weighting scheme to a universe consisting of the largest stocks rather than to a universe including both large and small cap stocks.

6 See for instance, Longin and Solnik (1995) and Goetzmann, Li and Rouwenhorst (2001).

7 $$GLR = \frac{Var(R_p)}{\sum_{i=1}^{N} W_i Var(R_i)}$$

where N is the number of stocks in the portfolio, R_p is the return of portfolio, W_i is the weight of stock and R_i is the return of stock i.

fact, this ratio can be viewed as the contribution of average pair-wise correlations to the volatility of the portfolio compared to that of a portfolio composed of uncorrelated stocks. The lower the GLR measure, the higher the diversification benefit of combining the set of stocks into a portfolio. Conversely, the diversification benefit will be minimal (and the GLR measure will be highest) in the extreme case where all pair-wise correlations are equal to one. Note that the computation of the GLR measure does involve both the individual stock volatilities and the pair-wise correlations. Disentangling the effect of correlations from the effect of individual asset volatilities has been widely discussed (see, eg, Amenc, Goltz and Stoyanov [2011]) and the Maximum Decorrelation weighting scheme tries to exploit the effect of risk reduction through 'decorrelation'. It is inspired by Christoffersen et al (2010), who focus on solely exploiting the correlation structure when measuring the benefits of diversification. They assess the diversification potential within a given global equity universe by minimising the total portfolio variance under the assumption that all individual assets' volatilities are identical, thus relying only on the information about their correlations.

In contrast with the three ad-hoc diversification strategies discussed above, the true Minimum Volatility portfolio lies on the efficient frontier. Indeed, the Minimum Volatility portfolio corresponds to a particular spot on the efficient frontier representing the portfolio that has the lowest level of volatility among all feasible portfolios. The Minimum Volatility strategy can be seen as an attempt to exploit information on risk parameters,

"The negative performance of equity markets following the 2008 financial crisis has spurred the demand for defensive equity strategies. The Minimum Volatility strategy is now a well-accepted solution among investors seeking low-risk equity investments"

including stock volatility and correlations across stocks. The fact that Minimum Volatility portfolios do not rely on expected returns estimates is an attractive feature as it is well documented that expected return estimates are unreliable (Merton [1980]) and from that viewpoint, the Minimum Volatility portfolio is sometimes considered to be an efficient and robust proxy for the optimal portfolio. Moreover, the negative performance of equity markets following the 2008 financial crisis has spurred the demand for defensive equity strategies. The Minimum Volatility strategy is now a well-accepted solution among investors seeking low-risk equity investments.

Nevertheless, a common problem cited for the Minimum Volatility strategy is that of concentration in low risk (low volatility or low beta) stocks, which in turn leads to pronounced sector biases towards defensive sectors such as utilities (see Chan et al [1999]). A possible remedy to this problem of concentration in low volatility stocks is to introduce weight constraints. Jagannathan and Ma (2003) show that weight constraints not only control the concentration but also improve the performance of Minimum Volatility portfolios. DeMiguel et al (2009a) go beyond considering rigid constraints at the individual stock level and introduce flexible out-of-sample risk and return properties of Minimum Volatility portfolios.[8] The Scientific Beta Efficient Minimum Volatility weighting scheme provides a proxy for the Minimum Volatility portfolio, and uses such flexible norm constraints within the optimisation procedure.

In addition to norm constraints, one can use a sector neutrality constraint – which is a more direct tool to control sector exposure of indices. Figure 1 shows that the Scientific Beta Developed World Efficient Minimum Volatility index is overexposed to utilities and non-cyclical consumer goods (>+7%) and under-exposed to the financial and technology sectors (<–4%). The sector neutrality constraints successfully bring excess exposures of all sectors close to zero.

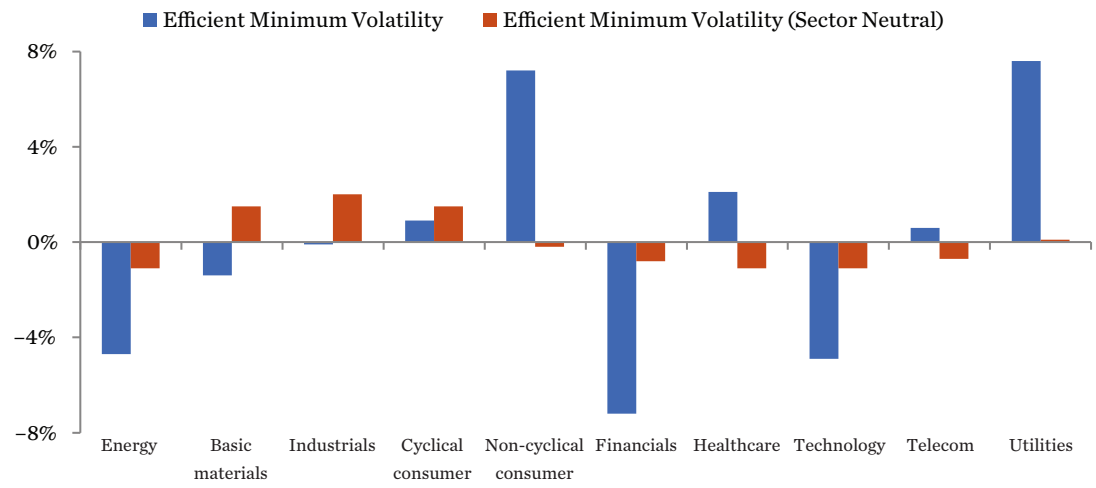
The Efficient Maximum Sharpe Ratio strategy is an implementable proxy for the tangency portfolio from Modern Portfolio Theory. As in any mean-variance optimisation, the estimation of input parameters is a central ingredient in the implementation of the methodology.

In contrast to minimum volatility strategies, the Maximum Sharpe ratio strategy relies on estimates of both risk parameters (volatilities and correlations) and expected returns. As direct estimation of expected returns is known to lead to large estimation errors (Merton [1980]), ERI Scientific Beta's Efficient Maximum Sharpe Ratio strategy estimates expected returns indirectly by assuming that they are positively related to a stock's semi-deviation (see Amenc et al [2011]).[9] More specifically, an extra step is added to the estimation process to provide more robustness: stocks are sorted by their semi-deviation into deciles and all stocks in a decile are then assigned the median value of the decile.

The Efficient Maximum Sharpe Ratio strategy can be an alternative to the minimum volatility approach, especially for investors who do not wish to hold a portfolio concentrated in low volatility stocks for long periods. Figure 2 shows that the Minimum Volatility strategy invests about 73% in the 40% least volatile stocks and just about 13% in the 40% most volatile stocks. The Maximum Sharpe Ratio strategy, on the other hand, features more homogeneous weight distribution across volatility quintiles.

Figure 3 summarises the description of ►

1. Excess sector exposures



The figure shows the excess sector exposures of the Scientific Beta Developed Efficient Minimum Volatility index and Scientific Beta Developed Efficient Minimum Volatility (Sector Neutral) index (over the Scientific Beta Developed Cap-Weighted benchmark) based on portfolio weights as of 21 December 2012. The total number of stocks in the Scientific Beta Developed World universe is 2,000.

2. Weight distribution of volatility

Scientific Beta Developed World	Low volatility	2	3	4	High volatility
Efficient Maximum Volatility	48.0%	24.8%	14.2%	8.7%	4.3%
Efficient Maximum Sharpe Ratio	31.8%	20.4%	17.1%	16.1%	14.6%

The table shows the weight distribution of the Scientific Beta Developed Efficient Minimum Volatility index and Scientific Beta Developed Efficient Sharpe Ratio index across volatility quintiles. The analysis is based on portfolio weights as of 21 December 2012. Stocks' volatilities over the past 104 weeks have been used to form volatility quintiles. The total number of stocks in the Scientific Beta Developed World universe is 2,000.

8 The authors show that using such flexible concentration constraints instead of rigid upper and lower bounds on individual stock weights (as in Jagannathan and Ma [2003]) allows for a better use of the correlation structure. The quadratic norm constraints used for the strategy can be written in terms of portfolio weights as:

$$\|w\|_2 = \sum_{i=1}^{N} w_i^2 \leq \frac{3}{N}$$

9 A number of studies show a positive relation between expected return and different measures of downside risk. Bali and Cakici (2004) and Huang et al (2012) find that a stock's expected return has a strong positive relation with its VaR and its extreme downside risk, respectively. Chen et al (2009) and Estrada (2007) show a positive relation between a stock's expected return and its semi-deviation. Ang et al (2006a) document a positive relation between a stock's downside beta (stocks that are strongly correlated with the market when it goes down) and its expected return.

3. Overview of popular equity diversification strategies

Strategy	Objective	Unconstrained closed-form solution	Required parameter(s)	Optimality conditions
Maximum Deconcentration	Maximise effective number of stocks	$w^* = \frac{1}{N}\mathbb{1}$	None	$\mu_i = \mu \ \forall\, i$ $\sigma_i = \sigma \ \forall\, i$ $\rho_{ij} = \rho \ \forall\, i$
Diversified Risk Parity	Equalise risk contributions under 'constant correlation' assumption	$w^* = \frac{diag\left(\boldsymbol{\sigma}^{-1}\right)}{\mathbb{1}' diag\left(\boldsymbol{\sigma}^{-1}\right)}$	σ_i	$\lambda_i = \lambda \ \forall\, i$ $\rho_{ij} = \rho \ \forall\, i$
Maximum Decorrelation	Minimise the portfolio volatility under the assumption of identical volatility across all stocks	$w^* = \frac{\boldsymbol{\Omega}^{-1}\mathbb{1}}{\mathbb{1}'\boldsymbol{\Omega}^{-1}\mathbb{1}}$	ρ_{ij}	$\mu_i = \mu \ \forall\, i$ $\sigma_i = \sigma \ \forall\, i$
Efficient Minimum Volatility	Minimise portfolio volatility	$w^* = \frac{\Sigma^{-1}\mathbb{1}}{\mathbb{1}'\Sigma^{-1}\mathbb{1}}$	σ_i, ρ_{ij}	$\mu_i = \mu \ \forall\, I$
Efficient Maximum Sharpe Ratio	Maximise portfolio Sharpe ratio	$w^* = \frac{\Sigma^{-1}\mu}{\mathbb{1}'\Sigma^{-1}\mu}$	$\mu_i, \sigma_i, \rho_{ij}$	Optimal by construction

The table indicates, for the diversification strategies, the optimisation objective (without taking into account any constraints, turnover control or liquidity rules), its unconstrained solution and the required parameters. The 'Optimality conditions' column indicates under which conditions each diversification strategy would result in the Maximum Sharpe Ratio portfolio of Modern Portfolio Theory. N is the number of stocks, μ_i is the expected return on stock i, σ_i is the volatility for stock i, ρ_{ij} is the correlation between stocks i and j, $\boldsymbol{\mu}$ is the (N×1) vector of expected returns, $\mathbb{1}$ is the (N×1) vector of ones, $\boldsymbol{\sigma}$ is the (N×1) vector of volatilities, Ω is the (N×N) correlation matrix and Σ is the (N×N) covariance matrix.

the five diversification strategies. Interestingly, since the diversification strategies differ from each other in the assumptions they make and the objectives they aim to achieve, the combination of these different strategies allows the risks that are specific to each strategy to be diversified away by exploiting the imperfect correlation between the different strategies' parameter estimation errors and the differences in their underlying optimality assumptions. Moreover, as the single strategies' performance will show different profiles of dependence on market conditions, a multi-strategy approach can help investors smooth the overall performance across market conditions.[10] For instance[11], Amenc et al (2012) form a combination of two diversification approaches[12] that leads to smoother conditional performance and higher probability of outperforming the cap-weighted index. In the same spirit, the ERI Scientific Beta Diversified Multi-Strategy weighting scheme combines in equal proportions the Efficient Maximum Sharpe Ratio, the Efficient Minimum Volatility, the Maximum Decorrelation, the Diversified Risk Parity and the Maximum Deconcentration weighting schemes.

10 This topic is discussed at more length in Badaoui and Lodh (2013).

11 Tu and Zhou (2010), Kan and Zhou (2007) and Martellini, Milhau and Tarelli (2013) among others also study whether portfolios of strategies can improve the performance of individual strategies.

12 Robust proxies for the Minimum Volatility portfolio provide defensive exposure to equity markets that does well in adverse market conditions, while robust proxies for Maximum Sharpe Ratio portfolios provide greater access to the upside of equity markets.

13 Gonzalez and Thabault (2013) present a more detailed performance and risk analysis of these diversification strategies. Amenc et al (2013) argue that investors should not only measure but also be allowed to control their risks at each step of the portfolio construction process: factor risks at the stock selection stage, and sector/country relative risks as well as tracking error risk against the cap-weighted reference index at the optimisation stage. Goltz and Gonzalez (2013) show how these risk control choices can be used by investors to tailor smart beta strategies to their needs (see also 'Risks of smart beta indices and customisation of these risks' in the present supplement – page 6).

14 Volatility concentration $= \sum_{i=1}^{N} C_i^2$ where

$$C_i = \frac{W_i.\sigma_i}{\sum_{j=1}^{N} W_j.\sigma_j}$$

where W_i is the weight of stock i in the index and σ_i is the volatility of stock i.

15 Weighted average market cap of $i = \sum_{k=1}^{N} W_{k,i}.MarketCap_k$

where $W_{k,i}$ is the weight of stock k in index i, N is the total number of stocks in the index, and $MarketCap_k$ is the float-adjusted market cap of stock k.

Performance and risk analysis

In this section, we briefly evaluate the performance and risks of the Scientific Beta Developed diversification strategy indices.[13] Figure 4 shows that all the diversification strategies tend to deliver higher returns than the cap-weighted reference index with annualised outperformance ranging from 1.93% (Maximum Decorrelation) to 2.71% (Efficient Minimum Volatility). Moreover all of the diversification strategy indices exhibit better risk-adjusted performance, with Sharpe ratios ranging from 0.40 to 0.55 (compared to 0.30 for the cap-weighted reference index).

Next we analyse the attainment of objective for each strategy in detail. The Efficient Minimum Volatility index delivers the least volatility; it has a volatility of 14.44% compared to 17.66% for the cap-weighted benchmark. Also, the Efficient Maximum Sharpe Ratio index results in a Sharpe ratio of 0.46 which is well above that of the cap-weighted index (0.30). However, the Efficient Minimum Volatility index achieves an even higher Sharpe ratio of 0.56, resulting from both higher returns and lower volatility than the Efficient Maximum Sharpe Ratio index over the analysis period, which tended to be favourable to defensive portfolios. As shown in the previous section, the Efficient Minimum Volatility index concentrates more in low volatility stocks and this defensive exposure is also confirmed by a low market beta of 0.81 as opposed to a market beta of 0.90 for the Efficient Maximum Sharpe Ratio index. As a result, both these strategies are potential tangency portfolio proxies, and the choice between them depends on the degree of defensiveness desired by the investor.

For heuristic strategies, the explicit index construction objective is not to maximise its Sharpe ratio directly. The idea is that achieving the objective set for this diversification scheme will enable the risk-adjusted performance of the benchmark in comparison to cap-weighted indices to be improved indirectly. Each of these schemes aims to correct a design flaw in cap-weighted indices which can often be summed up in terms of overconcentration. This overconcentration can be understood in an initial analysis as an excessively low effective number of stocks (ie, too much concentration of the value of the investment in a small number of stocks). In this case, the approaches that explicitly aim to maximise the effective number of stocks are the most effective. As such, the effective number of stocks for the Maximum Deconcentration index is 1,358, which is significantly higher than the mere 365 for the cap-weighted index.

If the analysis of the flaw in the cap-weighted index relates more to concentration of the risks, defined as an excessive volatility concentration, rather than the stocks, we can define the Volatility Concentration[14] (VC) as the Herfindahl index of the relative weighted average of stock volatilities. A lower value of this statistic means the portfolio is closest to an inverse-volatility weighted portfolio out-of-sample. As expected, the Diversified Risk Parity index shows a significantly low VC (0.08%)

"Each of these schemes aims to correct a design flaw in cap-weighted indices which can often be summed up in terms of overconcentration. This overconcentration can be understood in an initial analysis as an excessively low effective number of stocks"

compared to the cap-weighted index (0.27%). Results also show that the Maximum Decorrelation index fulfils its objective of reducing the GLR measure.

One-way annual turnover of all diversification strategies is close to 30%, showing the effectiveness of turnover rules. The strategies are adequately liquid as their weighted average market capitalisation[15] is about one-third of that of the cap-weighted index, which is highly liquid by construction.

The Diversified Multistrategy index, being an equal-weighted average of the five indices, provides close to average values of statistics such as returns, volatility, Sharpe ratio and market beta. As discussed before, combining strategies allows outperformance to be smoothed out across different market conditions (Amenc et al [2012]). Figure 5 shows how the Diversified Multistrategy approach averages out the excess returns

4. Absolute and relative performance of diversification strategy indices

	Scientific Beta Developed World Indices						
	Maximum Deconcentration	Diversified Risk Parity	Maximum Decorrelation	Efficient Minimum Volatility	Efficient Maximum Sharpe Ratio	Diversified Multistrategy	Cap-Weighted
Annual returns	8.94%	8.91%	8.88%	9.66%	9.15%	9.17%	6.95%
Annual volatility	18.06%	16.90%	16.72%	**14.44%**	16.13%	16.45%	17.66%
Sharpe ratio	0.40	0.43	0.43	0.55	**0.46**	0.46	0.30
Volatility concentration	0.08%	0.08%	0.13%	0.14%	0.12%	0.09%	0.27%
GLR measure	33.8%	33.0%	**28.7%**	28.8%	29.2%	30.6%	40.2%
Efficient number of stocks	**1,358**	1,264	875	699	836	1,122	365
CAPM beta	1.01	0.95	0.94	0.81	0.90	0.92	1.00
Maximum drawdown	59.07%	57.04%	56.05%	50.65%	55.08%	55.65%	57.27%
Cornish Fisher 5%	1.74%	1.63%	1.63%	1.37%	1.56%	1.59%	1.67%
Annual 1-way turnover	29.1%	21.3%	33.3%	32.0%	32.9%	25.8%	3.7%
Weighted average market cap	15,763	17,373	15,946	18,820	16,956	16,854	62,637

The statistics are based on daily total returns (with dividend reinvested) over the analysis period from inception date (21 June 2002) to 31 December 2012. All statistics are annualised and performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding period returns for investors. The total number of stocks in the Scientific Beta Developed World universe is 2,000.

5. Conditional relative returns of diversification strategy indices

	Scientific Beta Developed World Indices					
Relative returns (over CW)	Maximum Deconcentration	Diversified Risk Parity	Maximum Decorrelation	Efficient Minimum Volatility	Efficient Maximum Sharpe Ratio	Diversified Multistrategy
Bull market	0.89%	0.45%	0.43%	–0.33%	0.25%	0.36%
Bear market	–0.19%	0.53%	0.57%	2.37%	1.05%	0.86%
High volatility market	0.13%	0.29%	0.29%	1.21%	0.57%	0.53%
Low volatility market	0.88%	0.69%	0.68%	0.08%	0.51%	0.56%

The table shows the excess returns (over the Scientific Beta Developed World Cap-Weighted index) of diversification strategy indices in bull/bear and high/low volatility market conditions. The analysis is based on daily total returns (with dividend reinvested) over the analysis period from inception date (21 June 2002) to 31 December 2012. Quarters with positive returns for the reference index are labelled 'bull market' and the remaining quarters are labelled 'bear market'. High volatility quarters are the top 50% of quarters sorted on the reference index's volatility and low volatility quarters are the rest. All returns are quarterly and geometric averaged. The total number of stocks in the Scientific Beta Developed World universe is 2,000.

over different market regimes. Unlike some of the diversification strategies, its performance in bull/bear markets and high/low volatility markets is not extreme.

Conclusion

In brief, the diversification strategy indices address the limitations of cap-weighted indices, such as their high concentration levels (in weight or risk contributions) or inefficient return-to-risk profiles. Although each strategy has its own benefits, it also has certain limitations that stem from its specific risks. Investors can diversify the strategy-specific risk by allocating across strategies in the form of a diversified multistrategy index. To investors who are agnostic about either their capacity to identify the model with superior assumptions, or their capacity to take the risk of choosing a particular model in the wrong market conditions, the Scientific Beta Diversified Multi-Strategy index presents itself as an interesting candidate.

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Risks of smart beta indices and customisation of these risks

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The advent of smart beta investing came with a plethoric offer of indices with embedded portfolio construction methods that represent alternatives to cap weighting. Whether those solutions are backed by financial theory (eg, through the use of scientific diversification such as minimum volatility portfolios) or based upon ad-hoc methodologies (eg, equal weighting or characteristic-based weighting such as fundamentally-weighted indices), alternative weighting schemes carry significant risks, in absolute terms and relative to their cap-weighted reference index, as shown in Amenc, Goltz and Martellini (2013). These risks include systematic risks (common factor exposures) and specific risks (risks linked to the underlying model and input variables). In this article, we illustrate those risks using Scientific Beta Indices, as well as the capability of managing those risks using the Scientific Beta Platform.

Risks of smart beta solutions

The choice of a given weighting scheme leads to a different set of risk and return properties relative to cap-weighted indices. In figure 1 we show for example that the Maximum Deconcentration strategy has a moderate tracking error against its reference benchmark (3.62%) since it does not take the risk and return properties of stocks into account, while the Efficient Minimum Volatility has a high tracking error (at 4.60%) because it takes into account their volatility and correlation with an objective of reducing portfolio volatility, leading to a tilt towards low-beta stocks.

Hence, specific and systematic risks should be identified so investors can make informed choices, and condition those choices upon their expectations of market and economic conditions.

Specific risks are related to the characteristics of a given portfolio construction methodology and encompass two competing sources of risk: parameter estimation risk (the distance between estimated and true risk and return parameters in the attempt to reach the point on the efficient frontier which has the maximum Sharpe ratio), and optimality risk (the risk that arises when making simplifying assumptions about the risk and return parameters). For a detailed explanation of the concept of specific risks, we refer the interested reader to the article on 'Analysis of the specific risks of diversification strategies" in this IPE supplement (page 15).

These new forms of indices are also exposed to systematic risk factors, depending on the methodological choices guiding their construction and on the universe of stocks to which their construction schemes are applied. For example, given that a cap-weighted index is typically concentrated in the largest-capitalisation stocks, any departure from it through deconcentration will necessarily lead to an increase in the exposure to smaller-cap stocks. Figure 2 shows that the choice of an alternative weighting scheme will induce a particular set of exposures to systematic factors, as illustrated using five different Scientific Beta diversification strategies.

Choosing risks

Our approach to risk management emphasises the measurement and control of risks at each step of the portfolio construction process.

Firstly, a clear distinction between the stock selection phase and the weighting phase allows correction of implicit factor tilts that may arise from the weighting scheme through an explicit choice of the universe in which the strategy invests.[1]

Stock selection can be viewed as a reduction of the investment universe. When it is performed upon a particular stock-based characteristic linked to stock-specific risks, such as size, stock selection allows this specific factor exposure to be shifted, regardless of the weights that will be applied to individual portfolio components. Amenc et al (2012) show that stock selection is able to correct the risk factor exposures of diversification-based weighting schemes by excluding stocks with the undesired characteristics prior to applying a diversification scheme.

For example, investors can modify the exposure of their strategy to value by using a simple value/growth stock selection prior to the optimisation. Our value selection picks the top 50% of stocks by book-to-market value; conversely the growth selection picks the bottom 50% of stocks by book-to-market value from the full USA universe.

Our value/growth selection scheme allows the exposure to the value factor to be modified. For example, as shown in figure 3 for the Maximum Deconcentration strategy, the value tilt

1. Relative performance of US Scientific Beta indices with regard to reference cap-weighted index

	Scientific Beta USA Maximum Deconcentration	Scientific Beta USA Diversified Risk Parity	Scientific Beta USA Maximum Decorrelation	Scientific Beta USA Efficient Minimum Volatility	Scientific Beta USA Efficient Maximum Sharpe Ratio
Relative return	2.02%	2.05%	1.53%	2.16%	1.72%
Tracking error	3.62%	3.08%	3.57%	4.60%	3.39%
CF 5% VaTER	0.36%	0.30%	0.37%	0.47%	0.36%
Historical VaTER	0.34%	0.29%	0.35%	0.44%	0.32%
Maximum relative drawdown	13.76%	10.39%	12.29%	7.12%	9.15%

This table shows the relative performance and risk statistics with respect to the reference cap-weighted index over the analysis period from inception date (21 June 2002) to 31 December 2012. The statistics are based on daily total returns (with dividend reinvested). All statistics are annualised and performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding period returns for investors. ERI Scientific Beta uses the Secondary Market US Treasury Bills (3M) as the risk-free rate in US dollars.

2. Risk factor exposures and risk-adjusted performance of US Scientific Beta Indices

	Scientific Beta USA Maximum Deconcentration	Scientific Beta USA Diversified Risk Parity	Scientific Beta USA Maximum Decorrelation	Scientific Beta USA Efficient Minimum Volatility	Scientific Beta USA Efficent Maximum Sharpe Ratio	Scientific Beta USA Cap-Weighted
Alpha	*0.40%*	*0.75%*	*0.37%*	***1.90%***	*0.77%*	*0.00%*
Market	**1.01**	**0.96**	**0.96**	**0.84**	**0.93**	**1.00**
Size (SMB)	**0.44**	**0.37**	**0.40**	**0.22**	**0.34**	**0.00**
Value (HML)	**–0.01**	**–0.01**	**–0.06**	**–0.06**	**–0.05**	**0.00**
Adjusted R-square	>99%	>99%	99%	98%	99%	100%
Sharpe ratio	0.28	0.30	0.28	0.36	0.30	0.21

This table shows the coefficient estimates and R-squared of the regression of the index's excess returns over the risk-free rate using the Fama-French three-factor model and Sharpe ratios over the analysis period from the inception date of the indices (21 June 2002) to 31 Deceember 2012. The coefficients statistically significant at the 95% confidence level are highlighted in bold. The data are daily total returns (with dividend reinvested). The Market factor is the daily return of the cap-weighted index of all stocks that constitute the index portfolio. SMB factor is the daily return series of a portfolio that is long the top 30% stocks (small market-cap stocks) and short the bottom 30% stocks (large market-cap stocks) sorted on market capitalisation in ascending order. HML factor is the daily return series of a portfolio that is long the top 30% stocks (value stocks) and short the bottom 30% stocks (growth stocks) sorted on book-to-market value in descending order. ERI Scientific Beta uses the Secondary Market US Treasury Bills (3M) as the risk-free rate in US dollars. The cap-weighted reference index used is the SciBeta USA cap-weighted index.

1 A distinction between stock universe selection and the selection of a diversification-based weighting scheme recognises that, in principle, methodological choices can be made independently in these two steps that are used in the construction of advanced beta equity strategies. Flexibly combining different possible choices in the two steps allows us to test the performance and risk of the possible methodologies and to assess commercially available advanced beta strategy indices by constructing strategies with similar objectives and constraints.

is shifted from close to neutral in the standard version to +0.19 in the value stock selection and, conversely, to –0.20 when choosing growth stocks only. Interestingly, this shift in value exposure does not impact other factors' initial exposures. The Sharpe ratios of the value and growth indices are still superior to that of the cap-weighted benchmark (0.21), showing that systematic risk control does not come at the cost of performance.

Secondly, our approach allows control over systematic risks through linear constraints on sector and country risk exposures within the optimisation process. Using Scientific Beta's Efficient Minimum Volatility index we illustrate the effects of controlling for sector risks (deviations from the cap-weighted index).

Chan et al (1999) show that minimum volatility portfolios are tilted towards low-volatility stocks and thus to low-volatility sectors such as utilities.[2] As displayed in figure 4, constraining for sector neutrality materially reduces sector deviations from the cap-weighted reference (eg, the +11.1% utility sector overweight is materially reduced to +0.20%). The aggregate effect of doing so across all sectors is equally dramatic. The sum of absolute sector deviations drops from 46% for the standard version of the US Efficient Minimum Volatility to 10% for the sector-neutral version.

"Given that a cap-weighted index is typically concentrated in the largest-capitalisation stocks, any departure from it through deconcentration will necessarily lead to an increase in the exposure to smaller-cap stocks"

Thirdly, as shown in Amenc et al (2012), departing from cap-weighting mechanically exposes investors to relative risk: notably that of severe underperformance against the reference cap-weighted indices.[3] This relative risk stems from the fact that the different risk factors to which the alternative advanced beta index is exposed have time-varying rewards (see, eg, Cohen, Polk and Vuolteenaho [2003]).

One solution to that issue is proposed by Jorion (2003) and applied by Amenc et al (2012): impose a hard constraint on the level of tracking error within the optimisation while minimising the volatility of that tracking error through time using a statistical factor model to ensure that the ex-post tracking error does not differ significantly from ex-ante tracking error objectives. Once the tracking error of the alternative index has been set to a stable constrained level, one can also use a core-satellite approach and mix that (satellite) index with the (core) cap-weighted reference index to adapt the level of tracking error of the final investment to the investor's tracking error budget.

ERI Scientific Beta implements this approach through an explicit tracking error constraint of 5% within the portfolio optimisation process, and offers investors the ability to further reduce tracking error through a core-satellite approach to 3% or 2%.

Finally, an important risk that is considered throughout the entire Scientific Beta portfolio construction process is the implementation risk (also referred to as investment risk, see, eg, Roncalli [2010]). In order to obtain investable indices, ERI Scientific Beta goes beyond deploying liquidity adjustments and turnover constraints[4] and also offers a liquidity risk management possibility, which, in a manner similar to the previously described feature on value/growth risk customisation, allows stocks to be selected on the basis of their liquidity characteristics. Here we illustrate how the Scientific Beta High Liquidity[5] stock selection allows liquidity risk to be controlled on the US Maximum Deconcentration index.

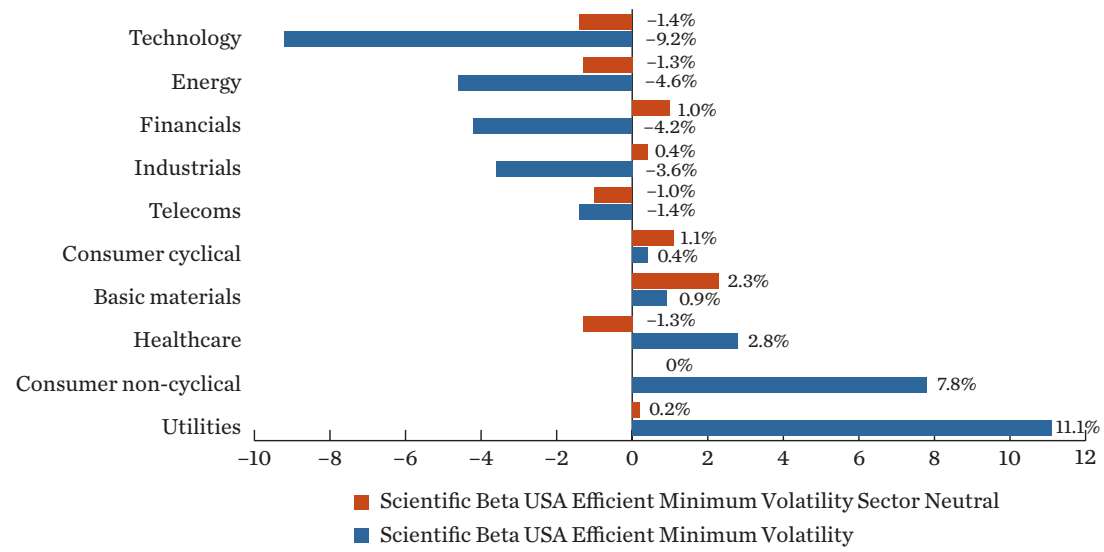
As shown in figure 5, the US Maximum Deconcentration index constructed using the full universe exhibits reduced liquidity, or investment capacity, against the cap-weighted ►

3. Controlling for the value factor exposure of the Scientific Beta Maximum Deconcentration Indices

Fama-French factors	Scientific Beta USA Value Maximum Deconcentration	Scientific Beta USA Maximum Deconcentration	Scientific Beta USA Growth Maximum Deconcentration	Impact of change in stock selection on factor exposure
	Coefficient	Coefficient	Coefficient	
Annualised alpha	*0.38%*	*0.40%*	*0.31%*	
Market	**0.98**	**1.01**	**1.03**	**Low impact**
Size (SMB)	**0.39**	**0.44**	**0.49**	**Low impact**
Value (HML)	**0.19**	**–0.01**	**–0.20**	**High impact**
Sharpe ratio	0.29	0.28	0.26	

This table shows the coefficient estimates and R-squared of the regression of the index's excess returns over the risk-free rate using the Fama-French three-factor model over the analysis period from inception date (21 June 2002) to 31 December 2012. The coefficients statistically significant at the 95% confidence level are highlighted in bold. The data are daily total returns (with dividend reinvested). The Market factor is the daily return of the cap-weighted index of all stocks that constitute the index portfolio. The SMB factor is the daily return series of a portfolio that is long the top 30% stocks (small market-cap stocks) and short the bottom 30% stocks (large market-cap stocks) sorted on market capitalisation in ascending order. The HML factor is the daily return series of a portfolio that is long the top 30% stocks (value stocks) and short the bottom 30% stocks (growth stocks) sorted on book-to-market value in descending order. ERI Scientific Beta uses the Secondary Market US Treasury Bills (3M) as the risk-free rate in US dollars. The cap-weighted reference index used is the SciBeta USA cap-weighted index.

4. Sector tilts of the Efficient Minimum Volatility standard and sector neutral indices



This chart shows sector exposures (in weight %) of the indices, based on their stock weight profile at the last rebalancing date (21 December 2012). We show the relative sector weights with respect to those of the reference cap-weighted index. The cap-weighted reference index is the Scientific Beta USA cap-weighted index. The sector classification used is the Thomson Reuters Business Classification.

2 For a thorough analysis of the Scientific Beta Efficient Minimum Volatility Strategy indices, we refer the reader to Scientific Beta's White Paper on this strategy found at www.scientificbeta.com/#/tab/article/scientific-beta-efficient-min-volatility-indices.

3 As explained in Amenc, Goltz and Martellini (2013), alternative benchmarks and indices have gained popularity due to their attractive absolute and relative returns, but (a) the new risks of those strategies and (b) the fact that such outperformance may be conditional on the choice of specific backtest periods, are not well understood.

4 Weight adjustments are implemented to achieve two objectives: we first impose a threshold for the weight of a stock and for the weight change at rebalancing, relative to the market-cap-weight of the stock in its universe. Second, ERI Scientific Beta indices are governed by an optimal turnover control technique based on rebalancing thresholds (Leland [1999], Martellini and Priaulet [2002]). This methodology consists in avoiding reviews when deviations between new optimal weights and current weights are relatively small, and brings down transaction costs to a large extent.

5 The liquidity stock selection scheme consists in ranking the stocks of the underlying universe according to their liquidity score and partitioning the resulting ranked stocks into two complementary and equally important sub-universes: High-liquidity stocks on the one hand, and mid-liquidity stocks on the other. The liquidity score of a stock is the average of its trading ratio z-score and trading volume z-score, both computed quarterly using data over the last four quarters. The median of the four quarterly values is considered for the z-score. The trading ratio of a stock is the ratio of number of days that the stock is actually exchanged to the total number of business days. The trading volume of a stock is its average traded daily dollar volume.

◄ reference (as measured by a weighted average market capitalisation of $22bn compared to $99bn for the reference cap-weighted index). Reducing the investment universe to the most liquid stocks, and thus customising the risk profile of the alternative index along the liquidity dimension allows that capacity gap to be reduced to reach $42bn average market capitalisation.

Conclusion

Because departing from the traditional cap-weighting portfolio construction will lead to different risk return profiles, as well as a specific set of risk exposures, investors need to be aware of the risks they bear when making the smart beta choice. It is important not only to measure those risks, but to enable users to control for most common exposures through separated, systematic, quantitative and well-identified portfolio construction steps: stock selection, weighting scheme and risk control. Moreover, controlling for risks with a Smart Beta 2.0 approach does not dilute the benefits of scientific diversification relative to cap-weighted indexing schemes.

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5. Risk factor exposures of the Standard and High Liquidity Maximum Deconcentration indices

	Scientific Beta USA Maximum Deconcentration	Scientific Beta USA High Liquidity Maximum Deconcentration
Fama-French factors	Coefficient	Coefficient
Alpha	*0.47*	*0.63*
Market	**1.01**	**1.08**
Size (SMB)	**0.44**	**0.35**
Value (HML)	**–0.01**	**–0.03**
Weighted average float	25,981	42,342

This table shows the coefficient estimates and R-squared of the regression of the index's excess returns over the risk-free rate using the Fama-French three-factor model over the analysis period from inception date (21 June 2002) to 31 December 2012. The coefficients statistically significant at the 95% confidence level are highlighted in bold. We also display the weighted average float-adjusted market capitalisation as an indication of liquidity (or investment capacity). The data are daily total returns (with dividend reinvested). The Market factor is the daily return of the cap-weighted index of all stocks that constitute the index portfolio. The SMB factor is the daily return series of a portfolio that is long the top 30% stocks (small market-cap stocks) and short the bottom 30% stocks (large market-cap stocks) sorted on market capitalisation in ascending order. The HML factor is the daily return series of a portfolio that is long the top 30% stocks (value stocks) and short the bottom 30% stocks (growth stocks) sorted on book-to-market value in descending order. ERI Scientific Beta uses the Secondary Market US Treasury Bills (3M) as the risk-free rate in US dollars. The cap-weighted reference index used is the SciBeta USA cap-weighted index.

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Beyond smart beta indexation

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Traditionally the asset management industry has separated product offerings into two distinct categories: passive management and active management. Simply put, passive management aims to deliver the aggregate market returns by simply replicating the cap-weighted or 'market' indices. The aim of active management on the other hand is to earn higher returns than the market, typically by exploiting the manager's security selection skills, in exchange for higher incentive (or management) fees compared to passive investment. Recently, however, with the growing realisation of the importance of systematic risk factors or betas, this demarcation has been gradually fading away. This article explains how it has given way to an increasing number of alternative equity index strategies or smart beta strategies (sometimes referred to as a 'third way' of managing equity portfolios). It shows the implication of smart beta indices in both passive and active management, with an emphasis on the benefits gained from a diversified allocation to a variety of different smart beta benchmarks.

Constructing a well-diversified benchmark: origins of smart beta

Be it the bursting of the tech bubble in 2000 or the financial crisis of 2008, the most important determinant of the performance of equity investments was not the stock selection but the associated exposure to systematic factors (or betas). For example, in the crisis of 2000, the stocks that were exposed to the new IT sector, growth or momentum styles underperformed severely while those exposed to the value style performed relatively well. Irrespective of the stock picking skill of managers, the long-term performance and the drawdowns of actively managed funds resulted from their choice of betas. The concept of systematic risk factors (betas) has been the foundation of modern portfolio theory and has undergone academic developments into what we call factor models. With betas being the key ingredients of active management, asset managers have become aware of the importance of managing betas and their diversity.

Indices or building blocks replicating micro-economic factors (like size, momentum, value, liquidity or volatility) and macroeconomic factors (like geographical region or industry sector) can be found in abundance in the market. Going beyond pure factor replication, one needs to address the issue of harvesting the risk premium of these betas. Plenty of empirical evidence shows that cap-weighted indices are not well diversified, efficient benchmarks – ie, they do not provide 'fair compensation' for the amount of risk taken (Haugen and Baker [1991], Grinold [1992]). The response from quantitative finance to this problem of extracting the right risk premia has come in the form of smart beta.

Smart beta strategies can be divided into two broad categories. Risk factor smart beta indices have the ability to extract the factor premiums most efficiently for a predominant risk factor. Diversified smart beta indices provide pre-packaged diversification-based solutions that can potentially replace the value created by a portfolio manager with the value of diversification. In both cases, smart beta strategies are based on the principle of systematic and transparent rules, deviating from pure buy

and hold strategies, and allowing for objectives other than representation of the market. Smart beta approaches, which make use of scientific diversification, were originally intended to address the common shortcomings of cap-weighted indices, such as concentration in a few stocks or the ignoring of correlations, to name only two. In the end, their ability to outperform cap-weighted indices through improved diversification is undoubtedly the main reason for the popularity of these smart beta strategies.

Passive management: where do smart beta strategies fit in?

In addition to improved performance, smart beta strategies maintain some of the advantages of cap-weighted indices (like systematic and transparent rules, low cost and low turnover) and can thus be used in passive management. We propose two kinds of use of smart beta in passive management – as a *replacement* for cap-weighted indices and as a *complement* to cap-weighted indices.

In its role as a replacement for a cap-weighted index, the smart beta index or a combination of smart beta indices becomes the strategic equity benchmark. The EDHEC-Risk North American and European Index surveys report that, although more than 40% of investment professionals have adopted alternative weighting schemes in their equity investments, very few of those see them as a replacement for cap-weighted indices (figures 1 and 2).[1] This is not surprising as the long standing monopoly and popularity of cap-weighted indices as benchmarks, owing to their simplicity, is not easy to replace. The surveys reveal that the smart beta techniques find rather broader application as a complement to cap-weighted indices.

"Smart beta approaches, which make use of scientific diversification, were originally intended to address the common shortcomings of cap-weighted indices, such as concentration in a few stocks or the ignoring of correlations, to name only two"

As a complement to a cap-weighted index, the smart beta indices can be used in two ways. Firstly, in what is termed enhanced indexation, smart beta strategies are used as a substitute for active benchmarked managers to outperform the cap-weighted benchmark. In this framework, the chief investment officers take considerable reputation risk. All advanced beta strategies need to deviate from the cap-weighted index, in terms of factor exposures and portfolio construction model,[2] to generate outperformance. The risk choices of smart beta strategies may be less rewarded than those of the cap-weighted index in certain periods, which makes them susceptible to periods of serious underperformance – ie, significant and lasting relative drawdowns (Amenc et al [2012]). Therefore, control of tracking error to hedge relative risk, including risk of extreme underperformance, becomes important. It is noteworthy that the first-generation smart beta index providers provide little risk assessment, let alone risk control. We go one step further and allow investors, in what we call Smart Beta 2.0, to control their risks, such as micro and macroeconomic factor exposures (through a characteristic-based stock selection option) and relative risk (through a tracking error control option) while maintaining the outperformance.[3]

Secondly, smart beta indices can be used to construct a blended benchmark where they are combined with the cap-weighted index to obtain a blended portfolio which becomes the strategic benchmark. The idea is to wisely combine a mix of smart beta strategies with the cap-weighted index to obtain a better-performing benchmark while respecting a relative risk budget. The key difference between enhanced indexation and the blended approach is that the former uses explicit tracking error control to hedge relative risk in the very construction of the smart beta indices, while the latter uses the cap-weighted benchmark itself in a core-satellite fashion to control the relative risk budget that is represented by the quantity invested in the satellite made up of the smart beta index (indices). Below we illustrate how smart beta can be used to diversify a large-cap index in a blended approach which is one of the most popular practices in passive investment in smart beta.

The diversification of a large-cap index is a two-step process. First, we construct a customised multi-smart-beta index by combining smart beta indices which outperform the cap-weighted benchmark (Scientific Beta USA CW index) and have dissimilar risk exposures and low correlations with each other. This is done because the combination of strategies provides two desirable qualities: i) smooth out outperformance across different market conditions and obtain lower tracking error overall (Amenc et al [2012]); ii) diversify away strategy-specific risk as the parameter estimation errors of the optimised strategies are not perfectly correlated (Kan and Zhou [2007]).

In this example, we select the Scientific Beta USA Value Maximum Decorrelation and Scientific Beta USA Low Volatility Efficient Minimum Volatility indices, as the correlation of their excess returns is found to be very low (0.089) and their Fama-French factor exposures are very different (panel A of figure 3 on page 10). Panel B shows that the Multi-Smart-Beta index, which is a simple equal-weighted combination of these two strategies, provides annual outperformance of 2.41%, which is around the average long-term outperformance of its constituents. However, its tracking error is way below the average tracking error; in fact, it is smaller than either constituent index tracking error, which clearly shows the benefits of diversifying across strategies. Panels B, C, and D of figure 3 show that the Multi-Smart-Beta index results in smoother out-performance across different periods.

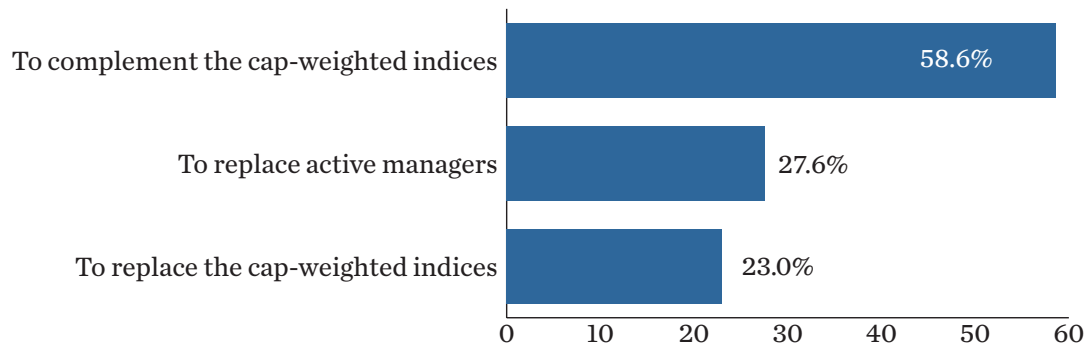
In the next step, we mix the Multi-Smart-Beta index with the Scientific Beta USA CW index to obtain a blended benchmark. Figure 4 shows the performance statistics of various blended indices obtained with different proportions of the Multi-Smart-Beta index and the Scientific Beta USA CW index. The results show that the investor faces a trade-off between performance (relative return and Sharpe ratio) and relative risk (tracking error and maximum relative drawdown) when selecting an appropriate blended benchmark. The important message, however, is that even for a tight tracking error target of 2%, a significant increase in Sharpe ►

1. Use of indices in investment

Question	North America	Europe
% of respondents who use indices in their equity investment	88.9%	91.4%
% of respondents who see significant problems with cap-weighted equity indices	53.2%	67.7%
% of respondents who have adopted any alternative weighting schemes in their equity investment	42.1%	45.2%

The table summarises response of investment professionals to the questions relating to the use of indices in investment in EDHEC-Risk Institute Surveys in Europe and North America.

2. Investors' purposes in using alternative weighting schemes



Results from the EDHEC-Risk North American Index Survey: The plot shows investors' purposes in using alternative weighting schemes. This question is only applicable to the respondents who have used, are going to use or are still considering using alternative weighting schemes.

1 Please see Amenc et al (2012), *EDHEC-Risk North American Index Survey 2011* and Amenc, N., F. Goltz and L. Tang (October 2011), *EDHEC-Risk European Index Survey 2011* for more details.

2 The choice of portfolio construction model itself has specific risks. Please refer to Amenc et al (2013) for more details on the specific risks of various smart beta weighting schemes.

3 Gonzalez and Thabault (2013) present a more detailed description of the methodology, performance and risk analysis of selected smart beta strategies. Goltz and Gonzalez (2013) show how the risk control choices can be used by investors to tailor smart beta strategies to their needs. Please also refer to the article 'Risks of smart beta indices and customisation of these risks' in this IPE supplement (page 6).

◄ ratio can be obtained over the Scientific Beta USA CW index (from 0.21 to 0.27). At the same time, extreme risks such as 95% tracking error and maximum relative drawdown do not exceed 2.93% and 2.62% respectively.

Benchmarked active management: where do smart beta strategies fit in?

Since smart beta is ultimately an element whose goal is to improve investment performance in an asset class through diversification, its application goes well beyond the framework of passive investment alone. A smart beta benchmark can be used as a better starting point in benchmarked active management. Irrespective of the benchmark used, a benchmark-constrained active manager with a given view on risk and return parameters will have the same active weights and same outperformance with respect to his benchmark (Roll [1992]).[4] The choice of benchmark is irrelevant, meaning that, by using smart beta benchmarks, active managers can benefit from the double added value of stock picking and smart beta's superior performance.

A smart beta strategy can be used as a completeness portfolio to modify a portfolio's risk profile. Many managers that beat their benchmarks specialise in certain factor tilts like value and high beta (Daniel et al [1997]). Changing this bias by requiring the manager to pick stocks outside his investment style is not likely to be a promising approach (Brown, Harlow and Zhang [2012]). However, one can manage an active portfolio's style tilts by adding an appropriate smart beta strategy, without changing the active investment strategy. Since smart beta indices are themselves market out-performing strategies, such an addition should not compromise the performance of the overall portfolio.

Smart beta strategies, when used in combination, have a role in multi-beta management to diversify across strategies. One can diversify across weighting schemes using the same stock selection – which is the case of the Scientific Beta Diversified Multistrategy index. Again, this combination of weighting schemes provides two desirable qualities: i) smoothes out outperformance across different market conditions and obtains lower tracking error overall (Amenc et al [2012]); ii) diversifies away strategy-specific risk as the parameter estimation errors of the optimised strategies are not perfectly correlated (Kan and Zhou [2007]). This effect is illustrated in figure 5.

Alternatively, one can diversify across stock selections within the same weighting scheme. A single weighting scheme may be preferred by the investor as a result of his own due diligence through which he finds the model or specific risks of a certain strategy most suitable for him. Since characteristic-based stock selection can be seen as an explicit choice of factor exposure, this kind of multi-beta diversification allows advantages in the form of reduction of tracking error and improvement of the information ratio, which is a key performance measure in benchmarked active management. Below, we illustrate this phenomenon using the example of ERI Scientific Beta USA Maximum Deconcentration indices. The choice is based on the fact that Maximum Deconcentration is the only smart beta strategy that does not require any

4 Roll (1992) has shown: If two managers have identical beliefs about expected stock returns, stock volatilities and pair-wise correlations, and aim to minimise tracking error volatility with respect to a benchmark for a fixed expected gain target, both would conduct the same trades relative to their respective benchmarks.

3. Fama-French factor exposures and risk-return analysis of the Scientific Beta USA Cap-Weighted, Value Maximum Decorrelation, Low Volatility Efficient Minimum Volatility and Multi-Smart-Beta indices

	Cap-Weighted	Value Maximum Decorrelation	Low Volatility Efficient Minimum Volatility	Multi-Smart-Beta (50/50 mix)
Panel A. Fama-French exposures (full period: June 2002–December 2012)				
Annualised alpha	0.00%	1.02%	2.51%	1.76%
Market beta	**1.00**	**0.94**	**0.73**	**0.83**
Small cap beta	**0.00**	**0.33**	**0.07**	**0.20**
Value beta	**0.00**	**0.14**	**–0.03**	**0.06**
r-squared	100%	97.66%	92.50%	97.32%
Panel B. Performance statistics (full period: June 2002–December 2012)				
Annualised returns	6.07%	8.73%	8.04%	8.48%
Relative returns	–	2.66%	1.98%	2.41%
Annualised volatility	21.31%	22.06%	16.27%	18.93%
Sharpe ratio	0.21	0.32	0.39	0.36
Tracking error	–	4.70%	7.23%	4.48%
Panel C. Performance statistics (first half: June 2002–September 2007)				
Annualised returns	11.35%	16.31%	11.76%	14.07%
Relative returns	–	4.96%	0.41%	2.72%
Panel D. Performance statistics (second half: September 2007–December 2012)				
Annualised returns	1.04%	1.64%	4.45%	3.17%
Relative returns	–	0.61%	3.42%	2.13%

The table shows the Fama-French factor exposures and risk-return analysis of the Scientific Beta USA CW, Value Maximum Decorrelation, Low Volatility Efficient Minimum Volatility and Multi-Smart-Beta indices. It also shows the absolute and relative returns of strategies in two sub-periods. Betas significant at the 95% confidence level are highlighted in bold. The yield on Secondary Market US Treasury Bills (3M) is a proxy for the risk-free rate. All statistics are annualised, all portfolios are rebalanced quarterly and the analysis is based on daily total returns (with dividends reinvested) from 21 June 2002 to 31 December 2012. The total number of stocks in the USA Scientific Beta universe is 500.

4. Absolute and relative risk-return analysis of different mixes of the Scientific Beta USA Cap-Weighted and Multi-Smart-Beta portfolios

% weight in Scientific Beta USA CW	**100%**	**80%**	**70%**	**60%**
% weight in Multi-Smart-Beta	**0%**	**20%**	**30%**	**40%**
Annualised returns	6.07%	6.56%	6.81%	7.05%
Relative returns	–	0.50%	0.74%	0.98%
Volatility	21.3%	20.8%	20.5%	20.3%
Sharpe ratio	0.21	0.24	0.25	0.27
Tracking error	–	0.90%	1.34%	1.79%
95% tracking error	–	1.46%	2.20%	2.93%
Maximu relative drawdown	–	1.31%	1.97%	2.62%

The table shows the absolute and relative risk-return analytics of different mixes of the Scientific Beta USA CW and Multi-Smart-Beta portfolios. The yield on Secondary Market US Treasury Bills (3M) is a proxy for the risk-free rate. All statistics are annualised, all portfolios are rebalanced quarterly and the analysis is based on daily total returns (with dividends reinvested) from 21 June 2002 to 31 December 2012. The total number of stocks in the USA Scientific Beta universe is 500.

5. Relative returns and tracking errors of five diversification-based Scientific Beta USA High Liquidity (flagship) indices and Scientific Beta USA High Liquidity Diversified Multistrategy index

Year	Maximum Deconcentration	Diversified Risk Parity	Maximum Decorrelation	Efficient Minimum Volatility	Efficient Maximum Sharpe	Diversified Multistrategy
Panel A. Annualised relative return over Scientific Beta USA cap-weighted index						
2012	0.39%	–0.16%	–0.85%	–1.99%	–1.50%	–0.79%
2011	–3.44%	–0.42%	–4.01%	5.26%	–0.38%	–0.63%
2010	3.53%	2.03%	2.35%	–2.61%	2.02%	1.46%
2009	16.94%	12.97%	12.01%	1.44%	10.67%	10.78%
2008	–3.78%	–1.25%	–3.56%	5.10%	–0.23%	–0.77%
2007	–0.02%	–1.26%	1.37%	–1.43%	1.35%	0.00%
2006	–2.98%	–1.78%	–5.35%	0.51%	–4.02%	–2.74%
2005	4.08%	3.09%	5.70%	3.10%	5.24%	4.25%
2004	3.64%	4.13%	6.08%	6.59%	5.90%	5.27%
Panel B. Annualised tracking error with respect to Scientific Beta USA cap-weighted index						
2012	3.00%	1.92%	2.53%	3.01%	1.92%	1.74%
2011	3.48%	1.99%	2.92%	3.86%	2.40%	1.90%
2010	3.13%	1.94%	3.04%	2.88%	2.77%	1.99%
2009	6.74%	4.71%	5.13%	5.02%	4.12%	3.86%
2008	5.80%	4.10%	5.66%	6.55%	5.04%	3.99%
2007	2.26%	1.65%	2.49%	2.62%	2.29%	1.76%
2006	3.18%	2.22%	3.18%	1.71%	2.43%	2.17%
2005	2.36%	1.75%	2.57%	1.91%	2.14%	1.86%
2004	3.75%	2.54%	4.68%	2.68%	3.34%	3.11%

The table shows relative returns and tracking errors of five diversification-based Scientific Beta USA High Liquidity (flagship) indices and the Scientific Beta USA High Liquidity Diversified Multistrategy index – an equal-weighted combination of the five indices. All statistics are annualised, all portfolios are rebalanced quarterly and the analysis is based on daily total returns (with dividends reinvested) from 1 January 2002 to 31 December 2012. The total number of stocks in the USA Scientific Beta universe is 500 and the high liquidity indices are constructed on the 50% most liquid stocks in the universe.

parameter estimation and is therefore free of estimation risk. Its out-of-sample performance is identical to its in-sample performance and thus allows us to examine the effect of stock selection in isolation.

In this example, we select the Scientific Beta USA Low Volatility Maximum Deconcentration, Scientific Beta USA Value Maximum Deconcentration and Scientific Beta USA High Liquidity Maximum Deconcentration indices as the correlation of their excess returns is found to be very low.[5] The multi-beta strategy, which is a simple equal-weighted combination of these three strategies, provides annual outperformance of 2.10% which is around the average long-term outperformance of its constituents (figure 6). However, its tracking error and extreme tracking error are significantly lower than those of either constituent, which clearly shows the benefits of diversifying across strategies. As a result, it achieves a high information ratio of 0.69.

Conclusion

Smart beta is a scientific way to add value through diversification but it comes with risks that are specific to the portfolio construction methodology. Investors, and not smart beta providers, should have the freedom to select the risks they want to be exposed to and to manage them. When used to outperform cap-weighted reference, the smart beta benchmarks should offer investors tracking error control. Active managers must realise that the best alpha is created by using the risk benchmarks – ie, betas that are well decorrelated or that reflect tactical macro or microeconomic bets. They should not see smart beta as a threat but rather as an opportunity. Smart beta strategies, which allow for flexibility in index construction to deliver a wide spectrum of risk choices, present new opportunities for active managers and multi-managers to enhance their performance at very low marginal cost.

5 The correlation of excess returns of different stock-selection-based maximum deconcentration indices are as follows: Low Volatility and Value = 0.14, Low Volatility and High Liquidity = -0.37, High Liquidity and Value = 0.59.

6. Risk-return analysis of the Scientific Beta USA Cap Weighted and Scientific Beta Maximum Deconcentration indices

	Scientific Beta USA Cap Weighted	Scientific Beta USA Maximum Deconcentration			
		Low Volatility	Value	High Liquidity	Multi-Beta
Annualised returns	6.07%	7.87%	8.52%	7.90%	8.17%
Relative returns	–	1.81%	2.46%	1.84%	2.10%
Annualised volatility	21.31%	19.02%	23.42%	24.01%	21.97%
Sharpe ratio	0.21	0.33	0.29	0.26	0.30
Tracking error	–	4.9%	4.8%	4.4%	3.0%
95% tracking error	–	7.1%	8.6%	7.5%	5.1%
Information ratio	–	0.37	0.51	0.41	0.69

The table presents risk-return analysis of the Scientific Beta USA CW and Scientific Beta Maximum Deconcentration indices. The yield on Secondary Market US Treasury Bills (3M) is a proxy for the risk-free rate. All statistics are annualised, all portfolios are rebalanced quarterly and the analysis is based on daily total returns (with dividends reinvested) from 21 June 2002 to 31 December 2012. The total number of stocks in the USA Scientific Beta universe is 500.

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Analysis of the conditional performance of smart beta strategies

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Recently there has been a significant increase in the number of alternative forms of equity indices, which are usually marketed on the basis of their outperformance over traditional cap-weighted indices. However, the promoters of these indices sometimes fail to present a complete performance analysis. A prime example is the analysis of performance in very different market regimes to test the robustness of the strategy. Market conditions such as bullish or bearish markets, as well as periods of high or low stock market volatility, may have a considerable impact on how different equity strategies perform. Ferson and Qian (2004) note that an unconditional evaluation made for example during bearish markets will not be a meaningful estimation of forward performance if the next period was to be bullish. Amenc et al (2012) show considerable variation in the performance of some popular smart beta strategies in different sub-periods, revealing the pitfalls of aggregate performance analysis based on long periods. They also show that certain sub-periods/market conditions favour some smart beta strategies while prove detrimental to others. The reason is that each smart beta strategy is exposed to a set of risk factors that have been shown to carry time-varying risk premia (Asness [1992], Cohen, Polk and Vuolteenaho [2003]).

Conditional performance analysis provides investors with a better understanding of the performance of smart beta indices in various economic conditions and allows them to make a selection that takes into account their view on future market conditions. In addition, analysing the dependence of performance on market conditions also provides a view on the robustness of a strategy's outperformance.[1] Also, allocating across smart beta strategies with contrasting conditional performance features may allow investors to diversify away the risk of ending up with a single strategy which may not deliver ►

1 Furthermore, because of data availability issues, the evaluation of the performance over the very long-term period (eg, over several decades) becomes infeasible for investable indices. Thus the introduction of the conditional performance analysis is important as it can be expected to be more robust to the limitations imposed by the short length of the sample used for the risk-return analysis.

◄ outperformance in the prevailing market conditions. Indeed, this type of contrasting allocation could prove to be useful to agnostic investors because it helps smooth the overall performance and thus provide positive outperformance regardless of the market conditions.

In this article we analyse the properties of strategies that perform well in bull markets and those that perform well in bear markets. We also show how one could use stock selection and tracking error control to modify the conditional performance of a particular strategy. Lastly, we analyse the conditional performance of a strategy which combines two different kinds of indices equally – bull market-favoured and bear market-favoured indices.

A selection of the best conditional performance strategies

Separating bull and bear market periods to evaluate performance has been proposed by various authors such as Levy (1974), Turner, Starz and Nelson (1989) and, more recently, Faber (2007). Quarters with positive returns for the reference index are called bull market and the remaining quarters are called bear market. We draw upon ERI Scientific Beta indices to conduct this analysis. The indices are based on popular diversification-based weighting schemes and allow a wide choice of stock selection, systematic risk control, and relative risk control.[1]

Strategies with best conditional performance: an overview

We present the average risk and return statistics of two sets of indices. The first set contains the 20% best-performing indices in bull markets and the second set is composed of the 20% best-performing indices in bear markets across the 2,442 indices available. Panel A of figure 1 shows that although the indices in both sets S1 and S2 are heavily affected by the condition of the broad market, their performance over the full period is impressive. The average outperformance for S1 and S2 is remarkable at 2.05% and 2.81% respectively. The indices in S1 tend to be more volatile however and those in S2 are more defensive (with –3.99% of excess volatility). The reason is that S1 is populated with high-volatility indices which mostly benefit from bull markets as indicated in panel B. However, S2 largely contains low-volatility indices that are protected in market crashes (or bear markets), which leads to high outperformance, as shown in panel C. Overall, both sets of indices show significant improvement over their reference cap-weighted benchmarks.

Analysis of strategies with best conditional performance

In this section, we discuss the risk and return properties of the five best-performing indices (among the developed world indices) for each of the bull and bear markets. Figure 2 shows the five best-performing strategies in bull markets (panel A) and bear markets (panel B). The table shows that the top-performing indices in bull markets mostly use high-volatility stock selection and the top performers in bear markets are mostly based on low-volatility stock selection and/or minimum volatility weighting. On the one hand, panel A1 shows that the strategies have higher markets betas (always >= 1.00) which means that they benefit the most from the market premium during a bull market regime, which is exactly what we observe in panel A2 as the strategies show higher (and significant) excess returns in bull markets than in bear markets (panel A3). On the other hand, panel B1 indicates that the strategies are associated with stocks resilient to market decline (ie, low volatility and high dividend yield stocks), which is in line with their low market beta (always <0.80). Furthermore, these same strategies have higher (and significant) excess returns in bear markets (panel B3) than in bull markets. Finally, the small size exposure of panel A1 indices is quite different from that of panel B1 indices, meaning that they are affected with a different magnitude by the time-changing small-cap premium.

Customising risk with stock selection and tracking error control

Investors can have a preference for a strategy (weighting scheme) that outperforms in certain market conditions and underperforms in others. However, if they wish to control the underperformance in unfavourable conditions, they can do so by choosing a stock selection that, in combination with the weighting scheme, results in outperformance in both market regimes. ►

1 Please visit www.scientificbeta.com to view the complete list of geographical regions, stock selection, weighting scheme, and risk control options.

1. Average risk and return statistics of the 20% best-performing strategies in bull and bear markets

	Selection of the 20% best-performing indices across all 2,442 indices on the platform	
	20% best in bull markets (S1)	20% best in bear markets (S2)
Panel A. Full period		
Annual excess returns	2.05%	2.81%
Annual excess volatility	0.92%	–3.99%
Maximum relative drawdown	19.15%	12.5%
Maximum time under water	1,214	951
Panel B. Bull markets		
Annual excess returns	7.03%	–2.65%
Annual excess volatility	2.58%	–1.66%
Panel C. Bear markets		
Annual excess returns	–2.97%	7.57%
Annual excess volatility	2.60%	–1.44%

The ERI Scientific Beta platform contains 2,442 indices – S1and S2 contain 488 indices each. The full period statistics in panel A are based on daily total returns (with dividend reinvested) for the period 21 June 2002 to 31 December 2012. The bull (bear) markets statistics in panel B (C) are based on quarterly total returns. Performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding period returns for investors. Calendar quarters with positive market index returns comprise bull markets and the rest constitute bear markets.

2. The five best-performing strategies in both bull and bear market regimes

Panel A: Top 5 indices in bull market conditions					
	High Volatility Maximum Deconcentration (5% TE)	**High Volatility Diversified Risk Parity**	**High Volatility Maximum Decorrelation (5% TE)**	**High Volatility Efficient Maximum Sharpe Ratio (5% TE)**	**Value Maximum Deconcentration**
Panel A1. Full sample period					
Annual returns	7.64%	8.13%	7.01%	6.86%	9.62%
Annual relative returns	0.69%	1.18%	0.07%	–0.09%	2.67%
Annual volatility	21.64%	20.91%	21.16%	21.00%	19.02%
Sharpe ratio	0.28	0.31	0.25	0.25	0.42
Market beta coefficient	**1.2**	**1.17**	**1.18**	**1.17**	**1**
SMB (Size) coefficient	**0.54**	**0.57**	**0.47**	**0.44**	**0.37**
HML (Value) coefficient	**–0.05**	**–0.08**	**–0.04**	**–0.05**	**0.18**
Panel A2. Bull markets					
Annual returns	9.13%	8.98%	8.81%	8.76%	8.59%
Annual relative returns	**1.84%**	**1.69%**	**1.51%**	**1.46%**	**1.29%**
Annual volatility	8.28%	7.95%	8.08%	7.99%	7.17%
Sharpe ratio	1.05	1.08	1.04	1.04	1.14
Panel A3. Bear markets					
Annual returns	–11.33%	–10.78%	–11.19%	–11.21%	–9.18%
Annual relative returns	–2.58%	–2.03%	–2.44%	–2.46%	–0.43%
Annual volatility	14.50%	14.08%	14.20%	14.13%	12.88%
Sharpe ratio	–0.81	–0.79	–0.81	–0.82	–0.74
Panel B. Top 5 indices in bear market conditions					
Low Volatility	**Low Volatility Efficient Min Volatility**	**Mid Liquidity Efficient Min Volatility**	**High Dividend Yield Efficient Min Volatility**	**Low Volatility Efficient Max Sharpe Ratio**	**Low Volatility Maximum Deconcentration**
Panel B1. Full sample period					
Annual returns	9.95%	10.31%	9.86%	9.62%	9.73%
Annual relative returns	3.00%	3.36%	2.91%	2.67%	2.78%
Annual volatility	12.91%	13.63%	13.98%	13.76%	13.92%
Sharpe ratio	0.64	0.63	0.59	0.58	0.58
Market beta coefficient	**0.72**	**0.77**	**0.77**	**0.77**	**0.78**
SMB (Size) coefficient	**0.12**	**0.38**	**0.12**	**0.18**	**0.19**
HML (Value) coefficient	**–0.07**	**–0.1**	**-0.01**	**–0.06**	**–0.06**
Panel B2. Bull markets					
Annual returns	6.41%	6.82%	6.75%	6.71%	6.81%
Annual relative returns	–0.88%	–0.47%	–0.54%	–0.58%	–0.49%
Annual volatility	4.65%	4.95%	5.08%	4.93%	4.99%
Sharpe ratio	1.29	1.29	1.24	1.27	1.28
Panel B3. Bear markets					
Annual returns	–5.21%	–5.69%	–5.86%	–5.95%	–6.04%
Annual relative returns	**3.54%**	**3.06%**	**2.89%**	**2.80%**	**2.70%**
Annual volatility	8.98%	9.43%	9.68%	9.59%	9.71%
Sharpe ratio	0.89	0.85	0.89	0.86	0.86

This table reports performance and risk statistics for selected Scientific Beta Developed Indices. The full period statistics (panels A1 and B1) are based on daily total returns (with dividend reinvested) for the period 21 June 2002 to 31 December 2012. The bull (bear) markets statistics in panels A2 and B2 (A3 and B3) are based on quarterly total returns. Performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding period returns for investors. Betas significant at the 95% confidence level are highlighted in bold. The excess returns highlighted in bold represent the index returns that are statistically significant (at the 95% confidence level) from the cap-weighted reference index where the significance has been determined using the paired t-test statistics. The yield on Secondary Market US Treasury Bills (3M) is used as a proxy for the risk-free rate in US dollars. The Scientific Beta Developed cap-weighted index which serves as a reference benchmark to compute the excess returns, comprises 2,000 securities weighted in proportion to their free-float market-capitalisation weights.

In fact, mechanisms like stock selection and relative risk (tracking error) control can allow for fine-tuning of the dependency of the performance of any smart beta weighting scheme.[2]

The minimum-volatility weighting scheme (in its standard form) has been shown to be favoured in bear markets and disrupted in bull markets. In figure 3, we show the conditional performance of different versions of the ERI Scientific Beta Developed Efficient Minimum Volatility Index that are the least affected by the changes in market conditions (ie, the difference between bull excess returns and bear excess returns is small), as compared to the cap-weighted reference index.

The figure shows that the Efficient Minimum Volatility index with no stock selection shows positive (+2.37%) and negative (–0.33%) excess returns in bear and bull markets respectively. These results are expected as the index typically overweights defensive stocks, which tend to perform better in bear markets

"The positive outperformance in bull markets that the value stock selection brought is due to the fact that value stocks tend to be less expensive and thus benefit most when investors believe in a market upturn and are seeking the cheapest stocks to take advantage of that upturn"

and worse in bull markets. The Scientific Beta Value Efficient Minimum Volatility index displays a positive excess return (+0.15%) in bull market conditions while maintaining the outperformance in bear markets (+1.85%). The positive outperformance in bull markets that the value stock selection brought is due to the fact that value stocks tend to be less expensive and thus benefit most when investors believe in a market upturn and are seeking the cheapest stocks to take advantage of that upturn. This enhanced the performance of the Value Efficient Minimum Volatility index in bull market conditions. For investors concerned by the capacity of an Efficient Minimum Volatility strategy to track rises in the market, the combination of stock selection and tracking error control could further improve the results as the table shows that the Mid-Cap Efficient Minimum Volatility index with 3% tracking error control can provide a more balanced and positive performance in both bull and bear market conditions with a slight improvement in tracking error (as compared to the other Efficient Minimum Volatility indices).

Conclusion

If investors do not have a strict preference for a certain weighting scheme, they could diversify across two completely different kinds of indices to limit the risk of big losses in unfavourable market conditions. In fact, they could create a smart beta index that combines in equal proportion the top ten best-performing indices in bull and bear markets. Figure 4 shows the risk and return statistics of two portfolios P1 and P2 each containing an equal weight of the top ten best-performing strategies in bull and bear markets respectively. The table shows that portfolio P1 (P2) outperforms the cap-weighted reference index in bull (bear) market conditions and underperforms in bear (bull) markets. The table also shows that the 50-50 mix of P1 and P2 portfolios achieves positive excess performance across distinct states of the market. It also indicates a significant decrease in tracking error due to the risk reduction resulting from the diversification across different weighting schemes (Amenc et al [2012]).

Our analysis of the conditional performances shows that portfolios that perform best in bull markets are riskier and have lower Sharpe ratios than portfolios that perform best in bear markets. Secondly, our study provides evidence that stock selection and tracking error control is useful in modifying the characteristics of a portfolio, which typically perform well in bear markets, in order to improve its performance in bull markets. Finally, we show that the contrasting characteristics of portfolios that perform best in bull/bear markets could work in favour of the investors if they are equally mixed into a single portfolio. This very simple and robust process helps smooth the overall performance, reduce tracking error, and achieve a positive excess return regardless of the state of the market.

2 Please refer to Goltz and Gonzalez (2013) for more information on stock selection and relative risk control methodology used for ERI Scientific Beta indices.

3. Conditional performances of the Efficient Minimum Volatility indices

	Scientific Beta Developed Efficient Minimum Volatility					
	Bull market	Bear market	Bull market	Bear market	Bull market	Bear market
Stock selection	None		Value		Mid cap (3% TE)	
Annual returns	6.96%	–6.38%	7.45%	–6.90%	7.93%	–7.81%
Annual relative returns	–0.33%	2.37%	0.15%	1.85%	0.64%	0.94%
Tracking error	1.71%	2.70%	1.69%	2.61%	1.49%	2.47%
Sharpe ratio	1.24	–0.68	1.24	–0.68	1.20	–0.73
Information ratio	–0.19	0.88	0.09	0.71	0.43	0.38

This table reports absolute and relative return statistics for the different versions of the Scientific Beta Developed Efficient Minimum Volatility Index in the periods of bull and bear market regimes respectively. The statistics are based on daily total returns (with dividend reinvested) for the period 21 June 2002 to 31 December 2012. All statistics displayed in this table are quarterly values and performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding period returns for investors. The yield on Secondary Market US Treasury Bills (3M) is used as a proxy for the risk-free rate in US dollars. The Scientific Beta Developed cap-weighted index, which serves as a reference benchmark to compute the excess returns, comprises 2,000 securities weighted in proportion to their free-float market-capitalisation weights.

4. Risk and return statistics of the top 10 best-performing strategies in bull and bear markets

Developed universe	Cap-weighted	Top 10 in bull markets (P1)	Top 10 in bear markets (P2)	50-50 mix of P1 and P2
Panel A. Bull markets				
Annual returns	28.73%	36.48%	27.19%	31.85%
Annual relative returns	–	7.75%	–1.54%	3.12%
Annual volatility	12.79%	15.70%	9.74%	12.50%
Sharpe ratio	0.27	0.35	0.26	0.30
Annual tracking error	–	4.17%	4.23%	1.78%
Panel B. Bear markets				
Annual returns	–29.52%	–37.24%	–20.95%	–29.43%
Annual relative returns	–	–7.72%	8.57%	0.09%
Annual volatility	25.25%	29.82%	19.95%	24.63%
Sharpe ratio	–0.31	–0.39	–0.23	–0.31
Annual tracking error	–	6.64%	6.68%	2.73%

This table reports the absolute and relative performance and risk statistics for two portfolios composed of the 10 top-performing strategies in the developed universe in bull and bear markets. The first (second) portfolio P1 (P2) equally weights the best-performing portfolios in bull (bear) market conditions. The '50-50 mix' portfolio equally weights portfolios P1 and P2. The statistics are based on daily total returns (with dividend reinvested) for the period 21 June 2002 to 31 December 2012. All statistics displayed in this table are quarterly values and performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding period returns for investors. ERI Scientific Beta uses the yield on Secondary Market US Treasury Bills (3M) as a proxy for the risk-free rate in US dollars. The Scientific Beta Developed cap-weighted index, which serves as a reference benchmark to compute the excess returns, comprises 2,000 securities weighted in proportion to their free-float market-capitalisation weights.

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Analysis of the specific risks of diversification strategies

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Every smart beta strategy carries risks, which can be divided into two categories: systematic risks and specific risks. Systematic risks come from the fact that strategy indices can be more or less exposed to dynamically rewarding systematic risk factors (such as value and size risk) depending on the methodological choices guiding their construction.[1] In contrast with systematic risks, which may be common among smart beta investments, specific risks are related to the characteristics of a given portfolio construction methodology. Indeed, any weighting scheme relies explicitly or implicitly on modelling assumptions and/or on parameter estimation that might lead to a risk of a lack of out-of-sample robustness. In that context, some heuristic strategies, whose objectives are not explicitly framed in terms of a formal optimisation objective, can in some cases turn out to be reasonable approaches due to the presence of an overwhelming amount of noise in parameter estimates. For example, DeMiguel, Garlappi and Uppal (2009) argue that mean-variance optimisation procedures do not consistently outperform, from an out-of-sample Sharpe ratio perspective, naive equally-weighted portfolio strategies.[2] It is therefore of importance to fully understand the modelling assumptions behind various diversification strategies.

The 'Overview of diversification strategies' article in this IPE supplement (page 2) describes the methodology and rationale behind the main diversification strategies available on the Scientific Beta platform. We thus briefly state their respective objectives here and refer the reader to the other article for more details.[3] The Maximum Deconcentration strategy aims at combining the set of stocks with the lowest possible portfolio concentration in nominal weights. The Diversified Risk Parity strategy aims at equalising the individual stock contributions to the total risk of the index[4], assuming the cross-correlations between stock returns are uniform. The Maximum Decorrelation strategy aims at minimising portfolio volatility under the assumption of identical volatility across stocks. The Efficient Minimum Volatility strategy aims at combining the set of stocks so as to generate the lowest possible total portfolio volatility. The Efficient Maximum Sharpe Ratio strategy aims at combining the set of stocks to achieve the highest possible risk-adjusted expected returns.

"Because risk is at the heart of portfolio diversification, equal weighting is a limited approach to diversification in the sense that it ignores all information on differences in risk parameters (volatilities and correlations) across stocks. As a consequence, the Maximum Deconcentration Strategy Indices generally exhibit a higher level of risks"

The risk that is specific to the construction of a strategy can be classified into two categories: parameter estimation risk and optimality risk. Indeed, Martellini, Milhau and Tarelli (2013) explain that the choice in risk and return parameter estimation for efficient diversification is between 'trying', which has a cost related to parameter estimation risk (ie, the risk of a substantial difference between the estimated parameter value and the true parameter value) or 'giving up', which has an optimality risk, related to the risk that a strategy can be very far from the mean-variance optimal portfolio – ie, the portfolio of risky assets with the maximum Sharpe ratio (MSR portfolio) – in terms of its risk-return profile. Each diversification strategy corresponds to either more or less restrictive assumptions regarding the true underlying parameters. Figure 1 provides a summary of the risks associated with each of the five methodologies we have introduced above (see page 16).

A trade-off occurs because an objective function that involves fewer parameters leads to a smaller amount of parameter estimation risk but a higher amount of optimality risk, since fewer dimensions are used for optimisation. In this sense, it is quite possible that a 'good' proxy (ie, a proxy based on parameters with low estimation risk) for a 'bad' target (ie, a target a priori far from the true maximum Sharpe ratio portfolio, based on true population values) eventually dominates a 'bad' proxy (ie, a proxy based on parameters plagued with substantial estimation risk) for a 'good' target (ie, a target a priori close to the true maximum Sharpe ratio portfolio, based on true population values).

In this decomposition of the specific risk, the parameter estimation risk measures the distance between the imperfectly estimated target and the true target (ie, the portfolio that would result from the given portfolio construction methodology if the true parameter values were known) whereas optimality risk is the distance between this true target and the true MSR portfolio. Martellini, Milhau and Tarelli (2013) provide a detailed empirical assessment of the opportunity costs (in terms of ex-ante Sharpe ratios) attributed to optimality risk and estimation risk.

Hence, different portfolios are intuitively expected to bear more estimation risk or more optimality risk.[5] For example, an equally-weighted portfolio is not subject to any estimation risk since it relies only on an observable quantity (the nominal number of stocks), but it arguably carries a high amount of optimality risk given that it is not a priori expected to be identical to the true MSR. The second extreme case is that of the MSR: the true MSR bears no optimality risk, but an investor trying to implement and estimate this optimal portfolio will probably commit estimation errors on volatilities, correlations and even more dramatically on expected returns.

In this context, Maximum Deconcentration indices for example are well suited to market conditions when all parameters are difficult to estimate because its construction methodology does not require any risk and return parameter estimation. Nevertheless, because risk is at the heart of portfolio diversification, equal weighting is a limited approach to diversification in the sense that it ignores all information on differences in risk parameters (volatilities and correlations) across stocks. As a consequence, the Maximum Deconcentration Strategy Indices generally exhibit a higher level of risks (ie, higher volatility, higher historical value-at-risk (VaR) and higher Cornish-Fisher VaR, which is a measure of VaR adjusted for skewness and kurtosis, as well as more significant Maximum Drawdowns) than the other Diversification Strategy Indices.[6] This goes back to a key difference in the methodology underlying Maximum Deconcentration Strategy Indices compared to the methodologies that form the basis of efficient diversification approaches (such as the Efficient Minimum Volatility and Efficient Maximum Sharpe Ratio strategies), and also compared to other ad-hoc methodologies such as Maximum Decorrelation or Diversified Risk Parity, all of which encapsulate information on risk parameters. More specifically, the Diversified Risk Parity strategy takes only stock level volatility information into ►

1 We refer the reader to the article 'Systematic risk factors and the robustness of smart beta strategies' in this IPE supplement (page 18) for more details on the systematic risks of smart beta strategies.

2 They evaluate the out-of-sample Sharpe ratio of 14 portfolio rules relying on various estimators for the covariance matrix and the expected returns that have been suggested in the literature. They find that none of these advanced strategies consistently outperforms the naive '1/N' rule across seven different datasets.

3 Note also that the discussion of the specific risks of ERI Scientific Beta Diversification Strategies focuses only on the weighting schemes for clarity, without taking into account the concentration adjustments, or the constraints on turnover or liquidity that ERI Scientific Beta applies to its strategy indices.

4 The risk contribution of a constituent is defined as the product of the constituent's weight and the marginal contribution of this constituent to the total portfolio volatility.

5 Nevertheless, there exist for each diversification strategy optimality conditions on parameter values, under which optimality risk would be zero. Under these conditions, each strategy has the same *ex-ante* Sharpe ratio (ie, based on true parameter values) as the true maximum Sharpe ratio portfolio.

6 See for instance the ERI Scientific Beta White Paper on the Overview of Diversification Strategy Indices.

1. Specific risks and conditions of optimality of the diversification strategies

Weighting scheme	Optimality cost	Parameter estimation cost	Required parameter	Optimality conditions
Maximum Deconcentration	Ignorance of cross-sectional differences in individual volatilities, expected excess returns and pair-wise correlations	None	No risk and return parameter required	$\mu_i \equiv \mu, \forall i$ $\sigma_i \equiv \sigma, \forall i$ $\rho_{ij} \equiv \rho, \forall i \neq j$
Diversified Risk Parity	Ignorance of cross-sectional differences in correlation levels and Sharpe ratios	Accuracy and robustness[1] of the estimated individual volatilities	Individual volatilities (and average pair-wise correlations)	$\lambda_i \equiv \lambda, \forall i$ $\rho_{ij} \equiv \rho, \forall i \neq j$
Maximum Decorrelation	Ignorance of cross-sectional differences in individual volatilities and expected excess returns	Accuracy and robustness of the correlation matrix estimate	Correlation matrix	$\mu_i \equiv \mu, \forall i$ $\sigma_i \equiv \sigma, \forall i$
Efficient Minimum Volatility	Ignorance of cross-sectional differences in expected excess returns	Accuracy and robustness of the covariance matrix estimate	Correlation matrix	$\mu_i \equiv \mu, \forall I$
Efficient Maximum Sharpe Ratio	None	Accuracy and robustness of the expected excess returns and the covariance matrix estimates	Expected excess returns and covariance matrix	Optimal by construction

The table lists the optimality and parameter estimation risks of the five diversification weighting schemes, together with the conditions on its parameters for the strategy to be identical to the Maximum Sharpe Ratio or optimality conditions. We denote here by σ_i the volatility of stock i, μ_i its expected return, λ_i its Sharpe ratio and ρ_{ij} the correlation between the i-th and j-th stocks.
1 Accuracy refers here to the unbiasedness of the estimators (their expectation is close to the true value) while robustness refers to their efficiency (the estimates have a small variance).

◄ account while the Maximum Decorrelation strategy solely focuses on correlations across stocks. The Efficient Minimum Volatility approach requires estimation of (and bears estimation risk for) both volatilities and correlations. It is thus critical to obtain a robust and accurate covariance matrix estimate to implement this strategy. On the other hand, the Efficient Minimum Volatility strategy does not require expected returns as inputs. This helps to avoid an important source of estimation risk and allows a reasonable proxy for the tangency portfolio to be obtained. Indeed, the presence of errors in expected return parameter estimates is particularly critical, since such estimates are more noisy compared to risk estimates due to a lack of convergence of sample-based expected return estimators (Merton [1980]), and to the fact that optimisation procedures are more sensitive to errors in expected return parameters versus errors in risk parameters (see, eg, Chopra and Ziemba [1993]).

By contrast, . Indeed the Efficient Maximum Sharpe Ratio does not ignore differences in expected returns across stocks: it indirectly estimates expected returns of the stocks by assuming they are related to their downside risk (measured by their semi-deviations[7]). Thus it is not exposed to the optimality risk, and its specific risk solely emanates from parameter estimation risk. Martellini, Milhau and Tarelli (2013) argue that parameter estimation risk can be further decomposed into (i) parameter model risk (arising for instance in the case of the Efficient Maximum Sharpe ratio weighting scheme from the assumption of a positive relationship between the expected returns and the downside risks of the constituent stocks) and (ii) parameter sample risk (due to the fact that semi-deviations must be estimated over a particular sample, and hence such estimates will contain some noise).[8] Overall, it must be noted that parameter estimation risk does not invalidate the two scientific diversification approaches that are Efficient Minimum Volatility and Efficient Maximum Sharpe as they respectively generally exhibit lower volatilities and higher Sharpe ratios than the heuristic approach to diversification that is Maximum Deconcentration.

"A key feature of the Efficient Maximum Sharpe Ratio strategy is that it takes into account all relevant information, including differences across stocks in terms of returns, volatility and correlations. As a result, this strategy also faces the corresponding risk of estimation of the necessary return inputs"

Interestingly, because the diversification strategies differ from each other in the assumptions they make and the objectives they aim to achieve, a combination of these different strategies will allow the risks that are specific to each strategy to be diversified by exploiting the imperfect correlation between the different strategies' parameter estimation errors and the differences in their underlying optimality assumptions (see Tu and Zhou [2010], Kan and Zhou [2007] and Martellini, Milhau and Tarelli [2013], among others, for theoretical and empirical evidence that portfolio of strategies dominate individual strategies in the presence of parameter estimation risk).

7 More specifically, an extra step is added to the estimation process to provide more robustness: stocks are sorted by their semi-deviation into deciles and all stocks in a decile are then assigned the median value of the decile..
8 An additional source of parameter risk is stationarity risk related to the fact that the true parameter values are time-varying.

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Systematic risk factors and the robustness of smart beta strategies

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The emergence of alternative forms of equity indices that no longer represent a market by using the capitalisation of its constituents, but rather different metrics, such as fundamentally-weighted indices, or achieve a risk/return profile, such as minimum-volatility indices, has led providers to market these indices on the basis of performance figures that, owing to the recent launch of these indices, are largely based on pre-live, back-tested performances.

While alternative index investors primarily buy into a novel portfolio construction process, they also more or less indirectly buy into systematic[1] and specific risks induced by the very peculiarities of the alternative underlying strategy.

In this article we explain how the risk/return profile of alternative indices can be materially driven by dynamically-rewarded systematic risks, putting the robustness of their performance across different time periods at risk. As we will show, those risks will not only jeopardise the robustness of strategies' performance across time periods: they will also expose the strategy to the potentiality of severely underperforming its cap-weighted reference during specific periods of time where the systematic risk factor will experience tremendously bad returns compared to cap-weighted index returns.

We then propose a framework to assess and improve the robustness of smart beta indices and to control the impact of systematic risks on that robustness.

We illustrate this reflexion with the Maximum Deconcentration[2] weighting scheme, which is a generalised version of the equal-weighting scheme, thus avoiding the issue of strategy-specific estimation risks and focusing on systematic risks instead. For a thorough discussion on the specific risks of smart beta indices, we refer the reader to the article in the present supplement on 'Analysis of the specific risks of diversification strategies' (page 15).

Systematic risks and the performance of smart beta indices

On the systematic risks of smart beta indices
Departing from the cap-weighting paradigm mechanically exposes investors to various risks. For instance, portfolio construction methodologies designed to diversify by exploiting the imperfect correlations among stocks, as prescribed by modern portfolio theory (see Markowitz [1952]), will notably be exposed to the size factor (SMB French factor). One explanation is provided by Petrella (2005) who shows that including small caps in an all-cap portfolio decreases its total average correlation as smaller caps tend to have a lower correlation with larger caps than larger caps with themselves. More generally, any scheme that is designed to reduce the concentration of weights compared to a cap-weighted index, which by construction grants more weight to the largest stocks, will mechanically introduce a tilt toward smaller capitalisation stocks.

Also, as evidenced in, eg, Chan, Karceski and Lakonishok (1999) and Chow et al (2011), risk-based strategies such as minimum volatility or low volatility exhibit both a low market beta and a bias towards defensive sectors such as utilities.

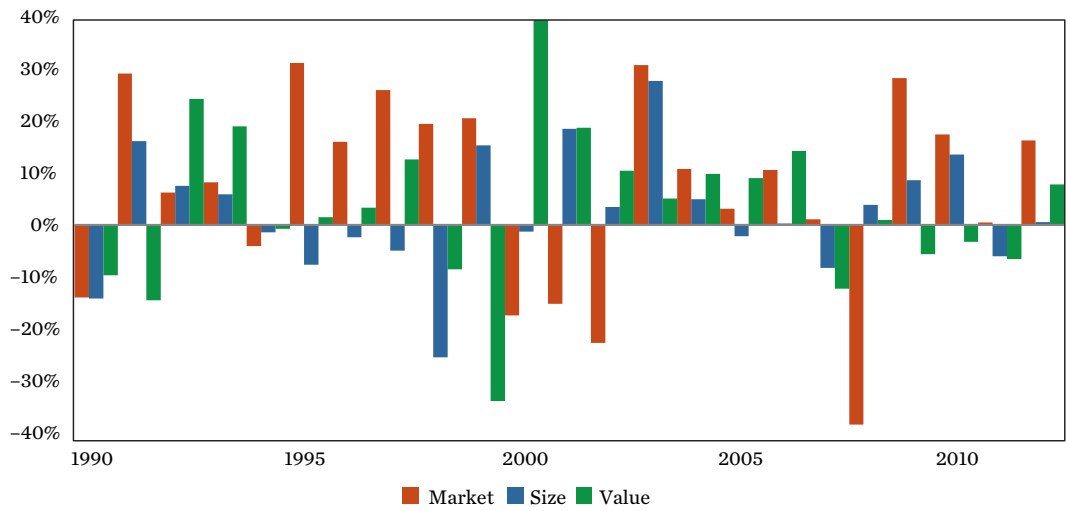
Alternatively, fundamentals-based indices that use fundamental measures of firm size such as book value or revenue as weighting criteria induce a positive value bias (as shown by Kaplan [2008], fundamentals-based strategies will overweight high-yielding stocks compared to low-yielding stocks, which materialises into an increased aggregated value exposure). This important result is in turn evidenced in the empirical studies of Arnott et al (2010) and Chow et al (2011).

In parallel, all risk factors have time-varying rewards. For instance, Asness, Friedman, Krail and Liew (2000), Cohen, Polk and Vuolteenaho (2003), show that equity, value and momentum premia do not reward investors constantly over time. Campbell and Vuolteenaho (2004) show that those time-varying commonalities in value and growth stocks are due to the pricing of future expected cash flows and changes in discount rates. Changes in those two state variables will thus induce changes in those factors' risk premia, as shown in figure 1.

There are two key implications of the exposure to the time-varying risk premium: firstly, the period used to back-test those indices, or to measure their live performance, will determine their exhibited risk/return profile. Secondly, attractive performances averaged over long periods might hide shorter-term factor-induced drawdowns, which relative to broad cap-weighted indices might prove to be unacceptable for alternative investors (especially benchmarked investments).

We turn to the magnitude of systematic risk

1. Annual returns of Fama-French factors from 1990–2012



The Fama-French factors are constructed using their 6 value-weight portfolios formed on size and book-to-market. Size (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios, Size = 1/3 (Small Value + Small Neutral + Small Growth) – 1/3 (Big Value + Big Neutral + Big Growth). Value (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios. Value = 1/2 (Small Value + Big Value) – 1/2 (Small Growth + Big Growth). 'Market' is the excess return on the market, value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t, good shares and price data at the beginning of t, and good return data for t minus the one-month Treasury bill rate (from Ibbotson Associates).
Source: Kenneth R. French website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html

"The risk/return profile of alternative indices can be materially driven by dynamically-rewarded systematic risks, putting the robustness of their performance across different time periods at risk"

1 In the rest of this article, we consider systematic risks as the loadings on the market, size and value factors defined by Fama and French (1992).
2 The interested reader can find a detailed description of the Maximum Deconcentration strategy in the associated White Paper at www.scientificbeta.com/download/file/scientific-beta-max-deconcentration-indices
3 For a more detailed overview of diversification strategies, please refer to the ERI Scientific Beta publication *Overview of Diversification Strategy Indices*, available on the www.scientificbeta.com website, and also to the article entitled 'Overview of diversification strategies' in the present supplement (page 2).

exposures of smart beta indices, as well as their influence on the performance of those indices.

Systematic risk exposures of popular diversification strategies[3]
In order to study the exposure to systematic risks, we have chosen to use the popular diversification strategies represented by the Scientific Beta indices. As shown in figure 2, any alternative weighting scheme induces a set of systematic risk exposures. For example, while the Maximum Deconcentration index exhibits a neutral exposure to both the market (1.01 market beta) and value factors (–0.01 HML beta), it presents the most positive exposure to the size factor (0.44 SMB beta).

As illustrated, alternative indices exhibit non-negligible exposures to a subset of systematic risk factors. Thus, their performances will inevitably be driven by the risk premium of those factors, and the variability of their performance will be lead by the dynamics in those risk premia.

We further illustrate this phenomenon by measuring the impact of the small-cap exposure over different time periods. Figure 3 shows the performance and factor exposures of the Scientific Beta USA Mid-Cap Maximum Deconcentration index[4] over two different periods where the small-cap factor earned negative (–12.63%) and positive (+17.54%) premiums respectively. The periods have been chosen such that they have a comparable length with quite different SMB (small-minus-big) premia. Due to mid-cap selection, the small size exposure of the index remained quite high between 0.55 and 0.67 across different periods. In the first period, the index underperformed its cap-weighted benchmark by 6.70% and the small-cap exposure contributed largely to this (–8.35%). In the second period, where the index outperforms the benchmark, a large part of the performance is driven by small-cap exposure (+9.80%). It is useful to see that although the long-term analysis shows the positive impact of small-cap exposure, the short-term impact of the risk factor can be dependent upon the prevailing economic conditions.

Improving the robustness of smart beta indices: towards best practice

Understanding the sources of performance
As shown earlier, the risk exposures of alternative equity indices have an impact on their performance and risk profile because they are rewarded differently through time, and thus on the consistency of this profile when measured at different time periods, ie, the robustness of this index profile.

A first step is thus to identify the sources of outperformance and the dynamics of their premium over time. Doing so allows a conscious choice of portfolio construction methodology to be made through the lens of the systematic risk exposure and the ground to be set for understanding the conditions under which the risk exposure will deliver positive or negative returns.

It is a documented fact that value stocks outperform growth stocks over the long run (see Fama and French [1992]). However, while holding a value-exposed portfolio, the investor takes the risk of short-term underperformance in the periods where the value factor earns negative returns. This is illustrated in figure 4, which shows the performance and factor exposures of Scientific Beta USA Value Maximum Deconcentration since its inception, and over a 16-month period leading to the 2008 financial crisis where the value factor underperformed heavily (HML premium = –14.68%). Figure 4 shows that in the full period, the index has a exposure of 0.19 to the HML factor, which contributes 78bps to its performance. However, in the sub-period, the performance contribution of the value factor is -2.30% (with exposure of 0.16).

Controlling for risk factor exposure: trade-off between performance and investment objective
Secondly, an investor might seek exposure to a specific methodology and objective (eg, minimum volatility) while reducing the relative risks induced by the underlying risk factor exposure (which as we showed has a time-varying risk premium and induces the risk of short-term underperformance or simply unstable performance due to this exposure). As part of the Smart Beta 2.0 approach promoted by EDHEC-Risk Institute, it is possible to control this factor exposure and as such be better able to control the risks of underperforming the reference cap-weighted index.

2. Risk factor exposures of Scientific Beta USA indices

	Scientific Beta USA Maximum Deconcentration	Scientific Beta USA Diversified Risk Parity	Scientific Beta USA Maximum Decorrelation	Scientific Beta USA Efficient Minimum Volatility	Scientific Beta USA Efficent Maximum Sharpe Ratio
Fama-French factors	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
Alpha	*0.40%*	*0.75%*	*0.37%*	*1.90%*	*0.77%*
Market	**1.01**	**0.96**	**0.96**	**0.84**	**0.93**
Size (SMB)	**0.44**	**0.37**	**0.40**	**0.22**	**0.34**
Value (HML)	**–0.01**	**–0.01**	**–0.06**	**–0.06**	**–0.05**
Adjusted R-square	>99%	>99%	99%	98%	99%

This table shows the coefficient estimates and R-squared of the regression of the index's excess returns over the risk-free rate using the Fama-French three-factor model over the analysis period from inception date (21 June 2002) to 31 December 2012. The coefficients statistically significant at the 95% confidence level are highlighted in bold.
The data are daily total returns (with dividend reinvested). The Market factor is the daily return of the cap-weighted index of all stocks that constitute the index portfolio. The SMB factor is the daily return series of a portfolio that is long the top 30% stocks (small market-cap stocks) and short the bottom 30% stocks (large market-cap stocks) sorted on market capitalisation in ascending order. The HML factor is the daily return series of a portfolio that is long the top 30% stocks (value stocks) and short the bottom 30% stocks (growth stocks) sorted on book-to-market value in descending order. ERI Scientific Beta uses the Secondary Market US Treasury Bills (3M) as the risk-free rate in US dollars. The cap-weighted reference index used is the SciBeta USA cap-weighted index.

3. Performance and risk factor exposure analysis of Scientific Beta USA Mid-Cap Maximum Deconcentration index

	First period (11 May 2006–21 Nov 2008)		Second period (24 November 2008–27 June 2011)		Full period (26 June 2002–31 December 2012)	
Annual returns of strategy	–22.32%		35.33%		8.63%	
Annual excess returns of strategy	–6.70%		12.87%		2.57%	
Annual risk premia of SMB	–12.63%		17.54%		3.85%	
	Coefficient	Performance	Coefficient	Performance	Coefficient	Performance
Annual alpha	0.62%	0.68%	1.42%	2.91%	0.45%	0.38%
Market beta	**1.00**	–18.57%	**1.05**	23.46%	**1.01**	4.36%
Size (SMB) beta	**0.66**	–8.35%	**0.56**	9.80%	**0.67**	2.58%
Value (HML) beta	**–0.08**	0.33%	**–0.14**	–0.96%	**–0.08**	–0.36%
R-square	0.99		0.99		0.99	

The table shows the performance and risk factor exposure analysis of the Scientific Beta USA Mid-Cap Maximum Deconcentration index in two different sub-periods, and in the full period. All statistics are annualised, all portfolios are rebalanced quarterly and the analysis is based on daily total returns (with dividends reinvested). The coefficients statistically significant at the 95% confidence level are highlighted in bold. The total number of stocks in the USA scientific beta universe is 500 and mid-cap indices are constructed on the bottom 50% market-cap stocks in the universe.

4. Performance and risk factor exposure analysis of Scientific Beta USA Value Maximum Deconcentration index

	Full period (26 June 2002–31 December 2012)		Sub-period (30 March 2007–16 July 2008)	
Annual returns of strategy	8.52%		–9.37%	
Annual relative returns of strategy	2.46%		–4.22%	
Annual volatility of strategy	23.42%		19.35%	
Sharpe ratio of strategy	0.29		–0.65	
Annual risk premia of HML	4.23%		–14.68%	
	Coefficient	Performance	Coefficient	Performance
Annual alpha	0.38%	0.33%	–0.42%	–0.63%
Market beta	**0.98**	4.26%	**0.98**	–7.99%
Size (SMB) beta	**0.39**	1.49%	**0.34**	–1.73%
Value (HML) beta	**0.19**	0.78%	**0.16**	–2.30
R-square	0.99		0.99	

The table shows the performance and risk factor exposure analysis of the Scientific Beta USA Value Maximum Deconcentration index in a selected sub-period, and in the full period. All statistics are annualised, all portfolios are rebalanced quarterly and the analysis is based on daily total returns (with dividends reinvested). The coefficients statistically significant at the 95% confidence level are highlighted in bold. The total number of stocks in the USA scientific beta universe is 500 and value indices are constructed on the top 50% book-to-market stocks in the universe.

4 The choice of weighting scheme is based on the fact that Maximum Deconcentration is the only smart beta strategy that does not require any parameter estimation and is therefore free of estimation risk. Its out-of-sample performance is identical to its in-sample performance and thus allows us to examine the effect of stock selection (in this case, small-cap) in isolation.

For example, we show in figure 5 how one can control for the size exposure of the US Maximum Deconcentration index through a large-cap selection, and measure the impact on relative risks (as measured by the time under water and the relative drawdown).

Basically, the main systematic risk driver of the strategy is its bias towards small caps because this is the main factor exposure (with a +0.44 SMB beta). Selecting only the larger-cap stocks reduces this small-cap bias (to +0.19) and consequently decreases the amount of the risk premium associated with this factor.

Doing so gives an exposure to the underlying Maximum Deconcentration methodology and at the same time materially reduces the relative risk compared to the cap-weighted index as measured by the maximum relative drawdown

"Ultimately, the best guarantee of the robustness of smart beta indices is good information from the market and investors on the systematic and specific risks of these alternative benchmarks"

(–13.76% before controlling for size and –7% after) and the maximum time under water (453 days before controlling for size and 48 days after). Thus, the robustness of the Maximum Deconcentration here is improved by reducing the relative (extreme) risks induced by the small cap factor exposure.

On the importance of transparency in robustness

In the context of equity indexing, being transparent not only consists in providing investors with sufficient information on the ex-post risk and return characteristics. Since the choice of an alternative indexation methodology induces (more often than not) an incidental exposure to systematic risks, transparency also consists in displaying the systematic risk and return drivers of the index in order for investors to assess the dynamics and conditions of those performances. For certain risk factors it is possible for an investor to be able to carry out due diligence simply on the basis of returns-based-type analysis, as is the case for example for Fama-French-type analysis. It is much more difficult however for the analysis of other risks such as sector or liquidity risk. For these types of risks it is essential to be able to access the historical compositions of the indices.

EDHEC-Risk Institute considers that it is only by allowing all stakeholders to have unrestricted access not only to the performance but also to the details on the methodologies and historical compositions of smart beta indices that it will be possible to avail of objective and contradictory information on strategies' risks. It is from this perspective that EDHEC-Risk Institute has constructed its smart beta index platform ERI Scientific Beta, which gives unrestricted access to all the information on the 2,442 smart beta indices that are representative of the different choices of weighing and risk factors.

Ultimately, the best guarantee of the robustness of smart beta indices is good information from the market and investors on the systematic and specific risks of these alternative benchmarks. This informational efficiency will lead providers to integrate devices for limiting and controlling these risks because they will know that investors will be able to compare the performances with the reality of the risks taken. Moreover, investors will no longer see a period of underperformance as evidence of the lack of out-of-sample robustness of methodologies that are often sold on the basis of a simulated historical track record. Instead, they will be able to understand the source of the underperformance, which, when it is related to a systematic risk, cannot be avoided irrespective of the quality and robustness of the index construction model.

5. Factor exposure control

	Scientific Beta USA Maximum Deconcentration	Scientific Beta USA Large-Cap Maximum Deconcentration
Fama-French factors	Coefficient	Coefficient
Alpha	0.40%	0.49%
Market	1.01	1.00
Size (SMB)	0.44	0.19
Value (HML)	–0.01	0.06
	Relative risk	
Maximum relative drawdown	13.76%	6.99%
Maximum time under water (number of business days)	453	48

The table illustrates the control of the implicit risk factor (SMB factor) exposure of Scientific Beta USA Maximum Deconcentration by selecting large-cap stocks in the Scientific Beta USA Large-Cap Maximum Deconcentration strategy, and its resulting impact on the relative risk of the strategy. The first panel shows the estimates of the coefficients and R-square from the regression of the index's excess returns over the risk-free rate using the Fama-French three-factor model over the period from inception date (21 June 2002) to 31 December 2012. The coefficients statistically significant at the 95% confidence level are highlighted in bold. The second panel of the table shows the impact of the control of implicit risk factor exposure on the relative risk (over the cap-weighted reference index) of the strategies.

The data are daily total returns (with dividend reinvested). The Market factor is the daily return of the cap-weighted index of all stocks that constitute the index portfolio. The SMB factor is the daily return series of a portfolio that is long the top 30% stocks (small market-cap stocks) and short bottom 30% stocks (large market-cap stocks) sorted on market capitalisation in ascending order. HML factor is the daily return series of a portfolio that is long top 30% stocks (value stocks) and short bottom 30% stocks (growth stocks) sorted on book-to-market value in descending order. ERI Scientific Beta uses the Secondary Market US Treasury Bills (3M) as the risk-free rate in US dollars. The cap-weighted reference index used is SciBeta USA cap-weighted index. The Maximum Relative Drawdown measures the maximum relative loss experienced by a strategy between a peak and a valley over a specified period. Maximum Time under Water is the length of time of the relative drawdown that lasted for the longest period.

6. Assessing the robustness of Scientific Beta strategies

	Scientific Beta USA Efficient Minimum Volatility (2% TE)[1]	Scientific Beta USA Mid-Cap Maximum Deconcentration[2]
Relative return over cap-weighted	0.50%	3.25%
Probability of outperformance (one year)	76.1%	64.0%

The table shows relative returns and probability of outperformance of the USA Efficient Minimum Volatility index (2% tracking error) and the USA Mid-Cap Maximum Decorrelation Index. The probability of outperformance is calculated using the performance in indices using a one-year rolling window with a one-week step size. The statistics are based on daily total returns (with dividend reinvested) for the period 21 June 2002 to 31 December 2012.

1 The Scientific Beta USA Efficient Minimum Volatility index (2% TE) has the highest probability of outperformance (one year) among the strategies that have yielded less than 1% excess return relative to the reference cap-weighted benchmark.

2 The Scientific Beta USA Mid-Cap Maximum Decorrelation index has the lowest probability of outperformance (one year) among the strategies that have yielded more than 3% excess return relative to the reference cap-weighted index.

Conclusion

The performance of an alternative index that is marketed on the basis of a smarter than cap-weighted portfolio construction process but mostly driven by some risk factor exposures that have different returns and risks between the back-track period and the live period, might not be reliable in terms of consistency and robustness, and expose investors to extreme short-term relative risks against a cap-weighted reference.

Put more simply, a strategy performing over a specific period, and exposed to a set of systematic risks, is not always guaranteed to behave the same way through time: it can simply mean that the exposure to a set of performing factors has driven the performance favourably over that period. More perniciously, even when consciously choosing an alternative index not only for its diversification benefits but also for a set of long-term positively rewarding factors (such as value and size), one is exposed to short-term risks of underperformance against a cap-weighted reference, and has to be made aware of it.

For example, as shown in figure 6, the USA Mid-Cap Maximum Decorrelation Index exhibits an attractive relative performance over its cap-weighted reference index (+3.25%) against the 2% tracking-error-controlled USA Efficient Minimum Volatility Index, which barely outperforms (+0.50%) the cap-weighted reference index during the same period. On the other end of the scale, the US USA Mid-Cap Maximum Decorrelation Index exhibits a lower probability of outperformance (64.0%) than the USA Efficient Minimum Volatility Index (2% TE), calculated using a one-year rolling window analysis. This means that choosing the mid-cap index on the basis of historical performance would only lead to being exposed to more systematic risks (mid-cap and defensiveness) and thus to time-varying rewards that will probably reduce the outperformance with respect to cap-weighted indices.

In this article, we have considered several ways to ensure the robustness of alternative equity indices. Measuring the robustness of smart beta indices' figures also improves the reliability and standards in alternative indices. This ultimately represents a manner in which to increase the confidence of investors in this new paradigm.

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