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Research Insights

SCIENTIFIC BETA SPECIAL

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Introduction
Noël Amenc

It is a great pleasure to introduce the latest Scientific Beta special issue of the Research Insights supplement to IPE. The aim of EDHEC-Risk Institute's Research Insights is to provide European institutional investors with informed conclusions from the research carried out by EDHEC-Risk Institute. In the case of the present supplement, this research has contributed to the ERI Scientific Beta venture that EDHEC-Risk Institute founded last year. It is important to stress, and we do so unreservedly, that the views expressed in this supplement are those of EDHEC-Risk Institute and ERI Scientific Beta and are not endorsed in any way by IPE or its journalists.

We first introduce a new approach to equity investing termed 'smart factor investing,' arguing that current smart beta investment approaches only provide a partial answer to the main shortcomings of capitalisation-weighted indices. The article assesses the benefits of simultaneously addressing the two main shortcomings of cap-weighted indices – undesirable factor exposures and heavy concentration – by constructing factor indices that explicitly seek exposures to rewarded risk factors while diversifying away unrewarded risks. The results suggest that such smart factor indices lead to considerable improvements in risk-adjusted performance. These smart factor indices are not the end point for investing in equities in a smart way, but instead the starting point, ingredients to construct smart beta allocation solutions while respecting risk objectives that can be expressed in absolute or relative terms. This first article provides a panorama of solutions that have been the subject of a considerable research effort conducted by EDHEC-Risk Institute with the support of Amundi ETF & Indexing.

On the major subject of risk allocation with smart factor indices, we conduct a case study with factor exposure constraints in a second article. We show, importantly, that it is possible to perform risk parity in the long-only world – ie, to have an exposure that is equal in terms of risk factors rewarded over the long term without necessarily having pure or orthogonal factors that are impossible to obtain in the long-only space. This point is all the more important in that often, under the pretext of purity, investors choose excessively concentrated factor indices that contribute neither purity nor diversification and therefore have a fairly low risk-adjusted return. Our argument is that by using well-diversified investable proxies for each factor (the Scientific Beta smart factor indices), it is possible to implement high-performance allocation between these indices while respecting factor risk parity constraints.

Finally, again as part of our solutions for allocating to smart beta, we find that value can be added through relative equal risk contribution and relative global minimum variance at the allocation stage, for investors with a tracking error budget. As a result, extremely substantial levels of risk-adjusted outperformance (information ratios) can be achieved even in the absence of views on factor returns

Naturally, the question that all investors pose for an innovative solution is its investability. The objective of the fourth article is to describe how to ensure the investability of smart beta indices by managing turnover control and capacity constraints. Investing in smart beta indices requires investors to have access to solutions where implementation costs and liquidity risks are thoroughly considered. A key implication is that the smart beta index turnover and capacity constraints need to be methodologically and carefully handled through the construction of the index.

The results presented in this supplement are sufficiently impressive for investors to raise the question of their robustness. We examine the robustness of smart beta strategies by defining this robustness according to two dimensions. The first – termed relative robustness – corresponds to the capacity for a smart beta index exposed to clearly identified systematic risk factors to always be exposed to the same outperformance compared to the return given by the market for that, or those, factor(s). Relative robustness can be improved by reducing all sources of unrewarded risks with the use of a consistent framework, robust parameter estimation techniques, weight constraints, and strategy-specific risk. The second dimension – which we refer to as absolute robustness – raises the question of the capacity of the smart beta index to outperform the market whatever the time period, or more specifically, whatever the market regimes or returns associated with such and such a systematic risk factor. Robustness can be achieved through allocating across several rewarded factors. Our results show that single factor indices have a high degree of relative robustness, but they are not robust in absolute terms. Multi-beta allocations, on the other hand, are highly robust in absolute terms.

To conclude this special issue, we provide a brief overview of equity factor index offerings from major index providers. Factor indices aim to provide explicit exposure to a common risk factor to harvest its long-term risk premia.

We hope you will find the articles on smart beta in the supplement informative and relevant. We extend warm thanks to our friends at IPE for their collaboration on the supplement.

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Risk allocation, factor investing and smart beta

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This article argues that current smart beta investment approaches only provide a partial answer to the main shortcomings of capitalisation-weighted (cap-weighted) indices, and introduces a new approach to equity investing referred to as smart factor investing. It provides an assessment of the benefits of simultaneously addressing the two main shortcomings of cap-weighted indices, namely their undesirable factor exposures and their heavy concentration, by constructing factor indices that explicitly seek exposures to rewarded risk factors while diversifying away unrewarded risks. The results we obtain suggest that such smart factor indices lead to considerable improvements in risk-adjusted performance.

For long-term US data, smart factor indices for a range of different factor tilts roughly double the Sharpe ratio of the broad cap-weighted index. Outperformance of such indices persists at levels ranging from 2.92% to 4.46%, even when assuming unrealistically high transaction costs. Moreover, by providing explicit tilts to consensual factors, such indices improve upon many current smart beta offerings where, more often than not, factor tilts result as unintended consequences of ad hoc methodologies. In fact, our research shows that by using consensual results from asset pricing theory concerning both the existence of factor premia and the importance of diversification, it is possible to go beyond existing smart beta approaches which provide partial solutions by only addressing one of these issues.

Designing efficient and investable proxies for risk premia

We focus on four well-known rewarded factors – the size and value factors (Fama and French [1993]), the momentum factor (Carhart [1997]) and the low volatility factor (Ang et al [2006, 2009]). For each rewarded factor, we introduce a corresponding smart factor index, which can be regarded as an efficient investable proxy for a given risk premium. In a nutshell, a risk premium can be thought of as a combination of a risk (exposure) and a premium (to be earned from the risk exposure). Smart factor indices have been precisely engineered to achieve a pronounced factor tilt emanating from the stock selection procedure (relevant risk exposure), as well as high Sharpe ratio emanating from the efficient diversification of unrewarded risks related to individual stocks (fair reward for the risk exposure). The access to the fair reward for the given risk exposure is obtained through a well-diversified, also known as smart-weighted, portfolio, as opposed to a concentrated cap-weighted portfolio, of the selected stocks so as to ensure that the largest possible fraction of individual stocks' unrewarded risks is eliminated.

"The combination of relevant security selection and appropriate weighting schemes in a two-step process leads to substantial improvements in risk-adjusted performance with respect to the use of a standard cap-weighted index"

The results in figure 1 confirm that the combination of relevant security selection and appropriate weighting schemes in a two-step process leads to substantial improvements in risk-adjusted performance with respect to the use of a standard cap-weighted index, which typically implies an inefficient set of factor exposures and an excess of unrewarded risk.

On the one hand, starting with a focus on the systematic risk exposure, we find that a higher Sharpe ratio can be achieved with the same weighting scheme, here a cap-weighting scheme, for stocks selected on the basis of their loadings on the value, size, momentum and low volatility factors, compared to the case where the full universe is held in the form of a cap-weighted portfolio.

The results we obtain, reported in figure 1, show that while the Sharpe ratio of the cap-weighted index is 0.24 on the sample period, it reaches values as high as 0.39 for a mid-cap stock selection, 0.30 for a high momentum stock selection, 0.29 for a low volatility stock selection or 0.35 for a value stock selection. These results suggest that a systematic attempt to harvest equity risk premia above and beyond broad market exposure leads to additional risk-adjusted performance. It should be noted at this stage that substantially higher levels of

1. Performance comparison of US cap-weighted factor indices and US multi-strategy factor indices

		Mid cap		High momentum		Low volatility		Value	
	Broad CW	CW	Diversified multi-strategy	CW	Diversified multi-strategy	CW	Diversified multi-strategy	CW	Diversified multi-strategy
Annualised return	9.74%	12.54%	14.19%	10.85%	13.30%	10.09%	12.64%	11.78%	14.44%
Annualised volatility	17.47%	17.83%	16.73%	17.60%	16.30%	15.89%	14.39%	18.02%	16.55%
Sharpe ratio	0.24	0.39	0.52	0.30	0.48	0.29	0.50	0.35	0.54
Historical daily 5% VaR	1.59%	1.60%	1.50%	1.64%	1.50%	1.42%	1.28%	1.59%	1.47%
Maximum drawdown	54.53%	60.13%	58.11%	48.91%	49.00%	50.50%	50.13%	61.20%	58.41%
Annualised excess returns	–	2.80%	4.45%	1.10%	3.56%	0.35%	2.90%	2.04%	4.70%
Annualised tracking error	–	5.99%	6.80%	3.50%	4.88%	4.44%	6.17%	4.74%	5.82%
95% tracking error	–	9.39%	11.56%	6.84%	8.58%	9.20%	11.53%	8.72%	10.14%
Information ratio	–	0.47	0.66	0.32	0.73	0.08	0.47	0.43	0.81

The table shows the absolute performance, relative performance and risk indicators for cap-weighted (CW) factor indices and multi-strategy factor indices for four factor tilts – mid cap, high momentum, low volatility and value. The complete stock universe consists of the 500 largest stocks in the US. The benchmark is the cap-weighted portfolio of the full universe. The yield on secondary market US Treasury bills (3M) is the risk-free rate. The return-based analysis is based on daily total returns from 31 December 1972 to 31 December 2012 (40 years). All weight-based statistics are average values across 160 quarters (40 years) from 31 December 1972 to 31 December 2012.

maximum drawdown are incurred for the mid-cap and value selections, confirming that the reward harvested through the factor exposure is a compensation for a corresponding increase in risk. In contrast, we note that high momentum and low volatility selections lead to lower levels of maximum drawdown compared to the no selection case, suggesting that the excess performance earned on these two factors has at best a behavioural explanation, and is not necessarily related to an increased riskiness.

On the other hand, shifting to the management of specific risk exposures, we find that even higher levels of Sharpe ratio can be achieved for each selected factor exposure through the use of a well-diversified weighting scheme, which we take to be an equally-weighted combination of five popular smart weighting schemes[1]. Thus, the Sharpe ratio of the so-called diversified multi-strategy combination reaches 0.52 for mid-cap stocks, 0.48 for high momentum stocks, 0.50 for low volatility stocks and 0.54 for value stocks.

These results suggest that multi-strategy factor-tilted indices obtain the desired factor tilts without undue concentration, which provides an explanation for their superior risk-adjusted performance with respect to the cap-weighted combination of the same selection of stocks.

Overall, it appears that the combined effects of a rewarded factor exposure ensured by a dedicated proper security selection process and an efficient harvesting of the associated premium through improved portfolio diversification leads to a Sharpe ratio improvement of around 100% compared to the broad cap-weighted index.

Risk allocation with smart factor indices

Once a series of smart factor indices have been developed for various regions of the equity universe, they can be used as attractive building blocks in the design of an efficient allocation to these multiple risk premia.

In an attempt to identify, and analyse the benefits of, the possible approaches to efficient risk allocation across the various smart factor indices, we identify four main dimensions that can be taken in consideration when designing a sophisticated allocation methodology (see figure 2).

The first, and arguably most important, dimension relates to whether risk is defined by the investor from an absolute perspective in the absence of a benchmark, or whether it is instead defined in relative terms with respect to an existing benchmark, which is more often than not a cap-weighted index. In the former situation, one would use volatility as a relevant risk measure, while tracking error with respect to the cap-weighted index would instead be used in the latter case.

The second dimension concerns whether one would like to incorporate views regarding factor returns in the optimisation process. While additional benefits can be obtained from the introduction of views on factor returns at various points of the business cycle, we focus in what follows only on approaches that are solely based on risk parameters, which are notoriously easier to estimate with a sufficient degree of robustness and accuracy (Merton [1980]). The third dimension is related to the objective of the allocation procedure. Indeed, there are several possible targets for the design of a well-diversified portfolio of factor exposure, depending upon whether one would like to use naive approaches (equal dollar allocation or equal risk allocation) or scientific approaches based on minimising portfolio risk (volatility in the absolute return context or tracking error in the relative return context). The fourth and last dimension related to the presence of various forms of constraints such as minimum/maximum weight constraints, turnover constraints, or factor exposure constraints, which are obviously highly relevant in the context of risk factor allocation.

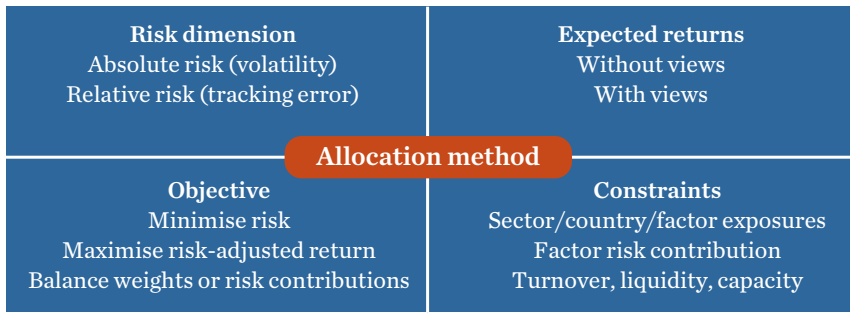
To illustrate the benefits of an efficient allocation to smart factor indices, we consider a second dataset over the 10-year period from 31 December 2003 to 31 December 2013 using five sub-regions of the global developed universe – US, UK, Developed Europe ex-UK, Japan and Developed Asia Pacific ex-Japan. Using the four smart multi-strategy indices as proxies for the value, size, momentum and volatility rewarded tilts in each region, we obtain a total of (5×4) 20 constituents.

Absolute return perspective

We start from the absolute return perspective and consider in figure 3 five allocation strategies to the 20 aforementioned smart factor indices – an equal dollar contribution portfolio (denoted by multi beta EW allocation), an equal risk contribution portfolio (denoted by multi beta ERC allocation), and then a global minimum variance portfolio (denoted by multi beta GMV allocation). Given that these allocation strategies lead in general to concentrated factor exposures (for example, the minimum variance portfolio heavily loads on the low volatility factor indices in each region), we also introduce factor risk parity constraints – that is, we restrict our analysis to portfolios such that each one of the four factors has the same contribution to the portfolio volatility. More precisely we consider a global minimum variance portfolio subject to factor risk parity constraints (denoted by multi beta GMV factor allocation), as well as a maximum deconcentration portfolio subject to factor risk parity constraints (denoted by multi beta MDecon factor allocation), a portfolio which can be regarded as the closest approximation to an equally-weighted portfolio that satisfies the factor risk parity constraints[2].

We note that the GMV allocation process leads to the lowest volatility, as expected. When analysing the performances in terms of bull versus bear market regimes (defined as positive versus negative returns for the cap-weighted index), we observe that the addition of risk parity constraints to the GMV allocation tends to stabilise the returns across market conditions. For example, in the absence of a factor risk parity constraint, the GMV allocation leads to a massive outperformance of 11.94% with respect to the cap-weighted index in bear markets, which is due to the almost exclusive domination of the low volatility factor, with a defensive ►

2. The various dimensions of allocation methodologies across assets or risk factors



3. Multi beta allocations across smart factor indices (Developed universe)

Developed (2004–13)		Diversified multi-strategy				
	CW (All stocks)	Multi beta EW allocation	Multi beta ERC allocation	Multi beta GMV allocation	Multi beta MDecon factor allocation	Multi beta GMV factor allocation
Annualised returns	7.80%	11.37%	11.07%	10.57%	11.17%	10.88%
Annualised volatility	17.09%	15.32%	14.33%	12.84%	17.23%	17.21%
Sharpe ratio	0.36	0.64	0.66	0.70	0.56	0.54
Maximum drawdown	57.13%	54.40%	51.82%	45.07%	55.22%	55.32%
Excess returns	–	3.56%	3.27%	2.76%	3.37%	3.07%
Tracking error	–	6.75%	7.51%	6.36%	3.08%	3.34%
95% tracking error	–	13.84%	14.89%	11.85%	5.19%	5.55%
Information ratio	–	0.53	0.44	0.43	1.09	0.92
Outperformance probability (3Y)	–	98.36%	89.34%	89.07%	100.00%	100.00%
Maximum relative drawdown	–	6.35%	9.54%	13.10%	4.03%	5.47%
Annualised relative return bull	–	2.50%	0.48%	-3.90%	3.81%	3.18%
Annualised relative return bear	–	4.65%	6.74%	11.94%	2.50%	2.66%

The table shows the allocations of the EW, ERC, GMV under geographical constraints, and both the max-deconcentration and GMV diversified multi-strategy indices under geographical and risk parity constraints, invested in the 20 diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value in the five sub-regions – US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan. The period goes from 31 December 2003 to 31 December 2013.

1 Diversified multi-strategy weighting is an equal-weighted combination of the following five weighting schemes – maximum deconcentration, diversified risk-weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio (see www.scientificbeta.com for more details).

2 So as to avoid introducing overly strong biases in country exposures, we also introduce a set of constraints dedicated to ensure that each one of the five regions is not too strongly under- or over-represented with respect to its market capitalisation in the cap-weighted global developed index.

bias that proves extremely useful in such market conditions.

On the other hand, the relative return in bull markets is negative at –3.90% due to the performance drag associated with exclusively holding defensive equity exposure in bull market conditions. In this context, one key advantage of the introduction of factor risk parity constraints is that it leads to a much more balanced return profile across market conditions with positive outperformance in both bear and bull markets (at 2.66% and 3.18% respectively).

We also find that the introduction of factor risk parity constraints has led to a substantial improvement in information ratios with an information ratio above 1 for the maximum-deconcentration allocation under risk parity constraints. Interestingly we note that the introduction of factor risk parity constraints leads to 100% outperformance probabilities over a three-year horizon. Overall, all tested strategies lead to extremely substantial levels of outperformance with respect to the cap-weighted index, with excess returns ranging between 276 and 356 basis points per annum.

Relative return perspective

It is often the case that investors maintain the cap-weighted index as a benchmark, which has the merit of macro-consistency and is well-understood by all stakeholders. In this context, a multi-smart beta solution can be regarded as a reliable cost-efficient substitute to expensive active managers, and the most relevant perspective is not an absolute return perspective, but a relative return perspective, with respect to the cap-weighted index.

"All tested strategies lead to extremely substantial levels of outperformance with respect to the cap-weighted index, with excess returns ranging between 276 and 356 basis points per annum"

In what follows, we focus on two approaches, a naive diversification approach leading to a relative equal risk allocation (R-ERC) portfolio, which focuses on equalising the contribution of the smart factor-tilted indices to the portfolio tracking error, and a scientific diversification approach leading to a relative global minimum variance (R-GMV) portfolio, also known as minimum tracking error portfolio, which focuses on minimising the variance of the portfolio relative returns with respect to the cap-weighted index.

From the results reported in figure 4, we note that the focus on relative return leads to lower tracking error levels compared to the portfolios that had an absolute return focus. For example, the ex-post tracking error is around 2.50% for these two portfolios (2.43% for the relative minimum variance portfolio and 2.56% for the relative equal risk contribution portfolio). Such low tracking error levels, associated with substantial outperformance (more than 300 basis points per annum for the R-ERC portfolio), eventually leads to exceedingly high information ratios. In particular, the relative ERC has an information ratio of 1.22, which is the highest level among all portfolio strategies tested so far, with an outperformance probability of 100% over any given three-year investment horizon during the same period.

4. Relative ERC and GMV allocation across smart factor indices (Developed universe)

Developed (2004–13)		Diversified multi-strategy	
	CW (All stocks)	Multi beta relative ERC allocation	Multi beta relative GMV allocation
Annualised returns	7.80%	10.92%	9.96%
Annualised volatility	17.09%	16.10%	16.64%
Sharpe ratio	0.36	0.58	0.50
Maximum drawdown	57.13%	54.14%	55.50%
Excess returns	–	3.12%	2.15%
Tracking error	–	2.56%	2.43%
95% tracking error	–	4.70%	4.27%
Information ratio	–	1.22	0.88
Outperformance probability (3Y)	–	100.00%	89.34%
Maximum relative drawdown	–	5.10%	4.95%
Annualised return bull	–	31.38%	31.02%
Annualised return bear	–	–25.25%	–26.93%

The table compares performance and risk of Scientific Beta diversified multi-strategy indices converted in US dollars. We look at relative ERC and relative GMV allocations invested in the 20 diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value in the five sub-regions – US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan. The period goes from 31 December 2003 to 31 December 2013.

Conclusion: from cap-weighted indices to smart factor indices

We find that well-rewarded factor-tilted indices constitute attractive building blocks for the design of an improved equity portfolio. First-generation smart beta investment approaches only provide a partial answer to the main shortcomings of cap-weighted indices. Multi-strategy factor indices, which diversify away unrewarded risks and seek exposure to rewarded risk factors, address the two main problems of cap-weighted indices (their undesirable factor exposures and their heavy concentration) simultaneously.

The results suggest that such multi-strategy factor indices lead to considerable improvements in risk-adjusted performance. For long-term US data, smart factor indices for a range of different factor tilts roughly double the Sharpe ratio of the broad cap-weighted index. Moreover, outperformance of such indices persists at levels ranging from 2.92% to 4.46%, even when assuming unrealistically high transaction costs. The outperformance of multi-strategy factor indices over cap-weighted factor indices is observed for other developed stock markets as well. By providing explicit tilts to consensual factors, such indices improve upon many current smart beta offerings where, more often than not, factor tilts result as unintended consequences of ad hoc methodologies.

Moreover, additional value can be added at the allocation stage, where the investor can control for the dollar and risk contributions of various constituents or factors to the absolute (volatility) or relative risk (tracking error) of the portfolio. As a result, extremely substantial levels of risk-adjusted outperformance (information ratios) can be achieved even on the absence of views on factor returns. The portfolio strategies we have presented in this publication can be regarded as robust attempts at generating an efficient strategic factor allocation benchmark in the equity space. Obviously, active portfolio managers may generate additional value on top of this efficient benchmark by incorporating forecasts of factor returns at various points of the business cycle in the context of tactical factor allocation decisions.

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Risk allocation with smart factor indices: a case study with factor exposure constraints

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In the context of generating a 'smart' (meaning efficient) allocation to smart factor indices, a natural first, albeit naïve, approach, consists in forming an equally-weighted portfolio of the selected smart factor indices, in this case the indices that serve as proxies for the value, small cap, momentum and low volatility risk premia.

While an equally-weighted scheme is the simplest approach one can use, it is likely that the use of more sophisticated weighting schemes could add additional value, in particular when it comes to the management of the risks relative to the cap-weighted (CW) benchmark. We shall sequentially consider in what follows the absolute return approach both with and without factor risk parity/budgeting constraints. We consider naïve approaches to diversification (maximum deconcentration in terms of dollar or risk contributions) and scientific approaches (minimum risk from the absolute return perspective). One of the important aims of this article will also be to show that it is possible to perform risk parity in the long-only world – ie, to have an exposure that is equal in terms of risk factors rewarded over the long term without necessarily having pure or orthogonal factors that are impossible to obtain in the long-only space. This point is all the more important in that often, under the pretext of purity, investors choose excessively concentrated factor indices that contribute neither purity nor diversification and therefore have a fairly low risk-adjusted return. Our argument here is that by using well-diversified investable proxies for each factor (the Scientific Beta smart factor indices), it is possible to implement high-performance allocation between these indices while respecting factor risk parity constraints.

All these methodologies will be implemented without any active views (expected return forecasts) on constituents or factors; they generate portfolios that can be regarded as attractive starting points, with very substantial risk-adjusted outperformance benefits with respect to cap-weighted indices, to which additional benefits could be added by asset managers possessing skills for actively timing factor exposures.

The Developed dataset extends over the 10-year period from 31 December 2003 to 31 December 2013 and uses five sub-regions of the global developed universe: US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan. Using four smart multi-strategy indices as proxies for the value, size, momentum and volatility-rewarded tilts in each sub-region, we obtain a total of 5×4 = 20 constituents. For US illustrations, long-term (40-year) data from 31 December 1973 to 31 December 2013 is used for the four smart multi-strategy indices.

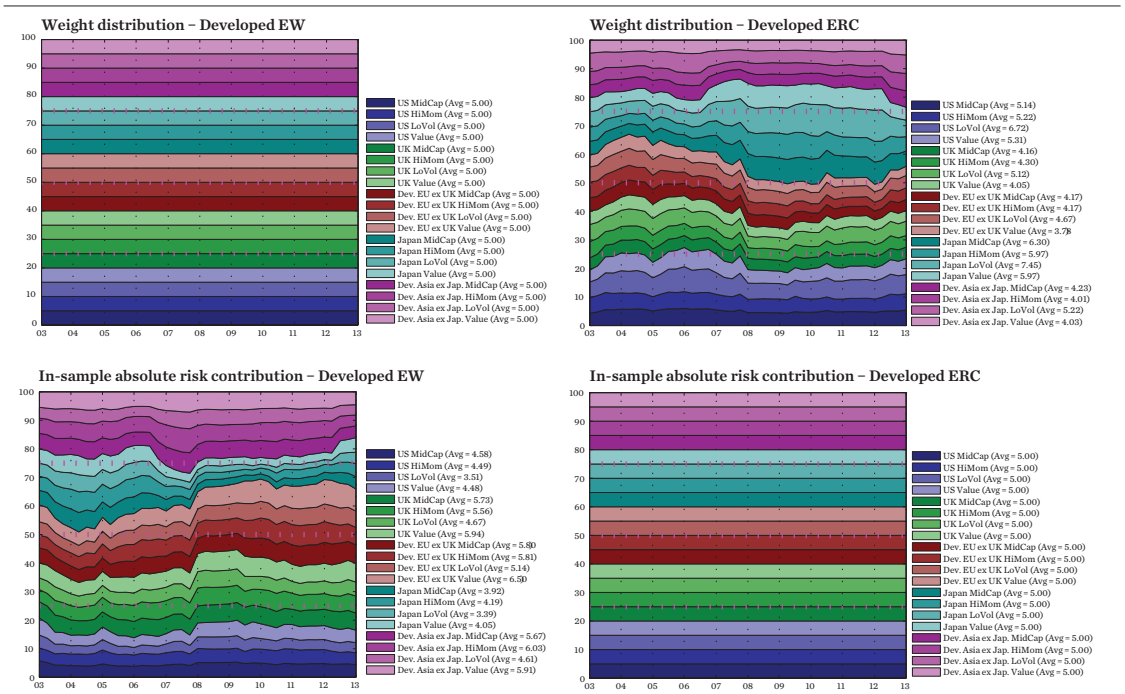
Following an equally-weighted allocation is equivalent to holding an equal dollar allocation, which does not necessarily lead to an equal risk allocation. Formally, the risk contribution of a stock to the total risk of a portfolio is given by the weight of the stock in the portfolio times the marginal contribution of the stock to total portfolio volatility. Qian (2006) shows that decomposing total portfolio volatility in terms of its constituents' risk contributions is also related to the expected contributions to the portfolio losses, particularly when considering extreme losses. In what follows, we consider two approaches to managing portfolio risk: one approach based on minimising portfolio volatility (global minimum variance or GMV approach) and another approach based on imposing equal contribution of all constituents to portfolio volatility (heuristic equal risk contribution or ERC approach).

Absolute risk management without factor risk exposure constraints

In our attempt to design an efficient allocation to smart factor indices, we first impose that all constituents in the portfolio have the same contribution to portfolio risk (ERC). If one makes the explicit assumption that all pairwise correlation coefficients across constituents are identical, then the equal risk contribution weights can be obtained analytically and are proportional to the inverse volatility of the smart factor indices. In the general case – ie, without the assumption of identical pairwise correlations across stocks – the risk parity methodology does not yield a closed-form solution. However, Maillard, Roncalli and Teïletche (2010) propose numerical algorithms to compute risk parity portfolios.

Overall, ERC and EW are two competing ways of implementing agnostic diversification. When looking at the empirical analysis performed in the global developed universe shown in figure 1, we find that the allocation between the equally-weighted and the ERC schemes can exhibit strong differences. For example, the largest average weight over the period under ►

1. EW and ERC allocations to smart factor diversified multi-strategy indices and risk contributions (Developed universe)



The graph compares the allocation and risk contributions of diversified multi-atrategy indices: the equal combination of the 20 diversified multi-strategy indices converted into US dollars with stock selection based on mid cap, momentum, low volatility and value in the five sub-regions US, UK, Developed Europe ex UK, Japan and Asia Pacific ex-Japan, and the ERC combination of the same 20 constituents. The period is from 31 December 2003 to 31 December 2013.

◄ study is given to the Japan low volatility smart factor index (7.45%), whereas the lowest weight is given to the Developed Europe ex-UK value smart factor index (3.78%). We also find that the equal risk contribution can lead to regional allocations that strongly deviate from the corresponding allocation within a cap-weighted index, where the larger markets (eg, the US) strongly dominate smaller markets, such as Japan for example.

We have also implemented an allocation between smart factor indices based on minimising the risk of the allocation, expressed by its volatility (GMV). In this case the GMV portfolio of the 20 index constituents, which is the efficient portfolio that requires only covariance matrix input, the sample covariance matrix is estimated using the past 18 months of weekly data as an input. Long-only constraints are applied to the standard minimum volatility problem, ie, minimise portfolio volatility as given by this expression:

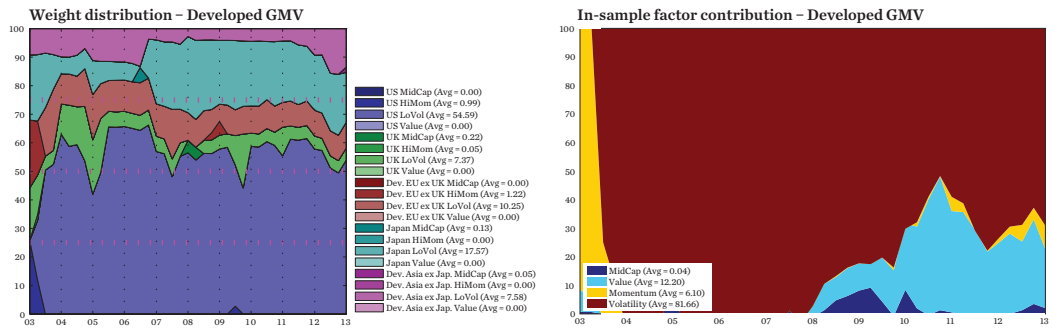
$$\underset{(w_i)_{i=1,\dots,N}}{\text{Min}} \; v(w) \equiv w'Cw$$

To avoid introducing excessively strong biases with respect to the CW index, and even though the focus is not on relative risk management in this illustration, we also introduce a set of constraints dedicated to ensuring that each sub-region is not too strongly under- or over-represented with respect to its market capitalisation in the CW global developed index – ie, we define the weight to lie between half the region's market cap weight and twice its market cap weight.

Figure 2 shows that the GMV allocation with geographical constraints leads to a portfolio that is almost exclusively invested in the lowest volatility smart index for each sub-region: on average, 52.47% low volatility smart factor US index, 8.60% low volatility smart factor UK index, 16.42% low volatility smart factor Developed Europe ex-UK index, 12.68% low volatility smart factor Japan index and 6.74% low volatility smart factor Asia Pacific ex-Japan index. In the end, this process leads to a dynamically managed portfolio of the 20 constituents that should achieve low volatility but that is highly concentrated.

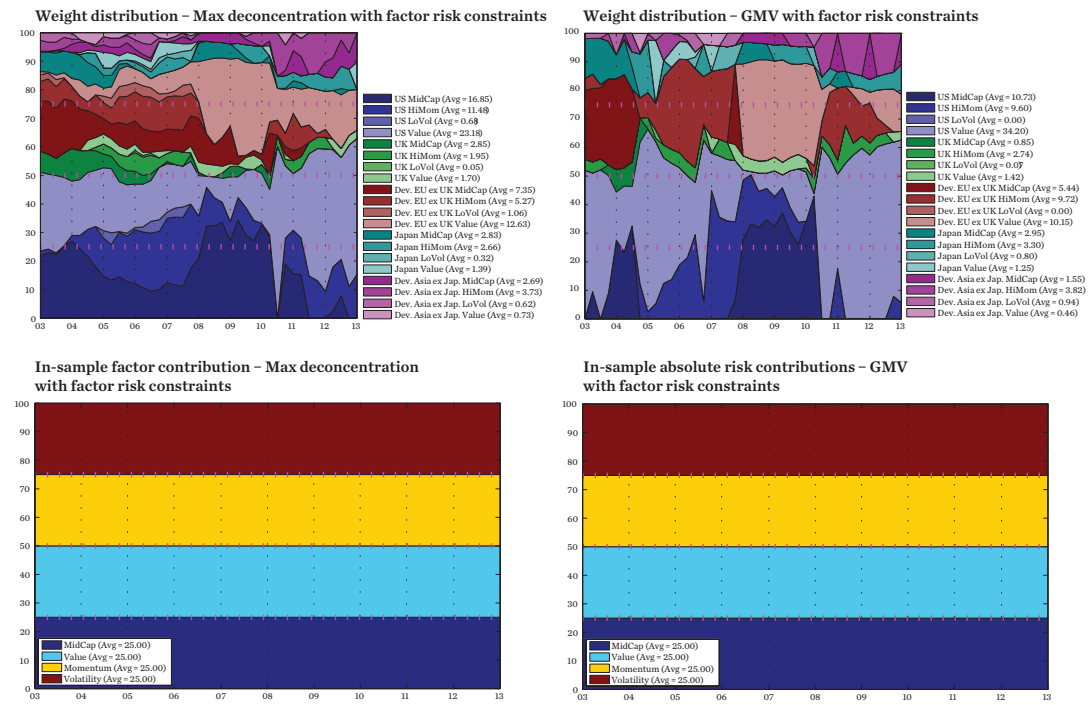
Figure 2 also shows that the portfolio variance is almost exclusively driven by the low volatility factor, an observation that stresses the need for the introduction of risk factor budgeting constraints in order to better balance the factor contributions to the risk of the portfolio.[1]

2. GMV allocation to smart factor diversified multi-strategy indices under geographical constraints and risk contributions (Developed universe)



The graph shows the allocation and risk contributions of the GMV allocation invested in the 20 diversified multi-strategy indices converted into US dollars with stock selection based on mid cap, momentum, low volatility and value in the five US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan sub-regions. Both risk parity and geographical constraints are imposed on the resulting portfolios. The period is from 31 December 2003 to 31 December 2013.

3. Maximum deconcentration and GMV allocations under risk factor and geographical constraints (Developed universe)



The graph shows the allocations and factor contributions of the maximum deconcentration and GMV diversified multi-strategy indices invested in the 20 diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value in the five US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan sub-regions. Both risk parity and geographical constraints are imposed onto the resulting portfolios. The period is from 31 December 2003 to 31 December 2013.

Introducing risk budgeting constraints

Having an equal contribution from the constituents to the overall portfolio risk is not identical to having an equal contribution from the factors. It is only if both the factors and the factor indices are perfectly 'pure' – that is, uncorrelated – that these two approaches coincide, which is not the case with smart factor indices. However, often it is the objective of investors to have an equal contribution to the underlying risk factors because risk contributions are perceived as indicators of the factor's expected contribution to future losses (see Qian [2006]). In this way, integration of factor risk constraints in the allocation process takes into account the imperfections of existing single (smart) factor indices.

In the following, we use the factor exposure of the smart factor indices to analyse the question. We will compute exposure with respect to the equally-weighted version of the factors, since they are the most neutral reference portfolios. As a neutral target, we may seek to impose an equal contribution of the factors to the variance coming from the factors. This extension of the equal risk contribution (ERC) approach from the constituents to the factors leads to linear constraints in the design of the portfolio. This method of ERC of factors (along with EW of factors) is a reasonable approach for investors who are agnostic about the future performance of any single factor and therefore don't want to take a bet on one factor over another. In practice, in the absence of any active views on factors, these approaches are quite robust allocation techniques.

We introduce factor risk budgeting constraints to the portfolio allocation process so as to avoid the domination of any one particular factor (such as the domination of the low volatility factor). When the number of constituents N is greater than the number of factor constraints K, and long-short solutions are allowed, an infinite number of portfolios satisfy a given set of factor risk budgets (eg, factor risk parity exposure). In a long-only context, we may have zero or multiple solutions. When no solution exists, one can start with the long-short version and rescale the weights to avoid short positions.

When multiple solutions exist, one can address the diversification of specific risks, eg, from a scientific perspective, by minimising portfolio variance subject to factor risk parity constraints. We may also maximise portfolio deconcentration, measured by the effective number of constituents, again subject to factor risk parity constraints.[2]

Figure 3 shows maximum deconcentration and GMV allocations under risk parity as well as geographical constraints.

1 The contribution of the low volatility factor is sometimes even greater than 100%, while other factors have a negative contribution to portfolio variance due to the presence of non-zero correlations between the smart factor indices and also between the long-short factors. For example, increasing the exposure to a factor that is negatively correlated with other factors may contribute to decreasing the portfolio variance.

2 Of course, in the absence of constraints, maximising deconcentration simply leads to giving a weight of $1/N$ to each constituent in the universe.

First of all, we notice that factor risk parity is satisfied, and that the portfolio is no longer simply invested in the low-volatility constituents. Similarly to the allocation we obtained in the previous case, we also notice that the aggregated weights in the different sub-regions appear to represent the sub-region market capitalisations more fairly due to the presence of regional constraints. We also note that the maximum deconcentration approach shows a more stable allocation over time compared to the GMV, which is still sensitive to changes in input parameters. Also we see that the addition of factor risk parity constraints forces the allocations to spread the country weight more evenly among the different tilts.

Figure 4 reports the risk and returns characteristics of various multi-smart-beta allocation portfolios, and compares the results. We note that the GMV allocation process leads to the lowest volatility. Also, we notice that the EW and ERC allocations have higher returns and higher volatilities than the GMV, as is often the case. We note further that the introduction of factor risk parity constraints has led to a substantial improvement in information ratios with an information ratio above 1 for the max-deconcentration allocation under geographical and risk parity constraints. This shows that the introduction of factor risk parity constraints leads to a stabilisation of the portfolio that has resulted in strong outperformance (3.37%) over the CW index, with a tracking error barely greater than 5%. The introduction of factor risk parity constraints leads to 100% outperformance probabilities over a three-year horizon.

"Simple allocations that do not balance their exposures to the factors may be too exposed to the low-volatility factor, which may lead to lower relative returns with respect to the cap-weighted index, particularly in bull market regimes"

In figure 5, we analyse the performances in bull versus bear market regimes (defined as positive versus negative returns for the CW index). We observe that the addition of risk parity constraints to the GMV allocation tends to stabilise the returns across market conditions. For example, in the absence of factor risk parity constraints, the GMV allocation leads to a massive outperformance of 11.94% with respect to the CW index in bear markets, which is due to the almost exclusive domination of the low volatility factor, with a defensive bias that proves extremely useful in such market conditions. On the other hand, the relative return in bull market is negative at −3.90% due to the performance drag associated with exclusively holding defensive equity exposure in bull market conditions. In this context, one key advantage of the introduction of factor risk parity constraints is that it leads to a much more balanced return profile across market conditions with positive outperformance in both bear and bull markets (at 2.66% and 3.18% respectively).

We have shown that simple allocations that do not balance their exposures to the factors may be too exposed to the low-volatility factor, which may lead to lower relative returns with respect to the cap-weighted index, particularly in bull market regimes.

4. Multi beta allocations across smart factor indices (Developed universe)

Developed (2004–13)		Diversified multi-strategy				
	CW (All stocks)	Multi beta EW allocation	Multi beta ERC allocation	Multi beta GMV allocation	Multi beta Max decon factor allocation	Multi beta GMV factor allocation
Annualised returns	7.80%	11.37%	11.07%	10.57%	11.17%	10.88%
Annualised volatility	17.09%	15.32%	14.33%	12.84%	17.23%	17.21%
Sharpe ratio	0.36	0.64	0.66	0.70	0.56	0.54
Maximum drawdown	57.13%	54.40%	51.82%	45.07%	55.22%	55.32%
Excess returns	–	3.56%	3.27%	2.76%	3.37%	3.07%
Tracking error	–	6.75%	7.51%	6.36%	3.08%	3.34%
95% tracking error	–	13.84%	14.89%	11.85%	5.19%	5.55%
Information ratio	–	0.53	0.44	0.43	1.09	0.92
Outperformance probability (3Y)	–	98.36%	89.34%	89.07%	100.00%	100.00%
Maximum relative drawdown	–	6.35%	9.54%	13.10%	4.03%	5.47%

The table compares performance of the EW, ERC, and GMV and both the max-deconcentration and GMV diversified multi-strategy indices under geographical and risk parity constraints, invested in the 20 diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value in the five US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan sub-regions. The period is from 31 December 2003 to 31 December 2013 (10 years). Outperformance probability is the probability of obtaining positive excess returns over CW if one invests in the strategy at any point in time for a period of three years. It is computed as the frequency of positive values in the series of excess returns assessed over a rolling window of three years and step size of one week covering the entire investment horizon.

5. Multi beta allocations across smart factor indices in bull/bear regimes (Developed universe)

Developed (2004–13)	Diversified multi-strategy				
	Multi beta EW allocation	Multi beta ERC allocation	Multi beta GMV allocation	Multi beta Max decon factor allocation	Multi beta GMV factor allocation
Annualised returns bull	31.58%	29.55%	25.18%	32.89%	32.26%
Annualised volatility bull	11.71%	11.09%	9.42%	12.85%	12.85%
Annualised relative returns bull	2.50%	0.48%	–3.90%	3.81%	3.18%
Tracking error bull	5.03%	5.94%	5.06%	2.53%	2.74%
Annualised returns bear	–24.51%	–22.42%	–17.23%	–26.67%	–26.50%
Annualised volatility bear	21.33%	19.79%	18.40%	24.42%	24.38%
Annualised relative returns bear	4.65%	6.74%	11.94%	2.50%	2.66%
Tracking error bear	9.64%	10.27%	8.62%	4.08%	4.43%

The table compares conditional performance of the EW, ERC, and GMV and both the max-deconcentration and GMV diversified multi-strategy indices under geographical and risk parity constraints, invested in the 20 diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value in the five US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan sub-regions. The period is from 31 December 2003 to 31 December 2013 (10 years). The quarters with positive market returns are considered bullish and the quarters with negative returns are considered bearish.

Long-term evidence in the US universe

The limited availability of data in the global stock universe caused us to restrict the analysis to a 10-year period. In order to test the robustness of the allocation schemes, we replicate the allocations in the US stock universe, for which data is available for 40 years. This period consists of varying degrees of market environments and therefore allows us to look at the performance of different allocations over time through a conditional analysis tool.

The first observation from figure 6 is that the results for the US are similar in nature to those for Developed. All allocations outperform the CW benchmark by a large margin (>3.8%). As expected, the information ratio of factor-risk-parity-constrained maximum deconcentration is 0.81, as compared to 0.76 for EW allocation, showing that the constraints fulfil ►

6. Multi beta allocations across smart factor indices (US universe)

US Long Term (1973–2012)		Diversified multi-strategy		
	CW (All stocks)	Multi beta EW allocation	Multi beta ERC allocation	Multi beta Max decon factor allocation
Annualised returns	9.74%	13.72%	13.63%	14.01%
Annualised volatility	17.47%	15.75%	15.67%	16.41%
Sharpe ratio	0.24	0.52	0.52	0.52
Maximum drawdown	54.53%	53.86%	53.62%	56.56%
Excess returns	–	3.98%	3.89%	4.27%
Tracking error	–	5.23%	45.25%	5.27%
95% tracking error	–	8.95%	9.10%	8.69%
Information ratio	–	0.76	0.74	0.81
Outperformance probability (3Y)	–	80.38%	80.43%	78.83%
Maximum relative drawdown	–	33.65%	43.46%	33.87%

The table compares the performance of the EW, ERC and maximum deconcentration indices with risk parity constraints, invested in the four diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value in the US. The period is from 31 December 1972 to 31 December 2012 (40 years). Outperformance probability is the probability of obtaining positive excess returns over CW if one invests in the strategy at any point in time for a period of three years. It is computed as the frequency of positive values in the series of excess returns assessed over a rolling window of three years and step size of one week covering the entire investment horizon.

their long-term objective. Figure 7 shows that all allocations are quite stable across different market conditions. They are able to outperform the CW benchmark in both bull and bear market conditions.

Adding value through allocation choices

We find that value can be added at the allocation stage, where the investor can control for the dollar and risk contributions of various constituents or factors to the absolute risk (volatility) of the portfolio. As a result, extremely substantial levels of risk-adjusted outperformance (information ratios) can be achieved even in the absence of views on factor returns. The portfolio strategies we have presented in this brief article can be regarded as robust attempts at generating an efficient strategic factor allocation process in the equity space. One of the important conclusions of our research is to show that it is possible to satisfy factor risk parity objectives in a long-only world by using long-only smart factor indices that by construction cannot be orthogonal to each other, but, while lacking 'purity', are well diversified. Other approaches which would extend the present illustrations could explicitly focus on the management of relative risk. Moreover, active portfolio managers may generate additional value by incorporating forecasts of factors returns at various points of the business cycle in the context of tactical factor allocation decisions.

7. Multi beta allocations across smart factor indices in bull/bear regimes (Developed and US universe)

US Long Term (1973–2012)	Diversified multi-strategy		
	Multi beta EW allocation	Multi beta ERC allocation	Multi beta Max decon factor allocation
Annualised returns bull	34.83%	34.57%	36.14%
Annualised volatility bull	12.94%	12.85%	13.42%
Annualised relative returns bull	3.03%	2.76%	4.34%
Tracking error bull	4.45%	4.46%	4.54%
Annualised returns bear	–20.17%	–20.04%	–21.13%
Annualised volatility bear	20.23%	20.14%	21.14%
Annualised relative returns bear	4.83%	4.96%	3.87%
Tracking error bear	6.57%	6.58%	6.53%

The table compares performance of the EW, ERC and maximum deconcentration indices with risk parity constraints, invested in the four diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value in the US. The period is from 31 December 1972 to 31 December 2012 (40 years). The quarters with positive market returns are considered bullish and the quarters with negative returns are considered bearish.

The research from which this article was drawn was produced as part of the Amundi ETF & Indexing research chair on ETF and Passive Investment Strategies at EDHEC-Risk Institute.

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Risk allocation with smart factor indices: a relative risk perspective

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Many investors are seeking to improve the performance of their equity portfolios by capturing exposure to rewarded factors. Investors may thus explore a variety of portfolio strategies which can be regarded as robust attempts at generating an efficient strategic factor allocation process in the equity space for different sets of objectives and constraints. Allocation can be done in the most simple manner, such as equal dollar contribution/equal weighting (EW) or equal risk contribution (ERC), or in a more sophisticated manner of diversification, such as volatility minimisation (GMV). For the objectives involving risk parameters, allocation methods can broadly be categorised into two groups depending on the risk dimension they deal with – absolute risk and relative risk. In this article, we focus on allocation across smart factor indices from a relative risk perspective.

It is often the case that investors maintain the cap-weighted index as a benchmark, which has the merit of macro-consistency and is well-understood by all stakeholders. In this context, a multi-smart-beta solution can be regarded as a reliable cost-efficient substitute for expensive active managers, and the most relevant perspective is not an absolute return perspective but a relative perspective with respect to the cap-weighted index. In what follows, we focus on two approaches:

- ***Naive diversification:*** a relative equal risk allocation (R-ERC) portfolio, which focuses on equalising the contribution of the smart factor-tilted indices to the portfolio tracking error.
- ***Scientific diversification:*** a relative global minimum variance portfolio (R-GMV), also known as minimum tracking error portfolio, which focuses on minimising the variance of the portfolio relative returns with respect to the cap-weighted index.

It should be noted that controlling for factor exposure biases from an absolute risk budgeting perspective is useful, but this is no longer a key required ingredient since the CW index already provides a proper anchor point that is an implicit, as opposed to explicit, reference set of factor exposures. In the same manner, we find that regional constraints are no longer needed, since a portfolio seeking to equalise the contributions of the 20 constituents to the portfolio tracking error, or seeks to minimise the tracking error, will not lead to a severe overweighting of smaller regions with respect to larger regions, in contrast to what has been found from an absolute risk perspective.

Methodology

Relative ERC is implemented in a way similar to ERC allocation, the only difference being that tracking error contributions are equalised instead of volatility contributions. If we define the contribution of component i to portfolio tracking error as:

$$C_i^{trk}(w) = \frac{1}{2}\frac{\partial TE^2}{\partial w_i} w_i$$

with $$\sum_{i=1}^{N} C_i^{trk}(w) = TE^2$$

The relative ERC portfolio is defined as the allocation w that satisfies the following identity:

$$\frac{C_i^{trk}(w)}{TE^2} = \frac{1}{N} \text{ for all } i$$

The relative GMV approach follows a mean

variance optimisation to minimise total portfolio tracking error under long-only constraints. Mathematically it can be written as (Σ is the covariance of excess returns over the CW benchmark):

$$Min(w^T.\Sigma.w) \text{ subject to } 1^T.w = 1 \text{ and } w_i \geq 0 \text{ for all } i$$

We discuss the composition and performance statistics of Developed and US portfolios. The Developed dataset extends over the 10-year period from 31 December 2003 to 31 December 2013 and uses five sub-regions of the global developed universe: US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan. Using four smart multi-strategy indices as proxies for the value, size, momentum and volatility-rewarded tilts in each sub-region; we obtain a total of 5×4 = 20 constituents.

Risk contributions and performance

In figure 1, we show the allocations of the relative GMV and relative ERC portfolios. First of all, we find again that the relative ERC allocation is more stable over time, which is due to the higher sensitivity of the relative GMV allocation to the parameter estimates, confirming a higher degree of robustness with the ERC approach. Even though both allocation strategies rely on risk parameter estimates, scientific diversification tends to over-use input information compared to the more agnostic risk budgeting diversification, which makes a more parsimonious use of input estimates (see Roncalli [2013] for more details and interpretations for the higher robustness of ERC portfolios with respect to errors in risk parameter estimates).

Secondly, by construction, we observe that the relative ERC leads to identical constituent contributions to the tracking error. However, the relative GMV portfolio involves non-equal time-varying contributions from various constituents to the tracking error of the portfolio. This observation is in line with the relative GMV objective – ie, the components that have large tracking error are under-weighted relative to ones that have lower tracking error.

Figure 2 displays the risk and return characteristics of the relative ERC and GMV allocation strategies. We note that the focus on relative return leads to low tracking error levels. For example, the ex-post tracking error is around 2.50% for these portfolios. Relative GMV, as per its objective, results in lower tracking error (2.43%) compared to relative ERC (2.56%). However, relative ERC exhibits greater outperformance (+3.12%) compared to relative GMV (+2.15%). Such low tracking error levels, associated with substantial outperformance, eventually leads to exceedingly high information ratios. In particular, the relative ERC has an information ratio of 1.22, which is the highest level among all portfolio strategies tested so far, with an outperformance probability of 100% over any given three-year investment horizon during the same period. We also find that the focus on relative risk leads to lower tracking errors in bull and bear market regimes compared to their absolute risk counterparts.

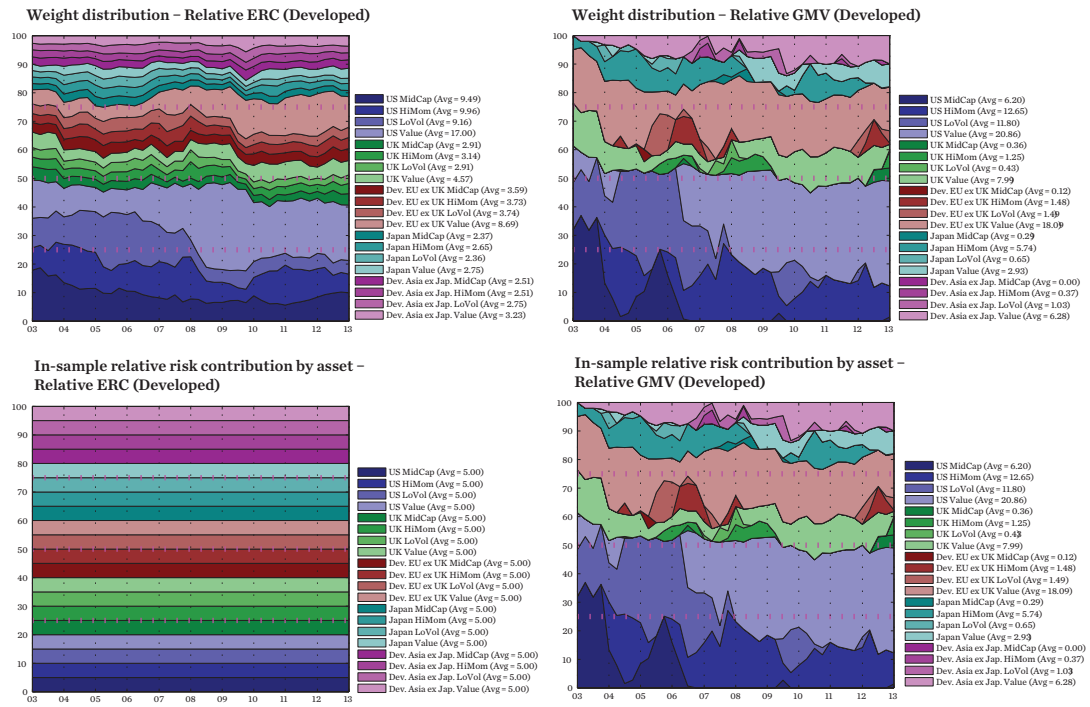
The benefit of exposure to multiple factors can be seen from conditional performance analysis. Both allocations are able to outperform the CW benchmark in both bull and bear market conditions. For example, relative allocation beats the CW benchmark by 2.30% in bull markets and by 3.92% in bear markets.

Relative risk allocation using long-term US factor indices

Since Developed track records are limited to a 10-year time period, we use the US stock universe (of the 500 largest market-cap stocks) to redo the relative risk allocation exercise to ensure the robustness of our results. The US universe gives us the advantage of a much longer history (40 years) but also limits us to using four smart factor indices (instead of 20 indices in the international domain). For the following illustrations, long-term (40-year) data from 31 December 1973 to 31 December 2013 is used for the four smart multi-strategy indices – mid cap, momentum, low volatility and value. All other construction principles remain the same as before.

Figure 3 shows that, over long periods, the weight distribution in the ERC allocation remains quite stable. The GMV allocation is relative time-varying, over-weighting the factor that is responsible for the lowest tracking error each time. ►

1. Relative GMV and relative ERC allocations to smart factor indices and risk contributions (Developed universe)



The graph compares the allocation and risk contributions of diversified multi-strategy indices: the relative GMV and relative ERC allocations invested in the four diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value. The period is from 31 December 2003 to 31 December 2013.

2. Relative ERC and relative GMV allocation to the CW index across smart factor indices (Developed universe)

Developed (2004–13)	CW (All stocks)	Diversified multi-strategy: Multi beta relative ERC allocation	Diversified multi-strategy: Multi beta relative GMV allocation	Developed (2004–13)	Diversified multi-strategy: Multi beta relative ERC allocation	Diversified multi-strategy: Multi beta relative GMV allocation
Annualised returns	7.80%	10.92%	9.96%	Annualised returns bull	31.38%	31.02%
Annualised volatility	17.09%	16.10%	16.64%	Annualised volatility bull	11.95%	12.25%
Sharpe ratio	0.36	0.58	0.50	Annualised relative return bull	2.30%	1.95%
Maximum drawdown	57.13%	54.14%	55.50%	Tracking error bull	2.09%	2.01%
Excess returns	–	3.12%	2.15%	Annualised returns bear	–25.25%	–26.93%
Tracking error	–	2.56%	2.43%	Annualised volatility bear	22.88%	23.75%
95% tracking error	–	4.70%	4.27%	Annualised relative return bear	3.92%	2.23%
Information ratio	–	1.22	0.88	Tracking error bear	3.41%	3.21%
Outperformance probability (3Y)	–	100.00%	89.34%			
Maximum relative drawdown	–	5.10%	4.95%			

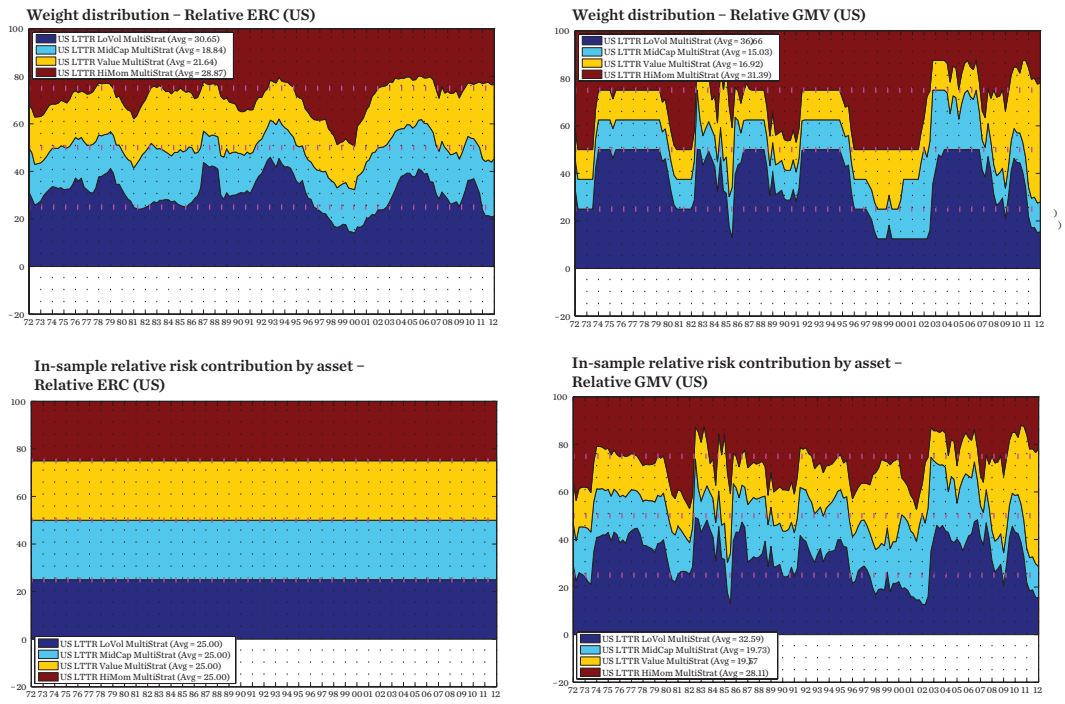
The table compares the performance and risk of Scientific Beta diversified multi-strategy indices converted into US dollars. We look at relative ERC and relative GMV allocations invested in the 20 diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility, and value in the five US, UK, Developed Europe ex-UK, Japan and Asia Pacific ex-Japan sub-regions. Outperformance probability is the probability of obtaining positive excess returns over CW if one invests in the strategy at any point in time for a period of three years. It is computed as the frequency of positive values in the series of excess returns assessed over a rolling window of three years and step size of one week covering the entire investment horizon. The quarters with positive market returns are considered bullish and the quarters with negative returns are considered bearish. The period is from 31 December 2003 to 31 December 2013.

◄ When analysing the risk and performance indicators in figure 4, we observe that the relative GMV, which is supposed to minimise the tracking error, achieves a tracking error of 4.79% compared to relative ERC, with a tracking error of 4.91%. As observed in the case of the Developed universe, both allocations result in high outperformance, with relative ERC slightly better (at +3.79%) than relative GMV (+3.71%). The conditional performance over the long term constitutes many market cycles, including the technology bubble and the financial crisis. The fact that both allocations outperform the benchmark in varying market conditions reconfirms the robustness of these strategies.

We find that value can be added through relative ERC and relative GMV at the allocation stage, for investors with a tracking error budget. As a result, extremely substantial levels of risk-adjusted outperformance (information ratios) can be achieved even in the absence of views on factor returns. The portfolio strategies we have presented in this article can be regarded as robust attempts at generating an efficient strategic factor allocation process in the equity space in the context of benchmarked investment management. While possibilities for adding value through smart beta allocation are manifold, the robust performance improvements obtained through relative ERC and relative GMV allocations to the four main consensual factors displayed above in this article, provide evidence that the benefits of multi-factor allocations exist in a context of strong relative risk constraints and are sizable.

The research from which this article was drawn was produced as part of the Amundi ETF & Indexing research chair on ETF and Passive Investment Strategies at EDHEC-Risk Institute.

3. Relative GMV and relative ERC allocations to smart factor indices and risk contributions (US universe)



The graph compares the allocation and risk contributions of diversified multi-strategy indices: the relative GMV and ERC allocations invested in the four diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value, and the ERC combination of the same four constituents. The relative GMV strategy has been derived with the following additional weight constraints: $1/\delta^{*}N<w<\delta/N$, where N=4 constituents and δ =2. The period is from 31 December 1972 to 31 December 2012.

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4. Relative ERC and relative GMV allocation to the CW index across smart factor indices (US universe)

US Long Term (1972–2012)		Diversified multi-strategy		US Long Term (1972–2012)	Diversified multi-strategy	
	CW (All stocks)	Multi beta relative ERC allocation	Multi beta relative GMV allocation		Multi beta relative ERC allocation	Multi beta relative GMV allocation
Annualised returns	9.74%	13.53%	13.45%	Annualised returns bull	34.72%	34.39%
Annualised volatility	17.47%	15.69%	15.60%	Annualised volatility bull	12.89%	12.97%
Sharpe ratio	0.24	0.51	0.51	Annualised relative return bull	2.92%	2.59%
Maximum drawdown	54.53%	53.30%	52.64%	Tracking error bull	4.20%	3.78%
Excess returns	–	3.79%	3.71%	Annualised returns bear	–20.44%	–21.02%
Tracking error	–	4.91%	4.79%	Annualised volatility bear	20.14%	20.15%
95% tracking error	–	8.11%	7.99%	Annualised relative return bear	4.56%	3.97%
Information ratio	–	0.77	0.77	Tracking error bear	6.12%	5.49%
Outperformance probability (3Y)	–	80.90%	81.31%			
Maximum relative drawdown	–	28.74%	27.00%			

The table compares the performance and risk of the Scientific Beta diversified multi-strategy indices. We look at relative ERC and relative GMV allocations in the four diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility and value respectively. All statistics are annualised and daily total returns from 31 December 1972 to 31 December 2012 are used for the analysis. The S&P 500 index is used as the cap-weighted benchmark. Yield on secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Outperformance probability is the probability of obtaining positive excess returns over CW if one invests in the strategy at any point in time for a period of three years. It is computed as the frequency of positive values in the series of excess returns assessed over a rolling window of three years and step size of one week covering the entire investment horizon. The quarters with positive market returns are considered bullish and the quarters with negative returns are considered bearish.

Investability of smart beta indices

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With the advent of smart beta equity indices, which represent alternatives to market-cap weighted indices, a major question has been raised on their investability: at what cost will investors be able to trade the index constituents in the same proportions as the underlying strategy? In fact, departing from the traditional cap-weighting investment scheme leads to risks that are sizable and significantly different, as shown in Amenc, Goltz and Lodh (2012) and Amenc, Goltz and Martellini (2013). These include common exposures to systematic risk factors such as size and liquidity.

Also, in contrast to cap-weighted indices, which are deemed to be buy-and-hold investments, and which are only marginally reviewed for the (often quarterly) addition and deletion of constituents as well as regular corporate events, smart beta indices exhibit higher levels of turnover than their cap-weighted counterparts (see, eg, Amenc et al [2011]). Importantly, for any level of liquidity, the level of turnover in the index will impact the performance of the tracking fund through the frequency of occurrence of transaction costs.

Clearly, investing in smart beta indices requires investors to have access to solutions where implementation costs and liquidity risks are thoroughly considered. A key implication is that the smart beta index turnover and capacity constraints need to be methodologically and carefully handled through the construction of the index. The objective of this article is to describe how to ensure the investability of the indices by managing turnover control and capacity constraints. The ERI Scientific Beta methodology has been used to exemplify best practices in this area.

Consistent framework

ERI Scientific Beta indices are derived from a consistent index design framework referred to as Smart Beta 2.0. An investor can select an index by making conscious and explicit choices of risks along the different dimensions of this framework. The objective of such a consistent framework is to avoid unintended risks due to ill-defined consequences of ad hoc methodologies.

Thanks to the consistent design framework, clear rules concerning liquidity and turnover can be aimed at facilitating implementation of the indices. Adjustments are performed through the main steps in the index construction process, which aim to ensure the investability of the indices, either by reducing implementation costs, or by allowing their liquidity profile to be improved.

Turnover control

The turnover of an equity index is an early indication of the aggregated trading activity of funds that will track that index. Although knowledge of a portfolio's turnover does not provide a precise measure for estimating the actual costs of trading, it provides an intuitive and parsimonious idea of the fund's trading activity and, as such, is a sensible indicator.

Turnover varies greatly from one index to another, and the methods for managing it are a complex problem in constructing indices. ERI Scientific Beta has opted for a conditional (or trigger) rebalancing approach described in Martellini and Priaulet (2002) and Leland (1999). The trigger approach activates rebalancing whenever the gap between the current index weights and new target weights reaches a specific threshold (eg, ±5% or ±10%). The main advantage of the trigger approach is that it avoids unnecessary rebalancing unless a significant amount of new information has been received since the last index rebalancing, hence avoiding rebalancing due to noise.

The threshold level of an index is determined through a calibration procedure over its back-test history. First, different versions of the index are constructed over the calibration period, each with a conditional rebalancing dictated by a threshold spanning 0% to 100%. Then, the smallest threshold that results in an average one-way annual turnover below or equal to 30% over the calibration period is used as the specific turnover threshold for that index in its live period.[1] Finally, irrespective of whether or not the threshold mentioned above is reached, suggested optimised weights will be used if the index has not been rebalanced optimally for seven consecutive quarters.

As shown in figure 1, before turnover control, Scientific Beta US indices exhibit levels of turnover that can exceed reasonable investability levels. This is notably the case ►

1 Index specific turnover thresholds may be re-calibrated at some point in time in order to reflect structural changes in market conditions.

1. Comparison of performance, turnover and liquidity before and after turnover control

US long-term track records		Maximum deconcentration	Diversified risk-weighted	Maximum decorrelation	Efficient minimum volatility	Efficient maximum Sharpe ratio	Cap-weighted
Before turnover control	Annualised return	12.77%	12.83%	12.76%	13.00%	12.86%	10.45%
	Annualised volatility	17.18%	16.40%	16.52%	14.03%	15.68%	17.12%
	Annualised one-way turnover	**23.48%**	**25.67%**	**59.51%**	**54.81%**	**65.02%**	**2.59%**
	Annualised return net of 20bps transaction costs	12.72%	12.78%	12.64%	12.89%	12.73%	10.44%
	Annualised return net of 100bps transaction costs	12.54%	12.57%	12.16%	12.45%	12.21%	10.42%
	Days to trade for $1bn initial investment (95% quantile)*	0.10	0.11	0.12	0.13	0.11	0.03
	Average float ($bn)	9.890	10.700	9.540	11.640	10.390	42.840
After turnover control	Annualised return	12.59%	12.67%	12.61%	12.69%	12.93%	10.45%
	Annualised volatility	17.02%	16.34%	16.20%	14.30%	15.55%	17.12%
	Annualised one-way turnover	**20.19%**	**22.15%**	**29.22%**	**29.83%**	**27.84%**	**2.59%**
	Annualised return net of 20bps transaction costs	12.55%	12.63%	12.55%	12.63%	12.87%	10.44%
	Annualised return net of 100bps transaction costs	12.39%	12.45%	12.32%	12.39%	12.65%	10.42%
	Days to trade for $1bn initial investment (95% quantile)*	0.10	0.11	0.12	0.13	0.12	0.03
	Average float ($bn)	10.110	10.860	10.120	12.030	10.820	42.840

The first panel reports the statistics for the indices before any turnover control is applied, whereas the second panel reports the statistics for the indices after turnover control is applied. The annualised one-way turnover and average float is calculated at the end of each quarter and are averaged over the analysis period. The net returns of transaction costs are obtained using two levels of transaction costs – 20 bps per 100% 1-W turnover and 100 bps per 100% 1-W turnover. The first case corresponds to the worst case observed historically for the large and mid cap universe of the indices while the second case assumes 80% reduction in market liquidity and a corresponding increase in transaction costs. The statistics are calculated over the period from 29 June 1970 to 21 December 2012.

* Days to trade is the number of days necessary to trade the total stock positions, assuming $1bn AUM and that 100% of the average daily dollar traded volume can be traded every day. Due to data availability, the period is restricted to the last 10 years of the sample for the Scientific Beta US indices.

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2. Comparison of performance, turnover and capacity before and after turnover control

US long-term track records		Maximum deconcentration	Diversified risk weighted	Maximum decorrelation	Efficient minimum volatility	Efficient maximum Sharpe ratio	Cap-weighted
Before capacity constraint	Annualised return	12.59%	12.67%	12.61%	12.69%	12.93%	10.45%
	Annualised volatility	17.02%	16.34%	16.20%	14.30%	15.55%	17.12%
	Annualised one-way turnover	20.19%	22.15%	29.22%	29.83%	27.84%	2.59%
	Days to trade for $1bn initial investment (95% quantile)*	0.10	0.11	0.12	0.13	0.12	0.03
	Average float ($bn)	10.110	10.860	10.120	12.030	10.820	42.840
	Index to cap weight ratio (low cap decile)	**14.42**	**11.10**	**15.78**	**9.80**	**12.77**	**1**
After capacity constraint	Annualised return	12.63%	12.70%	12.67%	12.73%	12.98%	10.45%
	Annualised volatility	17.13%	16.45%	16.34%	14.42%	15.67%	17.12%
	Annualised one-way turnover	20.20%	22.15%	29.23%	29.84%	27.85%	2.59%
	Days to trade for $1bn initial investment (95% quantile)*	0.10	0.11	0.10	0.12	0.11	0.03
	Average float ($bn)	10.160	10.910	10.170	12.080	10.880	42.840
	Index to cap weight ratio (low cap decile)	**9.66**	**8.31**	**8.55**	**6.35**	**7.67**	**1**

The first panel reports the statistics for the indices before any capacity constraint is applied, whereas the second panel reports the statistics for the indices after capacity constraints are applied. The annualised one-way turnover and average float is calculated at the end of each quarter. The turnover and average float figures reported here are the average of the annualised one-way turnover and average float over the analysis period. The statistics are calculated over the period from 29 June 1970 to 21 December 2012.

*Days to trade is the number of days necessary to trade the total stock positions, assuming $1bn AUM and that 100% of the average daily dollar traded volume can be traded every day. Due to data availability, the period is restricted to the last 10 years of the sample for the Scientific Beta US indices.

for the maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio indices, which respectively exhibit 59.51%, 54.81% and 65.02% average annualised one-way turnover over the period from 29 June 1970 to 21 December 2012. After controlling for turnover, the same indices exhibit a much more reasonable level of turnover (respectively 29.22%, 29.83% and 27.84%).

Most interestingly, the reduction in turnover is accompanied by no, or at most a marginal, loss in returns and volatility reduction. Indeed, the reduction in turnover of the five strategies analysed (respectively maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio indices) are marginally changed from respectively 12.77% to 12.59%, 12.83% to 12.67%, 12.76% to 12.61%, 13% to 12.69% and 12.86% to 12.93% (which is even an increase in returns). In parallel, the changes in volatilities are also very small. In short, reducing the turnover does not alter the benefits of Scientific Beta strategies over the long run to a significant extent, and brings implementation costs down substantially.

Capacity constraints

Capacity is a key ingredient in the constitution and construction of an equity index. To achieve it, adjustments in stock weights can be implemented post-optimisation, primarily with the use of cap-weight multipliers. The principle used to make such adjustments is to impose a threshold for the weight of a stock and for the weight change at rebalancing, relative to the market cap weight of the stock in its universe. Specifically, we define the cap-weight multipliers rules as follows:

- ***Holding capacity constraints:*** Each stock weight is capped at a multiple of 10 of its free float-adjusted market cap weight to avoid big investment in the smallest stocks.
- ***Trading capacity constraints:*** Change in weight of each stock is capped to its free-float-adjusted market cap weight to avoid large rebalancing in small stocks.[2]

As shown in figure 2, the capacity constraints do not have a substantial impact on overall performance and turnover metrics over the selection of indices. The Scientific Beta US maximum decorrelation index exhibits an estimated number of days to trade at 95% of 0.12 days and an average market capitalisation of $10.12bn before the capacity constraints are applied. These figures are marginally improved to 0.10 days to trade and $10.17bn average float after the capacity constraints are applied. Nevertheless, this marginal shift in performance and liquidity is one side effect of the capacity constraints, which are primarily designed to address deviations in weights of smaller cap stocks between the index and its cap-weighted reference. Indeed, it has been highlighted, eg, in Goltz and Gonzalez (2013), that smart beta indices can exhibit a bias toward smaller cap stocks relative to their cap-weight reference.

"Even over long-term horizons, a very reasonable ex-post turnover level that is in line with the ex-ante targets can be maintained with the use of a threshold-based method"

Figure 2 shows that the capacity constraint has a big impact on the ratio decile weight of index and decile weight of CW, especially in the 'low market cap' decile. For example, in the case of the Scientific Beta US maximum deconcentration index, weights before adjustments are equally distributed to each market capitalisation bucket. The ratio between sum of weights of the maximum deconcentration strategy to cap deciles and sum of cap weights in the 'low market cap' decile, where the maximum deconcentration strategy tends to concentrate 14.42 times more than the cap-weighted index on average historically, is above the 10-multiple threshold set in the capacity rule explained earlier. Hence the adjustment observed on the 'low' decile. After applying the capacity constraint, we observe the ratio has decreased to 9.66, respecting the capacity holding constraint.

2 After capping weights and weight changes following the two adjustments, index weights are renormalised so they sum again to one. As a consequence, the effective multiple will change again and, eventually, the index can hold some stocks at a higher multiple of their cap-weight.

Conclusion

In this article we have described the different aspects of implementation management and cost control. In particular, we have shown how the adjustments have specific impacts on key target implementation metrics. The article shows that, even over long-term horizons, a very reasonable ex-post turnover level that is in line with the ex-ante targets can be maintained with the use of a threshold-based method. The average annual one-way turnover of the indices we presented was reduced from 46% to 26% through the turnover rules, a level which is shown to have a small impact on the performance.

The capacity constraints allow us to manage the deviations from the cap-weighted reference index in terms of individual component market capitalisation both at the trading and the holding levels. Notably, we showed how the capacity constraint has an impact on controlling the imbalance between the weight allocated to smaller market cap stocks and their corresponding cap-weight, from, on average across the analysed indices, a 12.77 ratio to an 8.11 ratio.

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Robustness of smart beta strategies

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Alternative forms of equity indices, which draw from a wide range of portfolio construction practices, have become popular in recent years. The smart beta indices are usually marketed on the basis of outperformance. However, more often than not, the issue of robustness, as in extreme risk and performance attribution to well-defined risk factors, is not dealt with by index providers. The results of an EDHEC-Risk Alternative Equity Beta Survey show that investors are wary of the robustness of outperformance provided by various smart beta strategies.[1]

In general, robustness refers to the capacity of a system to perform effectively in a constantly changing environment. In the context of smart beta strategies, two kinds of robustness need to be taken into account – relative robustness and absolute robustness.

A strategy is assumed to be 'relatively robust' if it is able to deliver similar outperformance in similar market conditions. Single factor indices aim to achieve this kind of robustness. For example, a value factor index is expected to outperform in times when the value factor is rewarded in the market and will underperform when the factor undergoes short-term losses. The value factor index would be deemed relatively robust if it aligns well with the value factor performance and does not suffer idiosyncratic losses due to any other causes including, but not limited to, stock-specific and sector-specific events.

Absolute robustness is the capacity of the strategy to deliver risk-adjusted performance in the future to a degree that is comparable with that of the past owing to a well-understood economic mechanism rather than just by chance. In other words, absolute robustness is the absence of pronounced state and/or time dependencies and a strategy shown to outperform irrespective of prevailing market conditions can be termed robust in absolute terms.

Potential causes of lack of robustness

Lack of robustness in smart beta strategies is mainly caused by exposure to four different risks in the strategy construction process – factor fishing, model mining, non-robust weighting schemes and strong factor dependencies.

Factor fishing risks

Investors who wish to benefit from factor premia need to address robustness when selecting a set of factors. Harvey et al (2013) document a total of 314 factors with a positive historical risk premium, showing that the discovery of the premium could be a result of data mining – ie, strong and statistically significant factor premia may be a result of many researchers searching through the same dataset to find publishable results. For example, when capturing the value premium one may use extensive fundamental data including not only valuation ratios but also information on, for example, the sales growth of the firm. Therefore, a key requirement in investors accepting factors as relevant in their investment process is that there is a clear economic intuition as to why exposure to this factor constitutes a systematic risk (Kogan and Tian [2013]). Failure to recognise a suitable proxy for the rewarded factor will harm the relative robustness of the strategy.

Model mining risks

Model mining risk is the risk of having an index construction methodology which results in a good track record in back testing. Many value-tilted indices include a large set of ad hoc methodological choices, opening the door to data mining. As an illustration, one can consider the impact of various specification choices on fundamental equity indexation strategies, which are commonly employed as a way to harvest the value premium.

Figure 1 summarises the maximum calendar year difference between any two variants of fundamental indices which make different choices for two methodological ingredients – variable selection and leverage adjustment. It is evident that the outperformance of a fundamental equity indexation strategy is highly sensitive to strategy specification choices.

The value factor performed poorly during the years 1999 and 2008. From a relative robustness viewpoint, two slightly different versions of a value-factor-targeting smart beta strategy are expected to display similar performance in those two years. The results show, however, that 'total leverage adjusted' portfolio returns are up 5.3% while 'operating leverage adjusted' portfolio returns are down −4.0%, indicating that the weighting scheme does not reliably capture the value premium. In addition to being exposed to the value factor, the strategy is also exposed to some latent undesired risks resulting from proprietary definitions.

Non-robust weighting schemes

All smart beta strategies are exposed to unrewarded strategy-specific risks. Specific risks correspond to all the risks that are unrewarded in the long run, and therefore not ultimately desired by the investor. In line with portfolio theory, among the non-rewarded risks we find specific financial risks (also called idiosyncratic stock risks) which correspond to the risks that are specific to the company itself. It is this type of risk that asset managers are supposed to be the best at recognising, evaluating and choosing in order to create alpha, but portfolio theory considers it to be neither predictable nor rewarded, so it is better to avoid it by investing in a well-diversified portfolio.

Specific risks can also correspond to important financial risk factors that do not explain, over the long term, the value of the risk premium associated with the index. The academic literature considers for example that commodity, currency and sector risks do not have a positive long-term premium. For example, value strategies often lead to pronounced tilts towards financial sector stocks. During the financial crisis (2008), exposure to the financial sector proved to be a major determinant of the performance of these strategies. It should be noted that the tilt towards the financial sector may not be desired, but it came as a by-product of holding value stocks. Figure 2 shows a performance comparison between the euro-zone value maximum deconcentration[2] index and its sector-neutral version. The euro-zone value maximum deconcentration index overweighted the financial sector by 9.1% in June 2008, which resulted in a loss of about 20% of portfolio value.

Model-specific risks that are specific to the implementation of the diversification model are also a form of non-rewarded risk. As per modern portfolio theory (MPT), each investor should optimally combine risky assets so as to achieve highest possible Sharpe ratio. Implementing this

1. Impact of data mining

Data mining aspects and their impact on returns	Best performance		Worst performance		Range	Year
Variable selection	Earnings	−12.2%	Dividends	−23.0%	10.8%	1999
Leverage adjustment	Total leverage	5.3%	Operating leverage	−4.0%	9.3%	2008

The table shows the returns of best and worst performing variants of each specification of the fundamental weighting schemes on the universe of the top 1,000 US stocks. Portfolios are formed using fundamental data from the period January 1982 to December 2010. Data is obtained from Datastream and Worldscope.

1 Among the reasons for not investing in smart beta strategies, "doubts over robustness of outperformance" is rated the highest (Badaoui et al [2014]).
2 Maximum deconcentration is an equal weighting (1/N) strategy with liquidity and turnover constraints.

objective, however, is a complex task because of the presence of estimation risk for the required parameters, namely expected returns and covariances. In practice, the costs of estimation error may entirely offset the benefits of optimal portfolio diversification. (DeMiguel, Garlappi and Uppal [2009] provide evidence that naively-diversified portfolios have higher out-of-sample Sharpe ratios than scientifically-diversified portfolios.) In other words, the choice in risk and return parameter estimation for efficient diversification is between 'trying', which has a cost related to estimation risk – ie, the risk of a substantial difference between the estimated parameter value and the true parameter value – or 'giving up', which has a cost related to optimality risk, that is the risk that the heuristic benchmark (such as minimum volatility or equal-weighted) can be far removed from the optimal maximum Sharpe ratio (MSR) benchmark.

Different portfolios are intuitively expected to incur more estimation risk or more optimality risk. Martellini, Milhau and Tarelli (2013) provide a quantitative analysis of the trade-off between optimality risk and estimation risk. They show that under the assumption of true parameter knowledge, an MSR portfolio exhibits a far superior Sharpe ratio to that of other strategies. But after estimation risk is taken into account, GMV and a mix of GMV and EW portfolios generate higher average Sharpe ratios.[3]

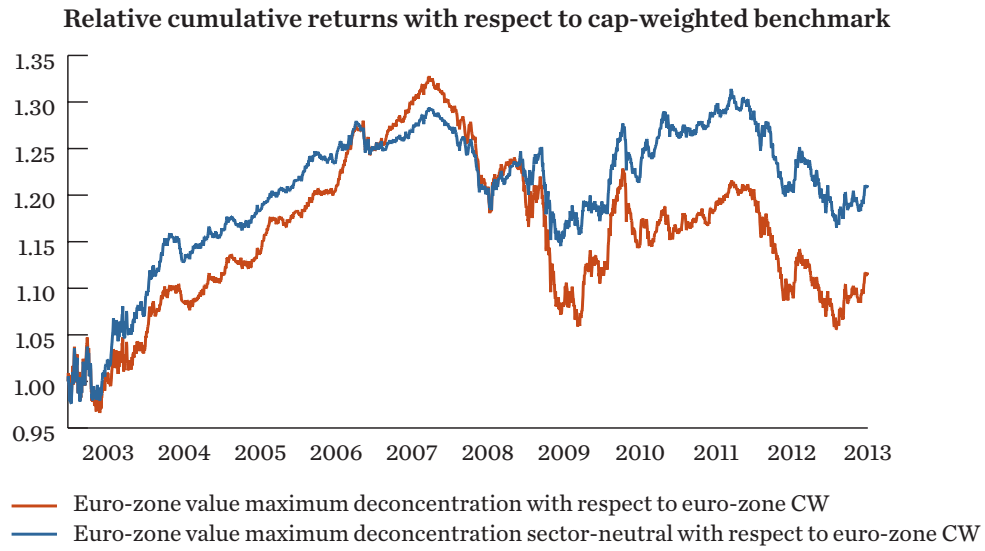
The failure to avoid unrewarded strategy-specific risk hampers the relative robustness of the strategy – ie, the strategy might not benefit to the maximum even in periods when the underlying risk factor is rewarded.

Dependency on individual factor exposures
Systematic risks come from the fact that smart beta strategies can be more or less exposed to particular risk factors, depending on the methodological choices guiding their construction (implicit), but also on the universe of stocks supporting this construction scheme (explicit). For example fundamental-weighted portfolios typically have a value tilt and minimum volatility strategies exhibit a low-beta tilt (see for example Scherer [2011], Blitz and Swinkels [2008], and Amenc, Goltz and Le Sourd [2008]). Each weighting scheme exposes investors to implicit risk factors which may or may not be consistent with their risk objective.

This is a major limitation of Smart Beta 1.0 strategies – the strategies which do not explicitly control for systematic risk factors. Following this drawback of Smart Beta 1.0 indices, 'factor indices' have gained popularity. Factor indices make sure that the portfolio is tilted towards the desired risk factor. Whatever the route to seeking systematic risk exposure, the elementary fact remains that stocks earn a risk premium through their exposure to certain rewarded factors (Ross [1976]).[4]

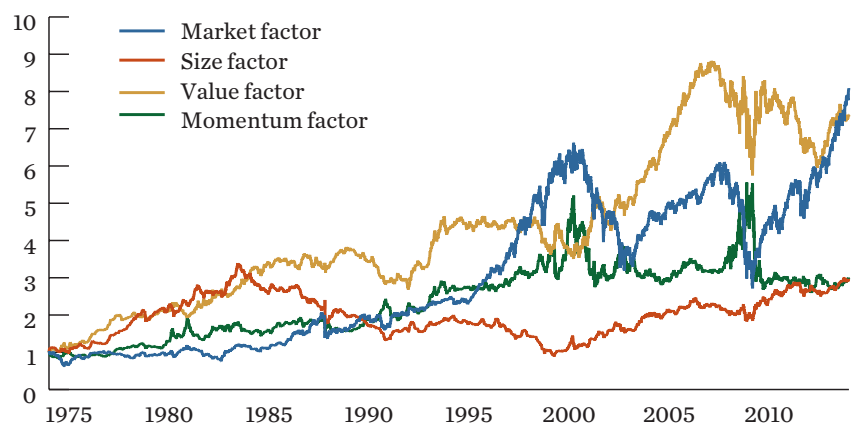
Figure 3 plots cumulative returns of long/short cap-weighted indices replicating factors such as market, size, value and momentum. Periods of poor performance in all factors are common throughout the 40-year time horizon and the underperformance occurs at different points in time. Therefore investing in a single factor is not a robust approach in absolute terms, as the performance will vary greatly across different time periods.

2. Performance of Scientific Beta euro-zone value maximum deconcentration and sector-neutral version during the financial crisis



The benchmark is the cap-weighted index on the Scientific Beta euro-zone universe, which consists of 600 stocks.

3. Cumulative returns of long/short cap-weighted factors



The market factor is the daily return of the cap-weighted index of all stocks that constitute the index portfolio in excess of the risk-free rate. The small size factor is the daily return series of a cap-weighted portfolio that is long the cap-weighted market portfolio deciles 6-8 (NYSE, Nasdaq, and AMEX) and short the 30% largest market-cap stocks from the top 500 stock universe. The value factor is the daily return series of a cap-weighted portfolio that is long the 30% highest and short the 30% lowest B/M ratio stocks in the S&P 500 universe. The Momentum factor is the daily return series of a cap-weighted portfolio that is long the 30% highest and short the 30% lowest 52 weeks (minus the most recent four weeks) past return stocks of the US 500 universe. The Secondary Market US Treasury Bills (3M) is the risk-free rate in US dollars. The analysis is based on daily total return data from 31 December 1973 to 31 December 2013 (40 years). The complete stock universe consists of the 500 largest stocks in the US. The S&P 500 index is used as the cap-weighted benchmark.

Improving robustness

ERI Scientific Beta proposes three ways by which the robustness of various smart beta strategies can be improved.

Avoidance of data mining through a consistent framework
A very effective mechanism to avoid data mining is by establishing a consistent framework for smart beta index creation, thus limiting the choices yet providing the flexibility needed for smart beta index creation. Consistency in the index framework has two main benefits. First, it prevents model mining by limiting the number of choices through which indices can be constructed. A uniform framework is the best safeguard against post hoc index design, or model mining (ie, the possibility to test a large number of smart beta strategies, and publish the ones that have good results).

Second, analysis across specification choices is vital because the range of outcomes gives a more informative view than a single specification, which could always have been picked. An index that performs well across multiple specification choices is more robust than an index that performs only in a single specification choice, which could very well have been by chance rather than because of the robustness of the strategy. Pre-packaged indices do not allow investors to allow comparison across specifications to obtain a view on the sensitivity of performance to index specification choices, thereby exposing investors to a risk of unintended consequences of undesired risks.

ERI Scientific Beta uses a consistent smart beta index design framework for the construction of its entire set of smart beta indices known as the Smart Beta 2.0 approach. Stock selection allows investors to choose the right (rewarded) risk factors to which they want ►

3 ERI Scientific Beta's Efficient MSR index was specifically designed to respond to the problem of estimation risk affecting the vector of returns when the vector is estimated in the traditional way, ie, on historical returns.
4 The economic intuition for the existence of a reward for a given risk factor is that exposure to such a factor is undesirable for the average investor because it leads to losses in bad times (ie, when marginal utility is high, see Cochrane [2001]).

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to be exposed. A well-diversified weighting scheme provides efficient access to the risk premia associated with this factor exposure. All the available variations (or choices) provided within the framework are based on proven academic or applied research allowing flexibility to accommodate various investor preferences. Figure 4 shows ERI Scientific Beta's design framework.

Figure 5 compares the design framework of the factor-based strategy indices offered by MSCI and ERI Scientific Beta. MSCI follows different stock selection schemes, weighting schemes and risk control options for different risk factors. Maximising the exposure to a factor by selecting stocks that are most exposed to the desired risk factor and the application of a cap-weighting scheme to this selection (MSCI High Dividend Yield) does not attend to the problem of poor diversification arising from high concentration. Weighting either the whole of the universe (MSCI Value) or a part of the universe (MSCI Momentum) by exposure to this factor, resulting in score/rank weighting, also misses out on diversification.

The Smart Beta 1.0 approach to a factor index (MSCI Minimum Volatility) does not guarantee either the highest exposure to low-volatility stocks or optimal diversification of this low-volatility portfolio. Moreover, it brings about other kinds of undesired risks, such as exposure to defensive sectors. Similarly, seeking exposure to the size factor through equal weighting of a broad universe is certainly less effective than selecting the smallest size stocks in the universe and then diversifying them, including with an equal-weighted weighting scheme. Also, no control for the undesired liquidity risk is implemented. Not only is the approach not optimal from the standpoint of a well diversified factor index, but the lack of uniformity in index design across factor indices may also introduce the data mining bias described earlier.

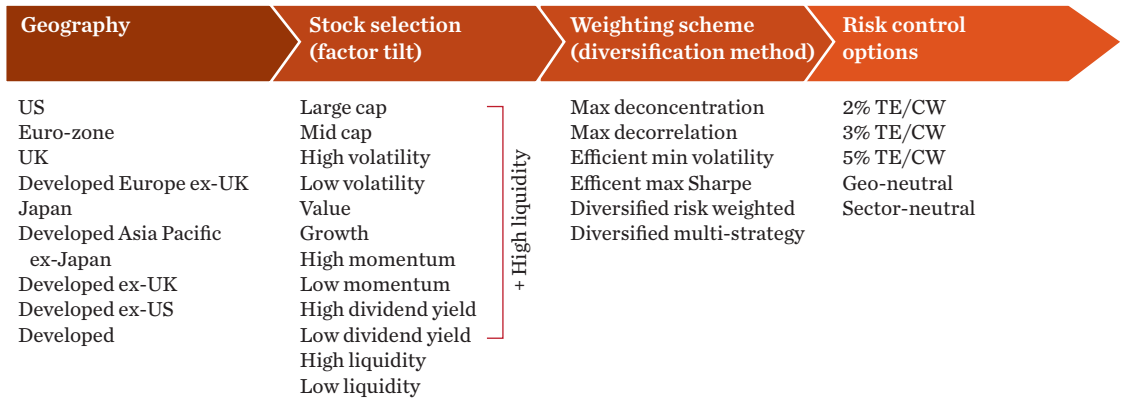
Another approach to the inconsistency of the conceptual framework, in addition to comparing construction methods for different factors as we did for MSCI, is by looking at the evolution or change of methodology over time for the same strategy or the same factor. Russell launched new factor indices to create a new brand known as 'High Efficiency' (HE) indices when it already had the following factor indices on the market – Russell 1000 High Momentum, Russell 1000 Low Volatility and Russell 1000 Value. The new indices have the same objective as the old ones but different construction principles.

Figure 6 shows the performance difference between the new set and the old set of Russell indices. Thus, an inconsistent framework (over time) is also a kind of model mining that allows index providers to launch new indices with better track records than previous ones.

Improving relative robustness by reducing unrewarded risks

Relative robustness can be improved by minimising the unrewarded risk as much as

4. ERI Scientific Beta's consistent index design framework



5. Comparison of consistency in index construction framework between MSCI and ERI Scientific Beta

Factor	Index	Stock selection	Weighting scheme	Risk controls
MSCI index methodologies				
Size	MSCI Equal-Weight index	All stocks in CW parent index universe	Equal-weighted	None
Value	MSCI Value-Weighted index	All stocks in CW parent index universe	Score adjusted by investability factor	None
Momentum	MSCI Momentum index	Selection by momentum score (fixed number of constituents to target 30% market cap coverage)	Market cap momentum score	Cap on weight of individual security
Low volatility	MSCI Minimum Volatility index	All stocks in CW parent index universe	Optimisation to minimise portfolio risk	Sector and country weight constraints Cap on multiple of market cap of individual security
Yield	MSCI High Dividend Yield index	Select stocks with dividend yield >1.3x parent index dividend yield	Market cap weighted	Cap on weight of individual security
Scientific Beta index methodologies				
Size	SciBeta Diversified Multi-Strategy Mid Cap index	Half the stocks by relevant score	Same weighting scheme for selected stocks (Diversifed multi-strategy by default)	Cap on multiple of market cap and weight of individual securities
Value	SciBeta Diversified Multi-Strategy Value index			
Momentum	SciBeta Diversified Multi-Strategy High Momentum index			
Low volatility	SciBeta Diversified Multi-Strategy Low Volatility index			
Yield	SciBeta Diversified Multi-Strategy High Dividend Yield index			

6. Russell factor indices performance comparison

US Russell factor index	Methodology	Time period	Annual returns	Annual volatility	Sharpe ratio
Russell 1000 High Efficiency Momentum	Tilt the portfolio based on momentum score taking marlet cap weight of stock in the Russell 1000 index as starting point	1 Jan 2005–31 Dec 2013	8.69%	21.62%	0.33
Russell 1000 High Momentum	Cap weight up to 200 highest momentum stocks in Russell 1000 index		8.05%	20.59%	0.31
Russell 1000 High Efficiency Low Volatility	Tilt the portfolio based on low volatility score taking market cap weight of stock in the Russell 1000 index as starting point	1 Jan 2005–31 Dec 2013	7.89%	17.73%	0.36
Russell 1000 Low Volatility	Cap weight up to 200 least volatile stocks in Russell 1000 index		7.69%	16.35%	0.37
Russell 1000 High Efficiency Value	Tilt the portfolio based on value score (B/M and E/P ratios) taking market cap weight of stock in the Russell 1000 index as starting point	31 Dec 2003–31 Dec 2013	9.76%	22.55%	0.36
Russell 1000 Value	Tilt the portfolio based on value probability (B/M, sales per share growth, I/B/E/S growth) taking market cap weight of stock in the Russell 1000 index as starting point		7.56%	21.96%	0.27

All statistics are annualised and daily total returns are used for the analysis

4.09%

is the average annual long-term outperformance observed with US data over 40 years of the Scientific Beta US Multi-Beta Multi-Strategy EW Index compared to a reference index based on the 500 largest market cap US stocks.

This index equalises the investment in four extremely well diversified smart factor indices (Value, Momentum, Mid-Cap and Low Volatility).

It combines the best of factor investing with the best of smart beta and has improved the Sharpe ratio with respect to a reference index based on the 500 largest market cap US stocks by 91%* over the last 40 years.

For more information, please visit www.scientificbeta.com
or contact Mélanie Ruiz on +33 493 187 851
or by e-mail to melanie.ruiz@scientificbeta.com

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*Overall percentage increase in Sharpe ratio observed between 31/12/1973 and 31/12/2013 for long-term track record Scientific Beta US Multi-Beta Multi-Strategy EW compared to its cap-weighted equivalent calculated on a universe of the 500 largest capitalisation US stocks. All the details on the calculations and the indices are available on the www.scientificbeta.com website.

possible. Among these unrewarded risks, 50 years of research in finance and the econometrics of finance have targeted the reduction of model-specific risk, notably estimation errors for the parameters used in the weighting scheme. There are numerous approaches to the estimation of risk parameters. The sample estimator of the covariance matrix produces extremely high estimation errors when the ratio of universe size to sample size is large (Kan and Zhou [2007]) – sample risk. One solution to this problem is to reduce the number of parameters to be estimated by imposing a structure on the covariance matrix (Chan et al [1999]). Although this method reduces sample risk, its drawback is that the estimator is biased if the risk model does not conform to the true stock return generating process – model risk.

The next generation of estimators aim to achieve a trade-off between sample risk and model risk by combining sample estimators and structured estimators (Ledoit and Wolf [2003]). Another way to reduce model risk, which is used by ERI Scientific Beta, and not necessarily at the cost of sample risk, is to use an implicit factor model such as principal component analysis (PCA). The factors from the PCA have the benefit of being uncorrelated and of providing the best summary of the information contained in the dataset (ie, zero model risk).

To reduce sample risk, the number of statistical factors is limited using a criterion from Random Matrix Theory in order to achieve parsimony and robustness (Plerou et al [2002]). Coqueret and Milhau (2014) show that minimum volatility strategies using Principal Component (PC) tend to have lower volatilities compared to other estimation techniques.

One serious concern with optimisation-based weighting schemes, is that the stocks with the highest estimation error may receive the highest weight– a process commonly known as 'error maximisation' – which is detrimental to the relative robustness of strategies. ERI Scientific Beta uses two types of constraints to improve diversification – a long-only constraint and a deconcentration constraint. Jagannathan and Ma (2003) provide empirical evidence that imposing non-negativity constraints removes large outliers and hence provides better performance through better diversification.Deconcentration constraints ensure sufficiently balanced weights across constituents.[4]

DeMiguel et al (2009) introduce flexible quadratic constraints that put limits on the overall amount of concentration in the portfolio (eg, on the sum of squares of portfolio weights) rather than limiting the weight of each stock in the portfolio, thus leaving more room for the optimiser while avoiding concentration overall. ERI Scientific Beta applies norm constraints for the minimum-volatility weighting scheme by placing a lower bound on the effective number of stocks of the portfolio – N_{eff}.[5]

7. Correlation of relative returns (over CW index) across factor-tilted multi-strategy indices

US Long Term (1974–2013)		Diversified multi-strategies			
		Mid cap	Momentum	Low volatility	Value
Diversified multi strategies	Mid cap	100%	69%	64%	86%
	Momentum		100%	63%	66%
	Low volatility			100%	71%
	Value				100%

The analysis is based on daily total return data from 31 December 1973 to 31 December 2013 (40 years). The S&P 500 index is used as the cap-weighted benchmark.

8. Maximum relative drawdown analysis

US Long Term (1974–2013)	Diversified multi-strategies					
	Mid cap	Momentum	Low volatility	Value	Multi beta multi-strategy EW	Multi beta multi-strategy ERC
Maximum relative drawdown	42.06%	17.28%	43.46%	32.68%	33.65%	28.74%
Start of max relative DD	24 Mar 94	23 Mar 94	20 Sep 93	22 Mar 94	24 Mar 94	25 Mar 94
Maximum loss point	27 Mar 00	22 Dec 99	10 Mar 00	23 Mar 00	27 Mar 00	27 Mar 00
Recovery completed on	6 Sep 01	3 Apr 01	6 Sep 01	2 Mar 01	4 Apr 01	4 Apr 01

The analysis is based on daily total returns data from 31 December 1973 to 31 December 2013 (40 years). The S&P 500 is used as the cap-weighted benchmark. Maximum relative drawdown is the maximum drawdown of the long-short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index.

This method was chosen for the minimum volatility approaches because they are very sensitive, due to their natural concentration in low-volatility stocks, to the definition of maximum and minimum weights. We felt it was less arbitrary to use the effective number of stocks, which would avoid an in-sample weighting constraint risk and would give a greater role in the weighting scheme to the proper use of the diversification properties of minimum volatility.

> *"The combination of different strategies allows diversification of risks that are specific to each strategy by exploiting the imperfect correlation between the different strategies' parameter estimation errors"*

Even though the different weighting schemes offer efficient diversification of stocks, there is an additional need for diversification of the weighting schemes to diversify away the strategy-specific risks – a concept called 'diversifying the diversifiers'.[6] ERI Scientific Beta proposes diversified multi-strategy – an equal combination of five weighting schemes – as its flagship strategy for factor indices. The combination of different strategies allows diversification of risks that are specific to each strategy by exploiting the imperfect correlation between the different strategies' parameter estimation errors. Thus, diversifying the model risks further reduces the unrewarded risks and renders the weighting scheme more robust (in a relative manner).

Improving absolute robustness by diversifying across factors

Investors who rely on a single factor exposure take the risk of the underlying factor underperforming over short periods. The reward for exposure to these factors has been shown to vary over time (see, eg, Harvey [1989], Asness [1992], Cohen, Polk and Vuolteenaho [2003]). While this time variation in returns is not completely in sync for different factors, allocating across factors allows investors to diversify the sources of their outperformance and smooth their performance across market conditions. Figure 7 shows the correlation of the relative returns of factor-tilted multi-strategy indices over a cap-weighted benchmark. The indices are not perfectly correlated with each other which shows a potential for diversification across factors in order to reduce risk and generate smoother outperformance over time.

ERI Scientific Beta offers two multi beta allocations – equal weight (EW) and equal risk contribution (ERC). The EW allocation, which is a simple and robust allocation in terms of absolute risk, invests one quarter in each of the four multi-strategy factor indices. The ERC allocation combines the four multi-strategy factor indices so as to equalise their contributions to the tracking error risk.

Measurement of robustness

ERI Scientific Beta proposes extreme risk measures including maximum relative drawdown analysis and a factor attribution exercise as measures of relative robustness and outperformance probability and conditional performance as tools to assess absolute robustness.

The maximum relative drawdown measures the maximum relative loss experienced by a strategy between a peak and a valley over a specified period. It is important to see if the losses can be explained through market fundamentals and if the reasons are in line with the index construction methodology. If not, then there are other unintended risks at play which bring down the relative robustness of the strategy. Figure 8 shows that maximum loss occurred during the late 1990s technology bubble when the cap-weighted benchmark was over-weighted in growth and technology stocks.

Many studies have underlined the importance of factor exposures in explaining part of the outperformance of portfolio strategies over cap-weighted indices (see Jun and Malkiel [2007] and Amenc, Goltz and Le Sourd [2008]). It is a particularly important robustness check in the case of single and multi-factor indices because it discloses what portion of a strategy's performance is indeed derived from its exposure to the intended risk factor and how much can ►

4 We impose an upper bound and a lower bound on the weight of each constituent security,

$$l_i = \frac{1}{3N} \le w_i^* \le u_i = \frac{3}{N}$$

where $i = 1,\ldots,N$ and N is the nominal number of constituents. Stock weights are bound to be below $3/N$ and above $1/3N$, where N denotes the number of constituents.

5 $N_{eff} \ge \frac{N}{3}, N_{eff} = \text{Effective number of stocks} = \frac{1}{\sum_{i=1}^{N} w_i^2}$,

where N is the number of constituent stocks in the index and w_i is the weight of stock i in the index.

6 See Timmermann (2006), Kan and Zhou (2007), Tu and Zhou (2010) and Amenc, Goltz, Lodh and Martellini (2012) on the benefits of combining portfolio strategies.

be attributed to other factors and unexplained alpha. The attribution exercise can be extended to tracking error to monitor the role of each factor in the deviation of the strategy from its benchmark.

Since the performance of smart beta varies over time, the analytics reported over long horizons – for example excess returns over 40 years – have limited information because of averaging over time periods. Probability of outperformance is a measure that overcomes this limitation. The probability of outperformance is defined as the empirical frequency of outperforming the cap-weighted reference index over a given investment horizon. It is an intuitive and relevant measure which shows how often and consistently the strategy would be able to outperform the cap-weighted reference index in the past for all possible entry points. Figure 10 shows the probability of outperforming the cap-weighted benchmark of multi beta multi-strategy EW, ERC and the simple average of the four component single beta indices with various investment horizons for two stock universes. This shows that a combination of factors indeed improves the chances of outperforming the CW benchmark (improves absolute robustness) compared to single factors in isolation.

Analysing the conditional performance of the smart beta strategies in bull-bear market conditions or in contraction-expansion business cycles is a powerful tool in robustness analysis because the performance of smart beta strategies is shown to vary over market phases (Gonzalez and Thabault [2013]). A strategy which performs well in different market conditions and shows little or no state and time dependency can be said to be robust in an absolute sense.

Figure 11 shows that multi-beta multi-strategy indices outperform the cap-weighted benchmarks in both the bull and bear regimes, whereas the component indices perform very differently in different market conditions. For example, the low-volatility index performs very poorly in bull markets but performs extremely well in bear markets and the mid-cap index performs well in bull markets but has a relatively poor information ratio in bear markets.

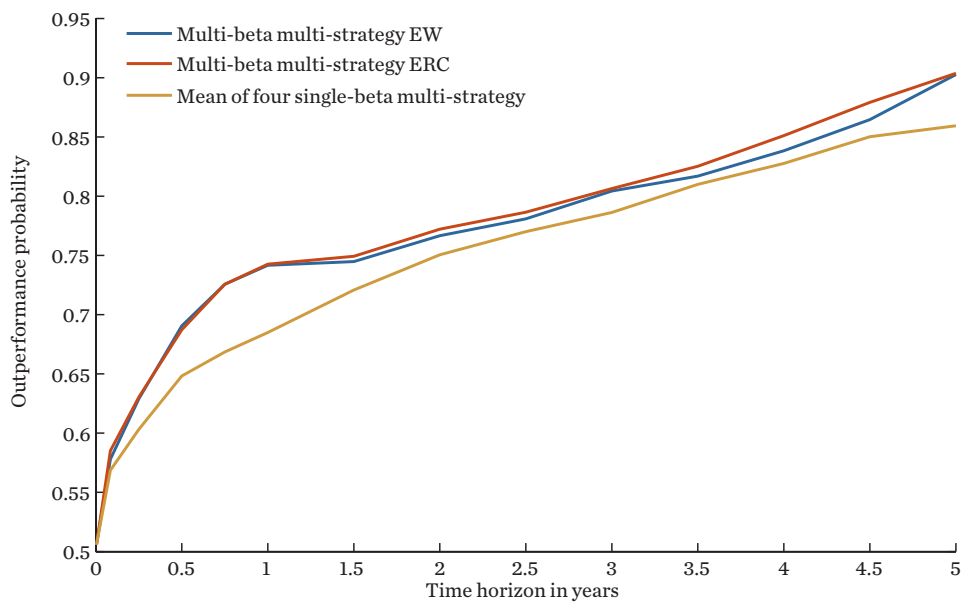
In conclusion, it is essential that smart beta strategy performance reporting be accompanied by a measurement of the relative and absolute robustness of its performance. The lack of relative robustness arises mainly from data mining, non-robust weighting methodologies and that of absolute robustness comes from undiversified factor exposures. Relative robustness can be improved by reducing all sources of unrewarded risks with the use of a consistent framework (to prohibit data mining), robust parameter estimation techniques, weight constraints, and diversification of strategy-specific risk. Absolute robustness can be achieved through allocating across several rewarded factors. Our results show that the single factor indices have a high degree of relative robustness, but they are not robust in absolute terms. The multi-beta allocations on the other hand are highly robust in absolute terms.

9. Exposure to equity risk factors – Carhart

US Long Term (1974–2013)				Diversified multi-strategies		
	Mid cap	Momentum	Low volatility	Value	Multi-beta multi strategy EW	Multi-beta multi-strategy ERC
Annual alpha	2.66%	1.84%	2.85%	2.33%	2.45%	2.35%
Market beta	0.93	0.94	0.78	0.91	0.89	0.89
SMB beta	0.31	0.16	0.02	0.16	0.16	0.15
HML beta	0.16	0.09	0.14	0.31	0.17	0.16
MOM beta	0.00	0.17	0.00	0.03	0.05	0.06
R-squared	92.20%	95.52%	90.14%	95.00%	94.76%	95.46%
Performance attribution						
Unexplained (alpha)	3.22%	2.44%	3.23%	2.90%	3.02%	2.91%
Market beta	4.96%	5.00%	4.16%	4.88%	4.75%	4.76%
SMB beta	0.85%	0.44%	0.06%	0.43%	0.44%	0.41%
HML beta	0.80%	0.44%	0.71%	1.56%	0.88%	0.80%
MOM beta	–0.01%	0.46%	–0.01%	0.08%	0.13%	0.15%

The complete stock universe consists of the 500 largest stocks in the US. The market factor is the daily return of the cap-weighted index of all stocks that constitute the index portfolio in excess of the risk-free rate. The small size factor is the daily return series of a cap-weighted portfolio that is long the smallest 30% of stocks (by market cap) and short the largest 30% of stocks (by market cap) of the extended universe (ie, including small caps). The value factor is the daily return series of a cap-weighted portfolio that is long the 30% highest and short the 30% lowest B/M ratio stocks in the investable universe. The momentum factor is the daily return series of a cap-weighted portfolio that is long the 30% highest and short the 30% lowest 52 weeks (minus the most recent 4 weeks) past return stocks in the investable universe. The Secondary Market US Treasury Bills (3M) is the risk-free rate in US dollars. All statistics are annualised. The analysis is based on daily total returns from 31 December 1973 to 31 December 2013. The statistics that satisfy a 95% significance level are highlighted in bold.

10. Outperformance frequency over different horizons



The analysis is based on daily total return data from 31 December 1973 to 31 December 2013 (40 years). The S&P 500 is used as the cap-weighted benchmark. Probability of outperformance is the historical empirical probability of outperforming the cap-weighted benchmark over an investment horizon of one month, three months, six months, nine months, one year, 18 months, two years, 36 months, and so on, up to five years irrespective of the entry point in time. It is computed using a rolling window analysis with window length corresponding to the investment horizon and one-week step size.

11. Conditional performance

US Long Term (1974–2013)				Diversified multi-strategies		
	Mid cap	Momentum	Low volatility	Value	Multi beta multi-strategy EW	Multi beta multi-strategy ERC
Bull markets						
Annual relative returns	5.12%	3.28%	–0.99%	3.54%	2.79%	2.71%
Annual tracking error	5.76%	4.04%	5.11%	5.00%	4.38%	4.13%
Information ratio	0.89	0.81	–0.19	0.71	0.64	0.66
Bear markets						
Annual relative returns	3.83%	3.77%	8.12%	5.99%	5.469%	5.14%
Annual tracking error	8.33%	6.26%	7.94%	7.12%	6.57%	6.12%
Information ratio	0.46	0.60	1.02	0.84	0.83	0.84

Calendar quarters with positive market index returns comprise bull markets and the rest constitute bear markets. All statistics are annualised. The analysis is based on daily total return data from 31 December 1973 to 31 December 2013 (40 years). The complete stock universe consists of the 500 largest stocks in the US. The S&P 500 index is used as the cap-weighted benchmark.

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The performance of smart beta indices: assessing factor indices

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This article provides a brief overview of equity factor index offerings from major index providers. Factor indices aim to provide explicit exposure to a common risk factor to harvest its long-term risk premia. Due to the predominance of long-only indices in practical application, we focus here on long-only indices and do not include long/short index offerings. To get a synthetic view on the performance of different factor indices, we have included one index each from the following index providers: ERI Scientific Beta, MSCI, Russell, FTSE and FTSE RAFI. We select indices from each of four main factors that seek exposure to the following factors: low volatility, momentum, size and value.

Index construction methodology

Traditional factor indices fall into two major categories. The first involves maximising the exposure to a factor by selecting stocks that are most exposed to the desired risk factor and the application of a cap-weighting (CW) scheme to this selection. This approach is included in the illustrations in the form of Scientific Beta tilted CW indices. While this approach brings the exposure to the desired factor, the problem of poor diversification arising from high concentration in a small number of stocks remains unanswered. The second method weights either the whole of the universe (Russell factor indices) or a part of universe (MSCI Momentum) by the exposure to this factor, resulting in score/rank weighting. Here again, the maximisation of the factor exposure does not guarantee that the indices are well diversified.

To overcome these difficulties, index providers that generally offer factor indices on the basis of the first two approaches have recently sought to take advantage of the development of smart beta indices to offer investors a new framework for smart factor investing (Bender et al [2013]). This approach recognises that smart betas have implicit risk exposures and aims to select and combine them according to these varying exposures. The drawback of this approach is that it maximises neither factor exposure nor diversification of the indices. For example, a minimum volatility index on a broad universe (eg, MSCI Minimum Volatility) does not guarantee either the highest exposure to low-volatility stocks or the best diversification of this low-volatility portfolio.

> *"Seeking exposure to the size factor through equal weighting of a broad universe is certainly less effective than selecting the smallest size stocks in the universe and then diversifying them"*

Similarly, seeking exposure to the size factor through equal weighting of a broad universe is certainly less effective than selecting the smallest size stocks in the universe and then diversifying them, including with an equal-weighted weighting scheme. Finally, seeking to be exposed to the value factor through a value-weighted index (for, eg, FTSE RAFI 1000 series and MSCI Value) will not produce a well-diversified index, simply because the integration of the attributes characterising the value exposure into the weighting does not take the correlations between these stocks into account.

In view of these problems, EDHEC-Risk Institute has promoted the concept of smart factor investing using the Smart Beta 2.0 approach. The idea is to construct a factor-tilted portfolio to extract the factor premia most efficiently and is based on two pillars: 1) explicitly selecting appropriate stocks for the desired beta and 2) using a diversification-based weighting scheme (Amenc et al [2013]). ERI Scientific Beta constructs smart factor indices by using diversified multi-strategy weighting on characteristics-based half universes – small size, high momentum, low volatility, and value.[1] In particular, the indices use a diversified multi-strategy weighting, which consists of an equal allocation to the five following weighting schemes: maximum deconcentration, risk-weighted, maximum decorrelation, minimum volatility and maximum Sharpe ratio.

This article performs a comparative study across major providers of factor indices – MSCI, Russell, FTSE and FTSE RAFI. S&P offers factor indices in the US and FTSE RAFI recently ►

1 The Scientific Beta US (Developed) stock universe consists of 500 (2,000) stocks, therefore each factor index is constructed on 250 (1,000) stocks. The index construction methodology remains identical across regions.

1. An overview of factor index construction methodologies

Factor	Index	Stock selection	Weighting scheme	Risk controls
MSCI index methodology				
Size	MSCI Equal-Weight	All stocks in CW parent index universe	Equal-weighted	None
Value	MSCI Value-Weighted	All stocks in CW parent index universe	Score adjusted by investability factor	None
Momentum	MSCI Momentum	Selection by momentum score (fixed number of constituents to target 30% market cap coverage	Market cap * momentum score	Cap on weight of individual security
Low volatility	MSCI Minimum Volatility	All stocks in CW parent index universe	Optimisation to minimise portfolio risk	Sector and country weight constraints; Cap on multiple of market cap of individual security
Scientific Beta index methodology				
Size	SciBeta Diversified Multi-Strategy Mid Cap	Half the stocks by relevant score	Same weighting scheme for selected individual securities	Cap on multiple of market cap and stocks (Diversifed multi-strategy by default)
Value	SciBeta Diversified Multi-Strategy Value			
Momentum	SciBeta Diversified Multi-Strategy High Momentum			
Low volatility	SciBeta Diversified multi-Strategy Low Volatility			
Russell index methodology				
Size	Russell Mid Cap	Smallest 800 companies from parent index	Cap-weighted	None
Value	Russell High Efficiency Value	Scoring based on non-linear probability method, results in approximately 50% of the stocks of the CW parent index	Conversion of the scores calculated in the stock selection stage into active weights by using the NLP method using cap weights as base	Turnover minimisation by calculating new stocks' weights using a banding process
Momentum	Russell High Efficiency Momentum	Scoring based on non-linear probability method, results in approximately 50% of the stocks of the CW parent index		
Low volatility	Russell High Efficiency Low Volatility	Scoring based on non-linear probability method, results in approximately 30% of the stocks of the CW parent index		
FTSE index methodology				
Size	FTSE Developed Size Factor	Selection based on size factor exposure score	Using cap weights as base, stock weights are tilted by their respective factor scores	Country and industry constraints
Value	FTSE Developed Value Factor	Selection based on value factor exposure score		
Momentum	FTSE Developed Momentum Factor	All stocks in CW parent universe		
Low volatility	FTSE Developed Low Volatility	Selection based on volatility factor exposure score		
FTSE RAFI index methodology				
Size	FTSE RAFI 1000	Selection of 1,000 stocks with the highest fundamental value from the CW parent index universe	Stocks' weights are calculated based on their investable RAFI fundamental value composite score	None

launched a low volatility index series, but due to limited history of publicly available data (less than 10 years) on these indices, we have not included them in the analysis, because it is not possible for us to draw conclusions from periods that are too short.
Figure 1 summarises construction principles for all the indices used in this study.

It should be noted that different index providers include implementation rules in their indices in order to avoid transaction costs for investors trying to capture the factor exposures. Russell indices have an implicit reference to market cap weights, which naturally tends to ease implementation. The MSCI factor indices use different rules across different factors. For example, no investability adjustments are made for equal-weighted indices. Other factor indices use different adjustments such as a weight cap in the momentum index, turnover constraints in the minimum volatility index, and smoothing over average fundamental variables in the value-weighted indices. The ERI Scientific Beta indices apply investability rules (capacity and turnover control) as well as turnover constraints.[2]

Absolute and relative performance

Figure 2 shows that all indices generally achieve outperformance over the broad cap-weighted index both in terms of returns and Sharpe ratio. This shows that the factor indices are able to benefit from the rewarded factor, some more than others. In the US universe, Scientific Beta factor indices achieve the highest outperformance in each factor category. For example, the Scientific Beta USA Low Volatility Diversified Multi-Strategy index has excess returns of 2.70% compared to just 1.39% for the closest competitor and the Scientific Beta USA Value Diversified Multi-Strategy index has excess returns of 3.16% compared to 2.38% for the closest competitor (Russell) and 1.94% for FTSE RAFI 1000 US.

> *"Investing in a single factor comes at the risk of short-term relative losses, which can be large at times. Therefore, maximum relative drawdown becomes an important measure of risk"*

We have seen that any portfolio tilted towards a rewarded risk factor will be rewarded by nature. This is why all Scientific Beta tilted CW indices, which make no effort to achieve diversification, have positive excess returns. The outperformance can sometimes be quite high, for example in the case of mid cap indices (3.03% and 2.65% for US and Developed indices respectively), but it comes at the cost of higher volatility. Therefore, the role of diversification can truly be demonstrated by comparing risk-adjusted performance – the Sharpe ratios. The Sharpe ratio of the Scientific Beta mid cap indices is far higher than that of their tilted CW counterparts. In general, the Sharpe ratios of Scientific Beta indices are also invariably superior in each factor category.

Investing in a single factor comes at the risk of short-term relative losses, which can be large at times (Asness [1992] and Cohen, Polk and Vuolteenaho [2003]). Therefore, maximum relative drawdown becomes an important measure of risk. This is the maximum relative loss, compared to the CW benchmark, experienced by a strategy between a peak and a valley over a specified period. The results show that Scientific Beta indices, with the exception of

2 While it is beyond the scope of this article to assess the implementation rules across providers, it is clear that factor indices potentially improve upon paper portfolios used to study factor premia in academic studies as far as investability is concerned.

2. Absolute and relative analytics of factor indices with respect to broad CW index in US and Developed universes

Panel A: US factor indices	Absolute analytics					Relative analytics (benchmark = S&P 500)				
	Annualised returns	Annualised volatility	Sharpe ratio	CF VaR (5%)	Maximum drawdown	Annualised relative returns	Annualised tracking error	Information ratio	Extreme (95%) TE	Maximum relative drawdown
S&P 500 index	7.38%	20.37%	0.29	1.77%	55.25%	–	–	–	–	–
SciBeta Mid Cap CW	10.41%	22.33%	0.40	2.08%	57.09%	3.03%	5.38%	0.56	9.18%	18.95%
MSCI USA Equal Weighted	9.51%	22.70%	0.35	2.07%	59.77%	2.13%	4.45%	0.48	10.12%	17.04%
Russell Mid Cap	10.19%	22.61%	0.38	2.12%	58.93%	2.81%	5.08%	0.55	9.31%	17.80%
SciBeta Mid Cap Multi-Strategy	10.80%	20.29%	0.45	1.87%	53.42%	3.42%	4.56%	0.75	7.92%	9.64%
SciBeta Momentum CW	8.64%	20.38%	0.35	1.70%	50.81%	1.26%	4.63%	0.27	9.21%	13.77%
MSCI USA Momentum	9.39%	20.83%	0.38	1.94%	55.94%	2.01%	7.79%	0.26	15.80%	23.59%
Russell High Efficiency Mom	9.06%	20.92%	0.36	1.85%	52.90%	1.68%	4.49%	0.38	7.61%	10.19%
SciBeta Mom Multi-Strategy	9.40%	20.07%	0.39	1.76%	53.25%	2.02%	5.50%	0.37	10.85%	16.22%
SciBeta Low Volatility CW	7.94%	17.82%	0.36	1.46%	51.10%	0.56%	4.01%	0.14	7.10%	12.12%
MSCI USA Minimum Volatility	8.77%	16.91%	0.43	1.48%	46.61%	1.39%	5.20%	0.27	8.39%	12.83%
Russell High Efficiency Low Vol	8.56%	17.06%	0.41	1.40%	47.50%	1.18%	4.79%	0.25	9.39%	11.36%
SciBeta Low Vol Multi-Strategy	10.08%	16.99%	0.50	1.45%	48.31%	2.70%	5.20%	0.52	9.67%	8.79%
SciBeta Value CW	7.56%	22.46%	0.27	2.03%	60.01%	0.18%	3.94%	0.05	8.03%	14.56%
MSCI USA Value Weighted	7.53%	22.12%	0.27	1.97%	60.81%	0.15%	3.33%	0.05	8.10%	14.51%
Russell High Efficiency Value	9.76%	22.55%	0.36	2.03%	59.34%	2.38%	4.50%	0.53	11.31%	11.59%
FTSE RAFI 1000 US	9.32%	22.16%	0.35	1.99%	60.22%	1.94%	4.31%	0.45	11.05%	12.71%
SciBeta Value Multi-Strategy	10.54%	20.63%	0.43	1.87%	53.75%	3.16%	3.56%	0.89	5.49%	5.97%

Panel B: Developed factor indices	Absolute analytics					Relative analytics (benchmark = MSCI World)				
	Annualised returns	Annualised volatility	Sharpe ratio	CF VaR (5%)	Maximum drawdown	Annualised relative returns	Annualised tracking error	Information ratio	Extreme (95%) TE	Maximum relative drawdown
MSCI World Index	7.53%	17.52%	0.34	1.63%	57.46%	-	-	-	-	-
SciBeta Mid Cap CW	10.18%	17.80%	0.48	1.74%	58.11%	2.65%	3.53%	0.75	5.99%	10.44%
MSCI World Equal Weighted	9.22%	17.44%	0.44	1.71%	59.66%	1.70%	4.47%	0.38	8.37%	13.40%
Russell Developed Small Cap	9.14%	17.41%	0.43	1.79%	60.64%	1.61%	5.95%	0.27	11.50%	17.95%
FTSE Developed Size Factor	9.41%	16.92%	0.46	1.69%	59.18%	1.88%	4.90%	0.38	9.24%	13.48%
SciBeta Mid Cap Multi-Strategy	10.45%	16.12%	0.55	1.57%	54.57%	2.92%	3.57%	0.82	6.80%	6.62%
SciBeta Momentum CW	8.90%	17.23%	0.43	1.56%	54.85%	1.37%	3.61%	0.38	6.70%	9.87%
MSCI World Momentum	10.80%	17.75%	0.52	1.67%	55.53%	3.27%	7.85%	0.42	15.67%	20.87%
Russell Dev HE LC Momentum	10.22%	18.24%	0.47	1.71%	56.69%	2.69%	3.99%	0.68	5.98%	9.97%
FTSE Dev Momentum Factor	8.29%	17.24%	0.39	1.59%	55.50%	0.76%	1.15%	0.66	1.78%	3.18%
SciBeta Mom Multi-Strategy	10.30%	16.09%	0.54	1.51%	54.35%	2.77%	4.23%	0.65	8.24%	12.65%
SciBeta Low Volatility CW	8.67%	15.07%	0.47	1.37%	52.59%	1.14%	3.48%	0.33	6.23%	9.54%
MSCI World Minimum Vol	8.36%	12.67%	0.54	1.19%	47.35%	0.83%	6.54%	0.13	10.95%	17.42%
Russell Dev HE Lge Cap Low Vol	9.52%	14.09%	0.56	1.28%	49.60%	1.99%	4.75%	0.42	8.95%	13.16%
FTSE Dev Volatility Factor	8.85%	15.98%	0.46	1.42%	52.44%	1.32%	3.20%	0.41	5.34%	9.89%
SciBeta Low Vol Multi-Strategy	10.54%	13.79%	0.65	1.30%	49.55%	3.01%	4.79%	0.63	9.24%	9.76%
SciBeta Value CW	7.82%	18.80%	0.33	1.76%	60.07%	0.29%	2.61%	0.11	4.83%	11.03%
MSCI World Value Weighted	7.53%	18.61%	0.32	1.75%	61.55%	0.00%	2.73%	0.00	5.82%	11.51%
Russell Dev HE Large Cap Value	9.70%	18.61%	0.44	1.77%	61.61%	2.17%	3.87%	0.56	8.71%	12.24%
FTSE Dev Value Factor Index	8.16%	18.74%	0.35	1.76%	60.47%	0.64%	3.52%	0.18	7.79%	12.03%
FTSE RAFI 1000 Developed	8.46%	19.15%	0.36	1.79%	61.00%	0.93%	3.92%	0.24	9.11%	13.50%
SciBeta Value Multi-Strategy	10.21%	17.23%	0.50	1.66%	57.32%	2.68%	2.55%	1.05	4.19%	5.68%

The table compares the absolute and relative performance of the Scientific Beta multi-strategy index for four factor tilts with the competing indices. Extreme tracking error (95%) is the 95th percentile of the distribution of one-year rolling window tracking errors. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. For US (Developed), the S&P 500 index (the MSCI World index) is used as the benchmark. Yield on secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The analysis is based on daily total returns of indices in the period 1 January 2004 to 31 December 2013 (10 years).

the momentum index, experienced maximum relative drawdown in the range of 6% to 10% during the analysis period – distinctly lower than the MSCI and Russell factor indices. The effect of risk reduction through diversification is particularly clear in the case of value indices. The diversification-based Scientific Beta factor index exhibits maximum relative drawdown of 5.97% and a Sharpe ratio of 0.43, in comparison with FTSE RAFI 1000 US, which has maximum relative drawdown of 12.71% and a Sharpe ratio of 0.35.

Panel B of figure 2 presents the same analysis in the Developed World stock universe. In each factor category, both the excess returns and Sharpe ratio of Scientific Beta factor indices is higher than those of competitors. For example, the Scientific Beta Developed Mid-Cap (Low Volatility) Diversified Multi-Strategy index has a Sharpe ratio of 0.55 (0.65) compared to just 0.46 (0.56) for the closest competitor. Other observations from the US universe, such as higher information ratio and lower maximum relative drawdown for Scientific Beta indices, remain qualitatively similar in the Developed universe.

"Relative to the broad CW index, size and value indices are favoured in bull markets, while momentum and low volatility indices perform better in bear markets. It is sometimes argued therefore that a combination of factor indices is better suited for an investor who desires more consistent outperformance across market cycles"

Robustness checks

It is understood that a view of average performance over some time frame does not reveal certain useful information, such as performance in extreme market conditions and consistency in beating the benchmark. These measures are indicative of the robustness of the strategies. Therefore, we compute 'outperformance probability' and performance in the top and bottom 25% of the market. The probability of outperformance is defined as the empirical frequency of outperforming the cap-weighted reference index over a given investment horizon. This calculation allows one to distinguish between strategies that are sensitive to the choice of period in achieving their performance and those that are less so. For conditional analysis, performance in the top and bottom 25% quarters is analysed by CW index returns.

The outperformance probability of Scientific Beta indices is higher in the case of size, low volatility and value indices in both the US and Developed universes. MSCI Value indices display around 25–40% outperformance probability for a three-year horizon, while for Scientific Beta indices this figure is around 90%. Except for US momentum, all Scientific Beta factor indices achieve 100% outperformance ►

probability for a five-year investment horizon, meaning that if one had invested in the strategy for a five-year period (as factors perform over longer time frames), one would have been sure to outperform the CW benchmark, irrespective of the entry time.

The information on extreme markets (25% bull vs 25% bear) shows that the dependence of the performance of all indices on market conditions is quite high. Relative to the broad CW index, size and value indices are favoured in bull markets, while momentum and low volatility indices perform better in bear markets. It is sometimes argued therefore that a combination of factor indices is better suited for an investor who desires more consistent outperformance across market cycles.

Our results show that factor indices in general deliver attractive performance, as they take advantage of risk factors that are supported by academic research. However, Scientific Beta's methodology, aiming at improved diversification within the factor space, provides better extraction of the related risk premia. Caution is in order when assessing any performance comparison that is based on a limited time period. That is why ERI Scientific Beta calculates long-term track records over 40 years on US data. Since the equivalent of these calculations is not disclosed by all the other index providers, it has not been possible to make this comparison. In fact, a key question that any index provider faces is potential criticism that indices could be a result of a data mining exercise, which in turn implies that performance may not be robust. Therefore, in addition to performance numbers, a key question is the methodological robustness of the different offerings. For index performance to be considered robust, index providers should follow transparent and consistent methodologies and build their indices on consensual factor definitions and models, which is unfortunately not always the case, as we show here. Therefore, beyond assessing the performance of backtests, the methodological robustness is likely to become a key consideration for investors going forward.

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3. Robustness analytics of factor indices in US and Developed universes

Panel A: US factor indices	Outperformance probability		Extreme bull (25% best)		Extreme bear (25% worst)	
	Three-year	Five-year	Excess returns	Tracking error	Excess returns	Tracking error
SciBeta Mid Cap CW	84%	98%	7.19%	4.18%	–1.34%	7.51%
MSCI USA Equal Weighted	85%	98%	11.82%	4.68%	–3.00%	6.44%
Russell Mid Cap	86%	100%	11.43%	4.76%	–2.53%	7.00%
SciBeta Mid Cap Multi-Strategy	95%	100%	1.06%	3.84%	3.04%	6.16%
SciBeta Momentum CW	56%	76%	–2.93%	3.59%	2.93%	6.82%
MSCI USA Momentum	58%	57%	1.28%	5.65%	2.45%	11.70%
Russell High Efficiency Momentum	91%	94%	2.51%	3.64%	0.03%	6.35%
SciBeta Momentum Multi-Strategy	60%	68%	–3.27%	4.36%	2.46%	8.21%
SciBeta Low Volatility CW	64%	71%	–6.91%	4.13%	5.47%	5.53%
MSCI USA Minimum Volatility	87%	94%	–9.12%	5.16%	10.98%	7.20%
Russell High Efficiency Low Vol	78%	97%	–8.69%	4.48%	8.04%	6.98%
SciBeta Low Vol Multi-Strategy	100%	100%	–7.15%	4.97%	8.79%	7.48%
SciBeta Value CW	28%	17%	3.80%	3.82%	–3.30%	5.81%
MSCI USA Value Weighted	39%	21%	6.96%	3.46%	–3.53%	5.29%
Russell High Efficiency Value	88%	100%	11.37%	4.99%	–1.14%	6.62%
FTSE RAFI 1000 US	82%	95%	15.17%	6.24%	–3.05%	4.65%
SciBeta Value Multi-Strategy	92%	100%	1.72%	2.90%	2.72%	4.54%

Panel B: Developed factor indices	Outperformance probability		Extreme bull (25% best)		Extreme bear (25% worst)	
	Three-year	Five-year	Excess returns	Tracking error	Excess returns	Tracking error
SciBeta Mid Cap CW	80%	100%	6.12%	2.70%	0.21%	4.93%
MSCI World Equal Weighted	67%	99%	9.48%	3.53%	–0.98%	6.22%
Russell Developed Small Cap	61%	88%	13.96%	4.57%	–1.73%	8.52%
FTSE Developed Size Factor	74%	98%	8.39%	3.78%	–1.37%	7.10%
SciBeta Mid Cap Multi-Strategy	97%	100%	0.80%	3.19%	4.14%	5.06%
SciBeta Momentum CW	76%	79%	–1.47%	3.04%	1.76%	5.11%
MSCI World Momentum	63%	82%	–3.90%	5.81%	5.61%	11.50%
Russell Dev HE LC Momentum	86%	100%	3.94%	3.15%	0.30%	5.31%
FTSE Dev Momentum Factor	100%	100%	–0.74%	0.95%	1.67%	1.61%
SciBeta Momentum Multi-Strategy	80%	100%	–1.00%	3.89%	3.58%	5.95%
SciBeta Low Volatility CW	81%	97%	–6.92%	3.09%	5.72%	4.80%
MSCI World Minimum Volatility	75%	93%	–13.19%	5.42%	12.85%	9.20%
Russell Dev HE Large Cap Low Vol	93%	96%	–8.52%	4.22%	8.75%	6.69%
FTSE Dev Volatility Factor	89%	97%	–6.99%	2.61%	6.06%	4.27%
SciBeta Low Vol Multi-Strategy	100%	100%	–6.49%	4.09%	8.87%	6.93%
SciBeta Value CW	29%	34%	3.03%	2.29%	–2.34%	3.67%
MSCI World Value Weighted	25%	20%	5.61%	2.50%	–3.04%	3.99%
Russell Dev HE Large Cap Value	68%	95%	10.88%	4.00%	–1.82%	5.57%
FTSE Dev Value Factor	33%	47%	4.42%	2.64%	–2.55%	5.44%
FTSE RAFI 1000 Developed	70%	74%	11.26%	5.04%	–3.12%	4.71%
SciBeta Value Multi-Strategy	88%	100%	3.65%	1.98%	1.45%	3.42%

The table compares the robustness and conditional performance of the Scientific Beta multi-strategy index for four factor tilts with the competing indices. Outperformance probability is the probability of obtaining positive excess returns over CW if one invests in the strategy at any point in time for a period of three (or five) years. It is computed as the frequency of positive values in the series of excess returns assessed over a rolling window of three (or five) years and step size of one week covering the entire investment horizon. The top 25% of quarters with highest market returns are considered extremely bullish and the bottom 25% quarters with the lowest returns are considered extremely bearish. For US (Developed), the S&P 500 index (the MSCI World index) is used as the benchmark. Yield on secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The analysis is based on daily total returns of indices in the period 1 January 2004 to 31 December 2013 (10 years).

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