ScientificBeta
An EDHEC-Risk Institute Venture

Research Insights

An EDHEC-Risk Institute Publication in association with IPE

Autumn 2015

EDHEC-Risk Institute
Research Insights

Introduction
Noël Amenc

It is a great pleasure to introduce the latest Scientific Beta special issue of the Research Insights supplement to IPE.

We begin by looking at 'quality' investing and more specifically the role of two separate equity risk factors related to balance sheet characteristics: low investment and high profitability. These factors rely on straightforward, parsimonious indicators, and can be expected to provide more robust performance benefits than ad-hoc stock picking indicators of quality used in the industry. Further value can be added by allocating across these two factors to exploit the low correlation across factor returns. Such combinations of the smart factor indices for high profitability and low investment have led to improved performance compared to various commercial indices which are based on ad-hoc definitions of quality.

One of the questions frequently asked about the quality factor, or factors, is the nature of its relationship to the value factor, and whether in fact quality might be redundant if one is already investing in value. We examine this question in detail and conclude that the profitability and investment factors are neither subsumed by other factors such as value and momentum, nor do they make these other factors redundant.

The performance of systematic equity investment strategies is typically analysed on backtests that apply the smart beta methodology to historical stock returns. Concerning actual investment decisions, a relevant question therefore is how robust the outperformance is. We examine the robustness of the first generations of smart beta indices on the basis of live track records and observe that differences in live performance are due to the attention given to the design of robust weighting schemes.

We compare the results of smart factor indices with several stylised examples of concentrated factor indices. We conclude that increasing concentration leads to high turnover levels and real investability hurdles which are not compensated by any performance advantages. The solution, as the title of the article suggests, is to go back to basics and focus on diversification when constructing factor indices.

We look at what academic research can teach investors about equity factors that are rewarded over the long term. Index providers emphasise the academic foundations of their factor indices, so it is useful to analyse what academic research has to say on equity factors. A minimum requirement for good practice in factor investing is to create a good match with academic factors. This can be achieved by referring to indicators for which academic research has provided thorough tests and economic explanations.

In addition to the question of selecting a suitable index as a stand-alone investment, the question of combining different smart beta strategies naturally arises in the context of an extensive range of smart beta offerings. Our article addresses the issue of combining several smart beta strategies, and clarifies the conceptual underpinnings and relevant questions arising when considering smart beta index combinations.

We show that, on the basis of existing smart factor indices, allocation between these indices can allow an investor who wishes to implement a defensive strategy to avoid concentration in a single factor and above all to benefit from the particular properties of volatility and its dissymmetric nature with respect to market conditions, and thereby adjust the portfolio's defensive bias to market conditions.

In our final article, we find that value, in terms of risk-adjusted relative performance, can be added through allocation across smart factor indices, for investors with a tracking error budget. The favourable factor tilts generate outperformance and two-fold diversification, one across factors and another across weighting schemes, reducing tracking error. Implementation of an allocation that guarantees a level of market beta equivalent to that of a cap-weighted index allows the benefits of this relative risk diversification to be optimised.

We hope you will find the articles on smart beta in the supplement informative and relevant. We extend warm thanks to our friends at IPE for their collaboration on the supplement.

Noël Amenc, Professor of Finance, EDHEC-Risk Institute, CEO, ERI Scientific Beta

Contents

IPE International Publishers Ltd, Pentagon House, 52-54 Southwark Street, London SE1 1UN, UK
Tel: +44(0)20 3465 9300, Fax: +44(0)20 7403 2788, Web site: www.ipe.com, ISSN 1369-3727
Investment & Pensions Europe is published monthly by IPE International Publishers Ltd. No part of this publication may be reproduced in any form without the prior permission of the publishers. Printed by Hastings Printing Company, Drury Lane, St Leonards-on-Sea, East Sussex TN38 9BJ, UK.

Quality investing

Felix Goltz, Head of Applied Research, EDHEC-Risk Institute, Research Director, ERI Scientific Beta; Ashish Lodh, Deputy Research Director, ERI Scientific Beta; Sivagaminathan Sivasubramanian, Quantitative Analyst, ERI Scientific Beta

Quality stock picking versus factor investing

Asset managers and index providers are increasingly touting the benefits of quality investing. Such strategies tilt portfolios to 'high quality' stocks, as characterised for example by high profitability, stable earnings or low leverage, to name but a few of the variables used in practice. However, asset managers and index providers do not use a common definition of 'quality', and a large variety of approaches exist. The premise of quality investing is that high quality stocks are not recognised by the market to a sufficient extent to increase their price to a level that fully reflects their superior quality – therefore such stocks offer a good investment opportunity. The concept has been traced back to fundamental stock pickers such as Benjamin Graham and Jeremy Grantham. De facto, systematic filters proposed today by many index providers aim to procure alpha in competition with traditional asset managers, without necessarily having all the same characteristics, and notably the capacity to take account of forecasts on evolutions in stock characteristics or new factors that can change the perception of those characteristics.

For academics and the proponents of a beta, rather than alpha, approach, which in our view is the only approach that is compatible with indexed investment management, the quality term refers to a completely different dimension: the factor approach.

Rational factor investing does not rely on finding underpriced stocks, but rather seeks to identify factors which lead to systematic risks which investors are unwilling to bear without a commensurate reward. It does not therefore require an ability to pick stocks by processing information in a better way than the market. Rather, it tries to identify risk factors with a strong economic rationale, and considerable empirical evidence in favour of a positive risk premium. Interestingly, recent research has identified fundamental characteristics that are similar to some of the descriptors of 'quality', namely high profitability and low investment. For example, Asness (2014) notes that quality measures tend to "overlap with the profitability and investment factors". Both these factors have been found to be relevant in explaining the cross section of stock returns. Such factors would be straightforward alternatives to ad-hoc definitions of quality currently used in the asset management industry. The advantage of these factors is that they have been widely documented, extensively tested in the data independently by many academics, and thoroughly explained in terms of the economic mechanisms underlying the associated premia.

In this article, we briefly review the high profitability and low investment factors and the economic rationale that lies behind their capacity to yield risk premia. Firstly, we summarise the empirical evidence of these two factors in explaining stock returns, and the rationale for a risk premium associated with these factors. Next, we construct well-diversified factor tilted portfolios using long-term US data for both factors and examine their risk and return characteristics. Lastly, we investigate if there are diversification benefits to combining the two factor indices.

Is there a premium to high profitability and low investment stocks? Empirical evidence

Several papers in the empirical asset pricing literature aim to find factors that are priced or command a premium in the securities market. Although these findings are consistent with asset pricing theory, which allows for multiple sources of priced systematic risks (see eg, Merton [1973] and Ross [1976]), recent research argues that claims about premia attached to most of the factors are not significant (Harvey et al [2014]). Perhaps, the most consensually accepted factors in academia – besides the market factor (Sharpe [1964]) – are size (Banz [1981] and Fama and French [1993]), value (Fama and French [1993]) and momentum (Carhart [1997]). The acceptance of these factors is attributed to economic explanations and sufficiently robust evidence of the risk premium associated with them over a long history.

More recently, authors have documented profitability and investment as factors which explain a cross-section of stocks returns (see eg, Fama and French [2014], Novy-Marx [2013], Cooper et al [2008] and Titman et al [2004]). Although the authors differ on characteristics that can be used as a proxy for the profitability or investment factor, they present robust evidence that there is a premium associated with these factors. They also emphasise that the profitability or investment factors are not manifestations of other well-documented factors, such as the value factor. For example, Novy-Marx (2013) notes that profitability exhibits negative correlation with the value factor. Similarly, Cooper et al (2008) note that the investment factor is a significant explanatory factor, even after controlling for factors such as value, size and momentum.

In figure 1 we report summary statistics of the five factors documented in Fama and French (2014). In panel A of the table, note that the monthly return on all five factors (market, size, value, profitability and investment) is positive over the last 50 years (July 1963–December 2013) and is statistically significant. Over this period, the average monthly return on the investment and the profitability factor is positive (0.17% and 0.22%) and statistically significant (at a 95% confidence interval), with t-statistics of 2.79 and 3.72.

In panel B, we report correlation between different factors over the last 50 years. Note that the profitability factor has negative correlation with the market (–0.13), size (–0.32) and investment factors (–0.19), whereas it has positive but low correlation with the value factor (0.04). The investment factor has negative correlation with the market (–0.43), size (–0.13) and profitability (–0.19) factors and positive correlation with the value factor (0.71). The low or negative correlation of the profitability and investment factors with other factors underlines the diversification benefit of combining these factors in a portfolio.

Profitability is usually defined by the gross profitability of the firm as the proxy variable. The outperformance of profitable over unprofitable companies has been documented in a recent paper by Novy-Marx (2013), who uses the ratio of gross profit (revenues minus cost of goods sold) to total assets as the measure of profitability. Cohen, Gompers and Vuolteenaho (2002) provide similar evidence showing that – when controlling for book-to-market – average returns tend to increase with profitability. Alternative definitions of profitability are for example the ratio of operating profit to book equity – ie, return on equity (Fama and French [2014]). Gross profitability has the advantage of avoiding reliance on accounting manipulations,

1. Summary statistics of factors

Panel A: Summary statistics					
	Market	**Size**	**Value**	**Profitability**	**Investment**
Average monthly return (in %)	0.50	0.30	0.28	0.17	0.22
t-statistics	2.74	2.33	3.22	2.79	3.72
Panel B: Correlation between factors					
	Market	**Size**	**Value**	**Profitability**	**Investment**
Market	1	0.30	–0.34	–0.13	–0.43
Size		1	–0.16	–0.32	–0.13
Value			1	0.04	0.71
Profitability				1	–0.19
Investment					1

Source: Fama and French (2014). Panel A of the table reports the average of monthly factor returns and their t-statistics. The market factor is the return on all sample stocks minus the one-month US treasury bill rate. The size, value, profitability and investment factors are created as returns on small minus large capitalisation portfolios, high minus low book-to-market portfolios, high minus low operating profitability portfolios and low minus high asset growth portfolios, respectively. The value, profitability and investment factors are constructed after controlling for size. All the portfolios are value weighted. The period and sample for analysis is July 1963 to December 2013 and the firms are incorporated in the US and listed on NYSE, AMEX or NASDAQ. Panel B reports correlation between the five factors. We refer readers to Fama and French (2014) for a detailed description of the construction of the five factors presented here.

as gross profits are a very high-level measure of profits.

Investment is usually defined as the growth rate of total assets of the firm (Fama and French [2014]). Cooper et al (2008) show that a firm's investment, characterised by the one-year total asset growth rate, determines its stock returns. In their analysis, low investment firms generate about 8% annual outperformance over high investment firms. A negative relation between investment and stock returns is also documented by Xing (2008), Lyandres, Sun and Zhang (2008), and Titman, Wei and Xie (2004). Ahroni, Grundy and Zeng (2013) show that even when controlling for profitability and book-to-market there is a negative relationship between investment and returns.

The empirically observed effects of investment and profitability have led other researchers to integrate these factors in multi-factor models of asset returns. Fama and French (2014) recently introduced a five-factor model which adds investment and profitability factors to their well-known three-factor model (containing the market, value and size factors). They find that this augmented model improves explanatory power for the cross-sectional variation in expected returns. Similarly, Hou, Xue and Zhang (2014) tested a four-factor model containing the market, size, profitability and investment factors and find that it is superior to the Fama and French three-factor model in explaining cross sectional return patterns and profits to many well-known profitable equity trading strategies. In the following section, we present the economic rationale behind the investment and profitability factors.

Why should the investment and profitability premia persist? Economic rationale

Asset pricing theory suggests that a factor is positively rewarded if and only if it performs poorly in bad times but more than compensates in good times, resulting in positive excess return over the entire market cycle – ie, when marginal utility is high (see eg, Cochrane [2000]). While substantial empirical evidence is a necessary condition for considering that a reward for a certain factor exposure exists, it is not sufficient. In fact, a widely-documented premium may disappear after it is being exploited by an increasing number of investors.

For the profitability and investment factors, several authors have provided an economic rationale for the existence of a risk premium. In fact, it is interesting to note that the premium for the profitability and investment factors can be explained directly using risk-based pricing, rather than using a behavioural explanation such as in the case of a premium associated with accruals (Sloan [1996]). Hou, Xue and Zhang (2014) argue that, since the investment and profitability factors should influence expected returns according to economic theory[1], using these factors "is less subject to the data-mining critique than the Fama-French model" – ie, the value and size factors. Two explanations suggesting a role for these factors are summarised below:

Dividend discount model

Fama and French (2006) derive the relationship between book-to-market ratio, expected investment, expected profitability and expected stock returns from the dividend discount model, which models the market value of a stock as the present value of expected dividends. The dividend discount model, together with a set of accounting identities, lead to the following three predictions:

- Controlling for expected earnings and expected changes in book equity, high book-to-market implies high expected returns;
- Controlling for book-to-market and expected growth in book equity, more profitable firms (firms with high earnings relative to book equity) have higher expected returns;
- Controlling for book-to-market and profitability, firms with higher expected growth in book equity (high reinvestment of earnings) have low expected returns.

The second and third predictions of the dividend discount model mentioned above justify the profitability and investment premium – ie, high return on profitable firms compared to less profitable firms and high return on low investment firms compared to high investment firms.

Production-based asset pricing

Hou, Xue and Zhang (2014) provide a more detailed economic model where profitability and investment effects arise in the cross section due to firms' rational investment policies (see also Liu, Whited and Zhang [2009]).

Concerning the explanation of high returns for low investment firms, it is useful to recall that a firm's optimal investment decision satisfies the first order condition that the marginal benefit of investment discounted to the current date should equal the marginal cost of investment. Put differently, the investment return (defined as the ratio of the marginal benefit of investment to the marginal cost of investment) should equal the discount rate. This optimality condition means that the relationship between investment and expected returns is negative: if expected investment is low, expected returns are high. Intuitively (given expected cash flows), firms with high cost of capital (and thus high expected returns) will have difficulty finding many projects with positive net present value (NPV) and will thus not invest a lot.

Concerning the explanation of the link between high profitability and high expected stock returns, it should be noted that the optimality condition further implies a positive relationship between profitability and expected returns. High profitability (ie, high expected cash flow relative to equity) at a given level of investment implies a high discount rate. Intuitively, if the discount rate were not high enough to offset the high profitability, the firm would face many investment opportunities with positive NPV and thus invest more by accepting less profitable investments.

Performance of smart factor indices for low investment and high profitability tilts

We construct factor-tilted portfolios, called smart factor indices, to extract the factor premia most efficiently using the two-step Smart Beta 2.0 process: 1) explicitly selecting appropriate stocks for the desired beta and 2) using a diversification-based weighting scheme (Amenc and Goltz [2013]). While various definitions of profitability exist in the literature, we use the gross profitability definition from Novy-Marx (2013) due to its parsimony and independence from accounting manipulations. For the low investment factor, we follow the factor definition used in the five-factor regression model developed by Fama and French (2014). The 50% of stocks with the lowest asset growth (past two-year growth rate of total assets) score are selected as low investment and the 50% of stocks with the highest gross profitability (gross profit/total assets) score are selected as high profitability stocks. Diversified multi-strategy weighting is applied to each stock selection.[2] The reasons for selecting 50% of stocks, instead of a stronger tilt such as 30%, are threefold. Firstly, a broader stock universe provides more room for diversification. Also, it allows for another level of independent screening such as 'highly liquid' stocks for investors who have liquidity constraints. Lastly, it ensures that there are sufficient stocks in the universe to accommodate constraints such as geographical neutral or sector neutral constraints in a long-only portfolio, in case they are desired by investors.

Figure 2 summarises the performance of portfolios tilted towards low investment and high profitability stocks. The tilted cap-weighted portfolios beat the broad cap-weighted (CW) benchmark, which shows that both low investment and high profitability factor tilts are rewarded in the long term. It is remarkable that the multi-strategy weighting scheme not only improves performance over tilted cap-weighted portfolios, but also does it with lower volatility.

2. Performance of portfolios tilted towards low investment and high profitability stocks

	Scientific Beta USA Long-Term Track Records				
Dec 1974–Dec 2014 (40 years)	All stocks	Low investment		High profitability	
	CW	CW	Multi-strategy	CW	Multi-strategy
Annual returns	12.16%	13.96%	16.05%	12.63%	15.49%
Annual volatility	17.12%	15.96%	15.34%	17.06%	15.95%
Sharpe ratio	0.41	0.55	0.71	0.44	0.65
Maximum drawdown	54.53%	53.38%	53.20%	52.29%	48.28%
Annual relative returns	–	1.80%	3.89%	0.47%	3.33%
Tracking error	–	3.85%	5.44%	3.34%	4.39%
Information ratio	–	0.47	0.72	0.14	0.76
Outperformance probability (one-year)	–	61.54%	71.86%	51.23%	70.58%
Outperformance probability (three-year)	–	75.21%	81.16%	58.59%	82.35%
95% tracking error	–	6.89%	10.06%	6.75%	7.58%
Maximum relative drawdown	–	26.47%	38.49%	20.27%	25.21%

All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). The Scientific Beta LTTR US universe consists of the 500 largest US stocks. Probability of outperformance is the probability of obtaining positive excess returns if one invests in the strategy for a period of three years (or one year) at any point during the history of the strategy. Rolling window of length three years (or one year) and a step size of one week are used. 95% tracking error is the 95th percentile of the one-year rolling tracking error computed using a one-week step size. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of strategy index to benchmark index.

1 Although a unified economic theory (investment-based asset pricing theory) can be used to explain the premium to value, as well as to the profitability and investment factors, the stocks that are selected using high book-to-market (or value firms), high profitability and low investment criteria are not the same (for example, highly profitable firms tend to have low book-to-market ratios and are growth firms).

2 Diversified multi-strategy weighting is an equal-weighted combination of the following five weighting schemes: maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. For more information on the weighting scheme, please refer to the white paper, Scientific Beta Diversified Multi-Strategy Index by Badaoui and Lodh (2013).

3. Performance indicators for low investment and high profitability factors (40 years)

		Scientific Beta USA Long-Term Track Records Multi-Strategy			
Dec 1974–Dec 2014 (40 years)	All stocks	Standard		Highly liquid selection	
	CW	Low investment	High profitability	Low investment	High profitability
Effective number of stocks	117	190	201	117	124
Capacity ($m)	50,222	11,108	15,221	15,744	22,533
Annual one-way turnover	2.68%	31.70%	22.21%	33.92%	24.08%
Information ratio	–	0.72	0.76	0.67	0.64
Annual relative returns	–	3.89%	3.33%	3.29%	2.54%
Net relative returns (20bps)	–	3.83%	3.29%	3.22%	2.49%
Net relative returns(100bps)	–	3.58%	3.11%	2.95%	2.29%

All statistics are annualised. The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). The Scientific Beta LTTR US universe consists of the 500 largest US stocks. The effective number of stocks (ENS) is defined as the reciprocal of the Herfindahl index, which is defined as the sum of squared weights across portfolio constituents. Mean capacity is the weighted average market capitalisation of the index in $m. Reported turnover is one-way annualised. Net relative returns are the relative returns after accounting for transaction costs (one-way turnover * 20 basis points (or 100 basis points)).

4. : Performance indicators and comparisons for low investment and high profitability factors (10 years)

		Scientific Beta USA			Competitors USA		
Dec 2004–Dec 2014 (10 years)	S&P 500 index	Low investment multi-strategy	High profitability multi-strategy	Custom EW combinations	MSCI USA Quality index	Russell 1000 Quality HEFI	S&P 500 High Quality Rankings
Annual returns	7.65%	10.88%	10.59%	10.66%	9.05%	9.50%	7.79%
Annual volatility	20.39%	18.81%	18.75%	18.92%	18.12%	19.34%	20.52%
Sharpe ratio	0.30	0.50	0.49	0.49	0.42	0.42	0.31
Maximum drawdown	55.25%	50.82%	47.58%	49.60%	44.03%	48.61%	57.68%
Annual relative returns	–	3.23%	2.94%	2.76%	1.40%	1.85%	0.14%
Tracking error	–	3.72%	4.45%	3.16%	4.65%	3.12%	4.81%
Information ratio	–	0.87	0.66	0.87	0.30	0.59	0.03
Outperformance probability (three-year)	–	100.00%	91.80%	99.45%	83.33%	90.71%	62.84%

All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The S&P 500 index is used as the broad cap-weighted benchmark. The analysis is based on daily total return data from 31 December 2004 to 31 December 2014 (10 years). The EW customised index is an equal-weighted combination of the Scientific Beta US Low Investment Multi-Strategy and Scientific Beta US High Profitability Multi-Strategy indices, rebalanced quarterly. The Scientific Beta US universe consists of 500 stocks. Probability of outperformance is the probability of obtaining positive excess returns if one invests in the strategy for a period of three years at any point during the history of the strategy. Rolling window of length three years and a step size of one week are used.

◄ Consequently, both low investment and high profitability smart factor indices deliver Sharpe ratios in excess of 0.55, compared to a mere 0.41 for the broad CW benchmark.

Since smart factor indices are constructed on half universes, their tracking error is substantial. However, information ratios of more than 0.72 imply that the indices are quite well compensated for this deviation from the benchmark. Like any other risk factor, the low investment and high profitability factors also experience relative drawdown. To access the robustness of these smart factor indices, we compute an 'outperformance probability', which is defined as the empirical frequency of outperforming the cap-weighted reference index over a given investment horizon. Both indices achieve more than 80% outperformance probability for a three-year investment horizon, indicating high consistency.

Investability of low investment and high profitability smart factor indices

Turnover rules and liquidity rules are applied to smart factor indices to overcome problems of high turnover and limited capacity, an issue which is usually cited in the case of factor investing. A conditional rebalancing approach is used such that it avoids unnecessary rebalancing unless a significant amount of new information has been received since the last index rebalancing, hence avoiding rebalancing due to noise. The capacity constraints allow us to manage the deviations from the cap-weighted reference index in terms of individual stock weights both at the trading and the holding levels.[3]

To foster more liquidity, investors have the option of making a highly liquid stock selection on top of the existing factor-tilted selection. These indices are constructed on the 70% most liquid stocks among the selected stocks and are called high liquidity smart factor indices. Figure 3 shows that the capacity of standard smart factor indices is sufficiently high at $11.1bn and $15.2bn compared to $50.2bn for the broad CW index. The 'highly liquid' filter improves capacity drastically and has little impact on performance or turnover. Two levels of transaction costs are used: 20bps per 100% one-way turnover and 100bps per 100% one-way turnover.[4] The excess returns net of transaction costs are still quite significantly high.

Combining low investment and high profitability factor tilts

Many commercial indices marketed under the umbrella of 'quality' indices do not make a distinction between the low investment and high profitability factors. Instead they use a composite of a wide range of scores. Most of them do not comply with either factor.[5] Figure 4 shows the benefit of a quarterly-rebalanced equal-weighted combination of the two smart factor indices and compares it with the standalone results of its component indices, but also to various 'quality' indices from well-known index providers over a 10-year period.

It should be noted that competitors mix a systematic approach to stock picking (alpha) with criteria used to define beta. For example, scoring stocks by a combination of return on equity (ROE) score and earnings variability score could be a good criterion if the objective is stock picking. However, reward for systematic risk does not exist for a combination of scores, meaning that making a stock selection based on a composite score does not tilt the portfolio to either beta. It selects stocks that are ranked moderately in both scores and therefore do not necessarily represent either systematic risk. This approach is therefore confusing and does not capture all possible risk premia, an example being the absence of any variable that explicitly provides exposure to the investment factor.

The competing 'quality' indices use a quality score to weight the portfolio in a variety of ways, ranging from using the quality score to tilt market-cap portfolios (MSCI's approach), converting quality scores into active weights using a probability algorithm (Russell's approach) to weighting stocks in proportion to their quality scores (S&P's approach). Irrespective of the definition of quality, ignoring stock correlations in the weighting results in a less-diversified portfolio, which in turn results in inferior performance. This is the reason why both the low investment and high profitability multi-strategy portfolios outperform the three competing indices. The combination of high profitability and low investment factor indices results in tracking error reduction, leading to an information ratio of 0.87, which is higher than or equal to the information ratio of either component. This suggests that combining the two factor tilts results in a diversification benefit, which naturally occurs with any set of separate factors which have low correlation with each other.

Compared to other commercially available indices, the high profitability and low investment combination displays a higher excess return (2.76%) and similar or lower volatility, resulting in a higher Sharpe ratio (0.49) over the 10-year period under analysis. From a relative risk perspective as well, this index displays better risk-adjusted performance as evidenced by its higher information ratio and outperformance probability for the 10-year sample period analysed.

Conclusion

Recent empirical studies document the role of two separate factors related to balance sheet characteristics: low investment and high profitability. These factors rely on straightforward and parsimonious indicators, and can be expected to provide more robust performance benefits

3 The target turnover for smart factor indices is 30% one-way annual. For more information on turnover and liquidity rules, please refer to the white paper, Overview of Diversification Strategies by Gonzalez and Thabault (2013).

4 The first case corresponds to the worst case observed historically for the large and mid-cap universe of our indices, while the second case assumes 80% reduction in market liquidity and a corresponding increase in transaction costs.

5 The MSCI Quality index uses ROE, debt-to-equity and earnings variability; the Russell Quality HEFI Index uses ROA, debt-to-equity and earnings variability; and the S&P 500 High Quality Ranking index uses growth and stability of earnings and recorded dividends to compute the quality score of stocks.

than ad-hoc stock picking indicators of 'quality' used in the industry. In fact, using these factors which are documented by independent research, avoids the risk of data-mining inherent in ad-hoc stock ranking methods. The performance of factor indices aiming to capture the high profitability and low investment premium can be improved by the use of a diversification based weighting scheme such as diversified multi-strategy weighting. Further value can be added by allocating across these two factors to exploit the low correlation across factor returns. Such combinations of the smart factor indices for high profitability and low investment have led to improved performance compared to various commercial indices which are based on ad-hoc definitions of 'quality'.

References

Ahroni, G., B. Grundy and Q. Zeng (2013). Stock Returns and the Miller Modigliani Valuation Formula: Revisiting the Fama French Analysis. *Journal of Financial Economics* 110: 347–57.

Amenc, N. and F. Goltz (2013). Smart Beta 2.0. *Journal of Index Investing* 4(3): 15–23.

Asness, C. (2014). Our Model Goes to Six and Saves Value from Redundancy. AQR Capital Management, available at : https://www.aqr.com/cliffs-perspective/our-model-goes-to-six-and-saves-value-from-redundancy-along-the-way

Badaoui, S. and A. Lodh (2014). Scientific Beta Diversified Multi-Strategy Index. ERI Scientific Beta White Paper.

Banz, R. W. (1981). The Relationship between Return and Market Value of Common Stocks. *Journal of Financial Economics* 6: 3–18.

Carhart, M. (1997). On Persistence in Mutual Fund Performance. *Journal of Finance* 52(1): 57–82.

Cochrane, J. (2000). *Asset Pricing*. Princeton University Press.

Cohen, R. B., P. A. Gompers and T. Vuolteenaho (2002). Who Underreacts to Cash-Flow News? Evidence from Trading between Individuals and Institutions. *Journal of Financial Economics* 66: 409–62.

Cooper, M. J., H. Gulen and M. J. Schill (2008). Asset Growth and the Cross Section of Stock Returns. *Journal of Finance* 63(4): 1609–1651.

Fama, E. F., and K. R. French (2014). A Five-Factor Asset Pricing Model. Working paper.

Fama, E. F., and K. R. French (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33: 3–56.

Gonzalez, N. and A. Thabault (2014). Overview of Diversification Strategies. ERI Scientific Beta White Paper.

Harvey, C. R., Y. Liu and H. Zhu (2014). ... and the Cross-Section of Expected Returns. Fuqua School of Business; National Bureau of Economic Research (NBER) working paper.

Hou, K., C. Xue and L. Zhang (2014). Digesting Anomalies: An Investment Approach. *Review of Financial Studies* forthcoming.

Liu, L., T. Whited and L. Zhang (2009). Investment-Based Expected Stock Returns.

Lyandres, E., L. Sun and L. Zhang (2008). The New Issues Puzzle: Testing the Investment-Based Explanation. *Review of Financial Studies* 21(6): 2825–55.

Merton, R. C. (1973). An Intertemporal Capital Asset Pricing Model. *Econometrica* 41(5): 867–87.

Novy-Marx, R. (2013). The Other Side of Value: the Gross Profitability Premium. *Journal of Financial Economics* 108: 1–28.

Ross, S. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory* 13(3): 341–360.

Sharpe, W. F. (1964). Capital Asset Prices – a Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance* 19(3): 425–42.

Sloan, R. G. (1996). Do Stock Prices Fully Reflect Information in Accruals and Cash Flows about Future Earnings? *Accounting Review* 71(3): 289–315.

Titman, S., K. C. J. Wei and F. Xie (2004). Capital Investments and Stock Returns. *Journal of Financial and Quantitative Analysis* 39: 677–700.

Xing, Y. (2008). Interpreting the Value Effect through the

The relationship between the value and quality factors

Noël Amenc, Professor of Finance, EDHEC-Risk Institute, CEO, ERI Scientific Beta; Kumar Gautam, Quantitative Equity Analyst, ERI Scientific Beta; Felix Goltz, Head of Applied Research, EDHEC-Risk Institute, Research Director, ERI Scientific Beta

The value factor is one of the most consensual and most widely-documented factors. There is ample evidence and numerous theoretical explanations suggesting that tilting an equity portfolio towards low valuation stocks allows above-market returns to be harvested. Traditionally, the value factor was seen as one factor among others that existed, while being distant from the latter factors. This is the case most notably in the Fama and French three-factor model or the Carhart four-factor model, where the value factor coexists with the size and momentum factors. While size and momentum are not uncorrelated with value, they do not create a dramatic overlap or question the role of value as a factor on its own right. Quite to the contrary, it has been shown that the momentum factor takes on the role of a diversifier of value-tilted portfolios with both factor tilts suitably complementing each other (see Asness, Moskowitz and Pedersen [2013]).

However, recent research in empirical finance has come up with new multi-factor models, augmenting the above-mentioned models with additional factors based notably on firms' investment decisions. These new factors are notably the low investment factor and the high profitability factor. More recently, authors have documented profitability and investment as factors which explain a cross-section of stocks' returns and presented robust evidence that there is a premium associated with these factors. The empirically-observed effects of investment and profitability have led other researchers to integrate these factors in multi-factor models of asset returns. Fama and French (2014) recently introduced a five-factor model which adds investment and profitability factors to their well-known three-factor model (containing the market, value and size factors). They find that this augmented model improves explanatory power for the cross-sectional variation in expected returns. Similarly, Hou, Xue and Zhang (2014) tested a four-factor model containing the market, size, profitability and investment factors and find that it is successful in explaining cross-sectional return patterns and profits for many well-known profitable equity trading strategies.

Given the emergence of these new factor models, discussion among researchers and practitioners has recently turned to the link between the well-known value factor on the one hand, and the profitability and investment factors on the other hand. This article aims to examine this link. In particular, a common question investors may have in practice is to ask if value is not redundant with the profitability or investment factors.

We should first note that using a value tilt does not mean taking a view on the best proxy for a true factor model of asset pricing. Factor-tilted indices for various factors allow investors to tilt towards – or, instead, away from – a large number of commonly-employed factors. Of those factor tilts, some can be expected to be rewarded in the long term, based on empirical evidence and economic rationale. Among these rewarded tilts, one can notably list the low-size tilt, value tilt, high-momentum tilt, low-volatility tilt, low-investment tilt and profitability tilt. Of course, these tilts are not entirely uncorrelated, and the empirical literature documenting the long-term premia on these factors has never argued that they are entirely uncorrelated. For investors who wish to harvest a premium associated with these factor tilts, the fact that they may be – to some extent – correlated does not in any way influence the expected return benefit that they can expect from taking on such tilts. However, from a diversification perspective, having tilts that are highly correlated will lower the benefit of using a multi-factor combination as opposed to a single-factor tilt. Conversely, having factor tilts with very low correlation will increase the diversification benefits of using multiple factors rather than a single factor. Therefore, from a diversification perspective it ►

is interesting to ask whether factors are – to some degree – correlated or overlapping.

Interestingly, for value and profitability, it has been widely documented that correlation is remarkably low. In that sense, profitability is often prescribed as a factor that combines well with a value factor tilt. See in particular Novy-Marx (2013), who writes: "Because strategies based on profitability are growth strategies, they provide an excellent hedge for value strategies, and thus dramatically improve a value investor's investment opportunity set. In fact, the profitability strategy, despite generating significant returns on its own, actually provides insurance for value; adding profitability on top of a value strategy reduces the strategy's overall volatility." The empirical evidence therefore suggests that high-profitability factor indices can be suitably combined with value factor indices to form multi-factor allocations with considerable diversification benefits.

We provide the following illustrations concerning overlap across different factor tilts. First, we look at the differences in composition between stock selections of the highest profitability versus value stocks (as defined by high book-to-market) and high-dividend-yield stocks. Second, we look at differences in the fundamental metrics of these different stock selections.

In figure 1, we display the time series of the percentage of overlap (as defined by the number of stocks that belong to both selections divided by the total number of stocks that belong to one of the two selections) between the high profitability selection and respectively the low investment, value and high dividend selections, in the Scientific Beta US Universe, between 21 June 2002 (index inception dates) and 19 December 2014 rebalancing dates.

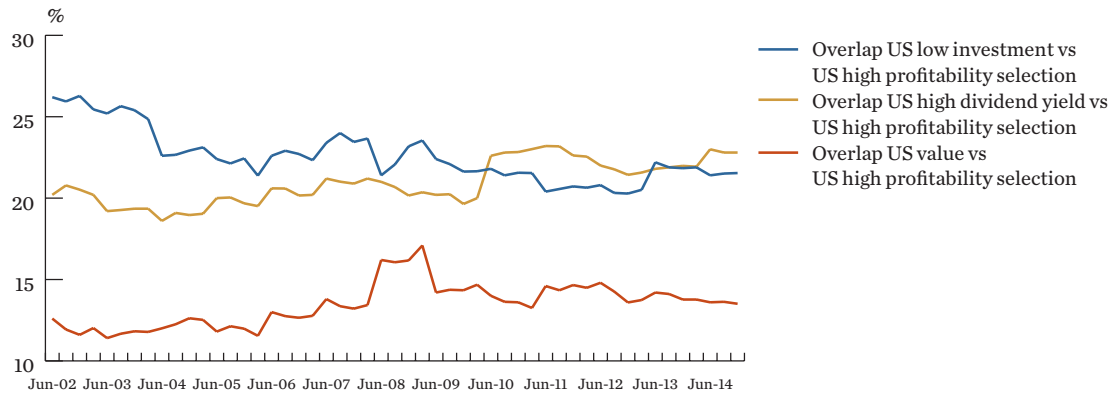
The lowest overlap overall is between the value and high profitability selections, at around 14% on average over the period. The highest overlap is between low investment and high profitability, but it still lies at a reasonable level, with an average of 23% over the period.

In figure 2, we summarise the average fundamental characteristics – price-to-earnings (including and excluding negative earnings), price-to-book, price-to-cash flow, price-to-sales and dividend yield – of maximum deconcentration indices that are based on equally weighting the stocks selected according to, respectively, high profitability, low investment, value and high dividend. Data is as of the December 2014 rebalancing date. We compare them to their SciBeta cap-weighted reference index. Those fundamental ratios are, for some investors, an indication of comparative average valuation level of indices against a reference.

We observe that the high profitability selection-based index exhibits the highest valuation ratios and the lowest dividend yield, compared to other selection-based indices as well as against the cap-weighted reference index. For example, the high profitability index exhibits a price-to-book of 4.6 while other indices exhibit a value of less than 3. This is an indication that the high profitability selection overlaps poorly with the value selection. Also, the high profitability selection exhibits the lowest dividend yield against other indices. We also observe that the low investment selection-based indices exhibit the lowest weighted average dividends yield, against other types of selection.

Overall, these two exhibits clearly illustrate that value-oriented stock selections differ considerably from high-profitability selections, thus providing ample diversification potential between value and high profitability tilts. Moreover, value selections do not appear redundant with low investment selections despite some commonality.

1. Percentage overlap of constituents in high profitability with low investment, value and high dividend yield (time series of the percentage overlap at each rebalancing date)



2. Fundamental metrics of stock selections for US SciBeta indices

Fundamental attribute	SciBeta US High Profitability Max Deconcentration index	SciBeta US Low Investment Max Deconcentration index	SciBeta US Value Max Deconcentration index	SciBeta US High Dividend Max Deconcentration index	SciBeta US Cap-Weighted Reference index
Price/earnings	22.48	19.01	18.53	19.64	19.50
Price/earnings (ex-negative earnings)	21.60	18.30	17.68	18.87	18.91
Price/cash flow	13.91	10.65	9.30	10.16	11.29
Price/sales	1.85	1.40	1.26	1.51	1.82
Price/book value	4.60	2.61	1.88	2.59	2.95
Dividend yield	1.45%	1.97%	2.02%	2.78%	1.92%

What is more, it should be noted that Fama and French (2014) when they develop an extended version with five factors of their three-factor model, include the market, size, value, profitability and investment factors in this new factor model, arguing that it is useful to include all five factors. In line with this research, some investors are using all of these five factors together. However, a widely-quoted result in the Fama and French (2014) paper is that value is a 'redundant factor' in the presence of the other four factors. What this means is that value does not carry any premium which could not be explained by its own exposure to the four other factors. From an investment perspective, this result only confirms that the premium for value should reasonably exist, given that it can be explained by exposure to well-documented rewarded factors. However, redundancy of the value factor would mean that there is not any additional diversification benefit from including value alongside the other four factors. It should be noted that the result derived by Fama and French hinges on two specific characteristics of the set-up they use. First, they exclude the momentum factor from their analysis. Asness (2014) and Asness, Frazzini, Israel and Moskowitz (2015) provide evidence that, if momentum is included in addition to the five factors in the Fama and French (2014) five-factor model, then value is not a redundant factor. This suggests that for an investor who tilts towards momentum, low investment, high profitability and momentum, adding a value tilt will have favourable diversification effects. Second, Asness, Frazzini, Israel and Moskowitz (2015) show that value is only redundant in the five-factor model excluding momentum when using a definition of value which is quite far removed from practical value implementations. In particular, when scoring stocks by a book-to-market ratio that uses the current price and the lagged book value, which corresponds to the approach used in practice by most providers of value indices, the value factor is not redundant relative to the other four factors. However, Fama and French (2014) use a value factor which is based on scoring stocks by a book-to-market ratio, which uses the lagged price coinciding with the end of the fiscal year for which book value is reported, and this same book value. While this approach can be justified from the perspective of matching the date of book value and price, in practical value investing, it arguably does not make much sense to ignore the current price when scoring stocks by valuation metrics. In the end – as Asness, Frazzini, Israel and Moskowitz (2015) write – "the value factor, rendered [...] redundant by the five factor model, is [...] easily resurrected".

Their key result appears clearly in figure 3, extracted from their paper, which is based on US data from 1963 to 2013. RMRF denotes the market factor, SMB the size factor, RMW the profitability factor, CMA the low investment factor, UMD the momentum factor, and HML is the standard Fama and French value factor, while HML-DEV is the more timely value factor. It appears clearly from the results that, in the presence of momentum, the timely HML-DEV factor generates excess returns which are not captured by the other five factors. The unexplained annualised return (Intercept) is 4.87% with a t-stat in excess of four.

Overall, the finding of value's redundancy is not particularly relevant for the practical benefits of using a value tilt. In fact, whether or not value is redundant, there is robust evidence that value investing leads to a long-term premium. Moreover, the finding of redundancy is not robust when including a momentum factor

3. The value factor in alternative multi-factor models

	Intercept	RMRF	SMB	RMW	CMA	UMD	R-squared
HML	–0.48%	0.01	0.02	0.23	1.04	–	52%
	(–0.46)	(0.37)	(0.81)	(5.38)	(23.03)		
HML	0.52%	–0.01	0.03	0.24	1.03	–0.11	54%
	(0.51)	(–0.35)	(1.04)	(5.96)	(23.37)	(–5.92)	
HML-DEV	0.23%	0.06	0.00	–0.02	0.95	–	28%
	(0.15)	(2.04)	(0.11)	(–0.30)	(14.24)		
HML-DEV	4.87%	–0.01	0.03	0.07	0.89	–0.52	68%
	(4.74)	(–0.32)	(1.15)	(1.60)	(20.01)	(–27.31)	

Source: Asness, Frazzini, Israel and Moskowitz (2015)

or using a more practical definition of value. Therefore, in a practical context, value is not even likely to be redundant.

Of course, a discussion of suitable combinations is useful. However, investors may consider taking an agnostic approach on this and consider factor indices for a range of factors, which can be used as suitable building blocks in allocations across various factors. In this sense, the low-investment and high-profitability factors are suitable additions to the investor's menu of factor tilts. These two factors are thoroughly documented as rewarded factors, both in terms of empirical evidence and economic rationale. Moreover, these factors are neither subsumed by other factors such as value and momentum, nor do they make these other factors redundant. Investors can thus choose to combine low-investment and high-profitability-tilted indices with other rewarded factor tilts (such as momentum, value, low size and low volatility) depending on their investment objectives, their constraints and their investment beliefs.

References

Asness, C. S. (2014). Our Model Goes to Six and Saves Value from Redundancy. AQR Capital Management, available at: https://www.aqr.com/cliffs-perspective/our-model-goes-to-six-and-saves-value-from-redundancy-along-the-way

Asness, C. S., T. J. Moskowitz and L. H. Pedersen (2013). Value and Momentum Everywhere. *Journal of Finance* 68: 929–985.

Asness, C. S., A. Frazzini, R. Israel and T. J. Moskowitz (2015). *Facts and Fiction about Value Investing*, working paper.

Fama, E., and K. French (2014). A Five-Factor Asset Pricing Model. Working paper.

Hou, K., C. Xue and L. Zhang (2014). Digesting Anomalies: An Investment Approach. *Review of Financial Studies*, forthcoming.

Novy-Marx, R. (2013). The Other Side of Value: the Gross Profitability Premium. *Journal of Financial Economics* 108: 1–28.

The robustness of smart beta and live performance

Noël Amenc, Professor of Finance, EDHEC-Risk Institute, CEO, ERI Scientific Beta; Ashish Lodh, Deputy Research Director, ERI Scientific Beta; Sivagaminathan Sivasubramanian, Quantitative Analyst, ERI Scientific Beta

There has been significant evidence that systematic equity investment strategies (so-called smart beta strategies) outperform cap-weighted benchmarks over the long run. These strategies are usually marketed on the basis of outperformance. However, it is important to recognise that performance analysis is typically conducted on backtests that apply the smart beta methodology to historical stock returns. Concerning actual investment decisions, a relevant question therefore is how robust the outperformance is.

In general, robustness refers to the capacity of a system to perform effectively in a constantly changing environment. In the context of smart beta strategies, two kinds of robustness need to be taken into account – relative robustness and absolute robustness. A strategy is assumed to be 'relatively robust' if it is able to deliver similar outperformance in similar market conditions. Single factor indices aim to achieve this kind of robustness. Absolute robustness is the capacity of the strategy to deliver risk-adjusted performance in the future to a degree that is comparable to that of the past owing to a well-understood economic mechanism rather than just by chance. Absolute robustness, in other words, is the absence of pronounced state and/or time dependencies and a strategy shown to outperform irrespective of prevailing market conditions can be termed robust in absolute terms.

Potential causes of lack of robustness

Lack of robustness in smart beta strategies can be caused mainly by exposure to four different risks in the strategy construction process – factor fishing and model mining, specific risks, and strong factor dependencies. While the first two issues can have a major influence on relative robustness, the last point is at the heart of the issue of absolute robustness.

Factor fishing and model mining risks as causes for lack of relative robustness

Investors who wish to benefit from factor premia need to address robustness when selecting a set of factors. Harvey et al (2013) document a total of 314 factors with a positive historical risk premium, showing that the discovery of the premium could be a result of data mining – ie, strong and statistically significant factor premia may be a result of many researchers searching through the same dataset to find publishable results. For example, when capturing the value premium one may use extensive fundamental data including not only valuation ratios but also information on, for example, the sales growth of the firm.

While there is an economic rationale for the value factor that is compatible with asset pricing theory, selection of stocks by fundamental data returns to the argument of mispriced or undervalued stocks, which is not based on any theoretical corpus. We perceive that this argument of mispricing for growth tech stocks favours the design of a fundamentals-based strategy after the tech bubble. This kind of weighting scheme consequently gives a sector bias to the strategy and is otherwise not based on any fundamental criterion that is associated with a long-term risk premium.

Therefore, a key requirement for investors to accept factors as relevant in their investment process is that there is a clear economic intuition as to why the exposure to this factor constitutes a systematic risk (Kogan and Tian [2013]). Failure to recognise a suitable proxy for the rewarded factor will harm the relative robustness of the strategy.

Model mining risk is the risk of having an index construction methodology which results in a good track record in backtesting. Many value-tilted indices include a large set of ad-hoc methodological choices, opening the door to data mining.

Exposures to specific risks as cause for lack of relative robustness

All smart beta strategies are exposed to unrewarded strategy-specific risks. Specific risks correspond to all the risks that are unrewarded in the long run, and therefore not ultimately desired by the investor. In line with portfolio theory, among the unrewarded risks we find specific financial risks (also called idiosyn- ►

cratic stock risks) which correspond to the risks that are specific to the company itself. It is this type of risk that asset managers are supposed to be the best at knowing, evaluating and choosing in order to create alpha, but portfolio theory considers it to be neither predictable nor rewarded, so it is better to avoid it by investing in a well-diversified portfolio.

Specific risks can also correspond to important financial risk factors that do not explain, over the long term, the value of the risk premium associated with the index. The academic literature considers for example that commodity, currency and sector risks do not have a positive long-term premium. For example, value strategies often lead to pronounced tilts towards financial sector stocks. During the financial crisis of 2008, exposure to the financial sector proved to be a major determinant of performance of these strategies. It should be noted that the tilt towards the financial sector may not be desired, but it came as a by-product of holding value stocks.

Model-specific risks that are specific to the implementation of the diversification model are also a form of unrewarded risk. As per Modern Portfolio Theory, every investor should optimally combine risky assets so as to achieve the highest possible Sharpe ratio. Implementing this objective, however, is a complex task because of the presence of estimation risk for the required parameters, namely expected returns and covariance parameters. In practice, the costs of estimation error may entirely offset the benefits of optimal portfolio diversification.

Dependency on individual factor exposures as cause for lack of absolute robustness

Systematic risks come from the fact that smart beta strategies can be more or less exposed to particular risk factors depending on the methodological choices guiding their construction (implicit) but also on the universe of stocks supporting this construction scheme (explicit). For example fundamentals-weighted portfolios typically have a value tilt and minimum-volatility strategies exhibit a low-beta tilt (see for example Scherer [2011], Blitz and Swinkels [2008], and Amenc, Goltz and Le Sourd [2008]). Each weighting scheme exposes investors to implicit risk factors which may or may not be consistent with their risk objective. It is important to note that periods of poor performance in all factors are common throughout long-horizon historical tests and the underperformance occurs at different points in time. Therefore investing in a single factor is not a robust approach in absolute terms, as the performance will vary greatly over time across different time periods.

Improving robustness

We propose three ways in which the robustness of various smart beta strategies can be improved.

Avoidance of data or model mining through a consistent framework

A very effective mechanism to avoid data mining is by establishing a consistent framework for smart beta index creation, thus limiting the choices while providing the flexibility needed for smart beta index creation. Consistency in the index framework has two main benefits. First, it prevents model mining by limiting the number of choices through which indices can be constructed. A uniform framework is the best safeguard against post hoc index design, or model mining (ie, the possibility of testing a large number of smart beta strategies and publishing the ones that have good results).

Second, analysis across specification choices is vital because the range of outcomes gives a more informative view than a single specification, which could always have been picked. An index that performs well across multiple specification choices is more robust than an index that performs only in a single specification choice which could very well have been by chance rather than because of the robustness of the strategy. Pre-packaged indices do not allow investors to compare across specifications in order to obtain a view on the sensitivity of performance to index specification choices, thereby exposing investors to a risk of unintended consequences of undesired risks.

Another approach to the inconsistency of the conceptual framework is to look at the evolution or change of methodology over time for the same strategy or the same factor. Some index providers have launched new factor indices when they already had factor indices for the same factor on the market. In this case, the new indices have the same objective as the old ones but different construction principles. This phenomenon has a striking resemblance to the practice of funds or asset managers of creating new funds or changing the strategy of funds in order to overshadow the poor track record of the old fund. Thus, an inconsistent framework over time is also a kind of model mining that allows the index providers to launch new indices with better track records.

Improving relative robustness by reducing unrewarded risks

Relative robustness can be improved by minimising the unrewarded risk as much as possible. There are numerous approaches to estimating risk parameters. The sample estimator of a covariance matrix produces extremely high estimation errors when the ratio of universe size to sample size is large (Kan and Zhou [2007]) – sample risk. One solution to this problem is to reduce the number of parameters to be estimated by imposing a structure on the covariance matrix (Chan et al [1999]). Although this method reduces sample risk, its drawback is that the estimator is biased if the risk model does not conform to the true stock-return-generating process – model risk. The next generation of estimators aims to achieve a trade-off between sample risk and model risk by combining sample estimators and structured estimators (Ledoit and Wolf [2003]). Another way to reduce model risk, and not necessarily at the cost of sample risk, is to use an implicit factor model such as principal component analysis (PCA), especially when implementing PCA while limiting the number of statistical factors using Random Matrix Theory in order to achieve parsimony and robustness (Plerous et al [2002]).

One serious concern with optimisation-based weighting schemes is that the stocks with the highest estimation error may receive the highest weight – a process commonly known as 'error maximisation' – which is detrimental to the relative robustness of the strategies. In practice, various kinds of deconcentration constraints are adopted to improve diversification. Jagannathan and Ma (2003) provided empirical evidence that imposing non-negativity constraints removes large outliers and hence provides better performance through better diversification. Deconcentration constraints ensure sufficiently balanced weights across constituents. DeMiguel et al (2009) introduce flexible quadratic constraints that put limits on the overall amount of concentration in the portfolio (eg, on the sum of squares of portfolio weights) rather than limiting the weight of each stock in the portfolio, thus leaving more room for the optimiser while avoiding concentration overall.

Even though different weighting schemes offer efficient diversification of stocks, there is a need for additional diversification of the weighting schemes to diversify away the strategy-specific risks – a concept called 'diversifying the diversifiers'.[1] The combination of different strategies diversifies risks that are specific to each strategy by exploiting the imperfect correlation between the different strategies' parameter estimation errors. Thus, diversifying the model risks further reduces the unrewarded risks and renders the weighting scheme more robust (in a relative manner).

Improving absolute robustness by diversifying across factors

As discussed before, investors who rely on single factor exposure take the risk of the likelihood of the underlying factor underperforming over short periods. The reward for exposure to these factors has been shown to vary over time (see eg, Harvey [1989], Asness [1992], Cohen, Polk and Vuolteenaho [2003]). If this time variation in returns is not completely in sync for different factors, allocating across factors allows investors to diversify the sources of their outperformance and smooth their performance across market conditions.

Overview: how to improve robustness in smart beta performance

To conclude the section on improvement of robustness, figure 1 summarises how to improve robustness in smart beta performance.

Assessing conditional performance, outperformance probability and live performance

When assessing the robustness of a smart beta strategy one necessarily needs to rely on a conceptual analysis of the strategy design. Purely evaluating performance data will not be conclusive on the question of the degree of robustness of a strategy. For example, a strat-

1. Best practices to improve robustness

Category	Best practices: requirements for robustness	Common practice: risk of a lack of robustness
Methodology	Consistent framework	Ad-hoc methodologies open the door for data mining/model mining
Factor definitions	Simple, tried and tested factors (eg, book-to-price for 'value')	Complex, proprietary and unproven factor definitions (eg, use of proprietary variables, adjustments or constraints)
Weighting scheme	Diversification of model risk and robust risk parameter estimation	Choice of a single weighting model and high sensitivity to input parameters
Transparency	Full transparency: free access to historical constituents and weights and unambiguous ground rules	Opaque and restricted or no access to backtest data with ambiguous ground rules

1 See Timmermann (2006), Kan and Zhou (2007), Tu and Zhou (2011) and Amenc, Goltz, Lodh and Martellini (2012) on the benefits of combining portfolio strategies.

egy that has been derived from extensive data mining may well be performing well in a long-term historical data set, if the time period and data set essentially correspond to those that had been used to design an over-fitted strategy. Even when assessing out-of-sample performance, one might not be able to detect a lack of robustness if the out-of-sample period is relatively short. In fact, over any short time period, a given strategy could generate performance benefits purely due to chance. At the end of the day, what would be needed for a conclusive assessment is long-term live performance, ideally spanning several decades, which simply is not available for any of the commercially-available smart beta indices. This does not mean however that we should not look at performance data to inform our evaluation of robustness. In fact there are essentially two ways in which we can assess robustness by deviating voluntarily from the backtest time frame which may have been used to data-mine a strategy prior to launch. First, we can exploit any reasonably long historical backtrack, and divide it into sub-samples reflecting certain market or factor conditions. Such an assessment specifically uncovers some of the sensitivities of performance to market factors that may be hidden in a longer term backtest average performance result. Second, we can assess robustness by looking at the historical probability of outperforming the cap-weighted reference index over a given investment horizon. This is an intuitive measure to show how often the strategy has managed to outperform the cap-weighted reference index in the past. It is calculated by computing the probability of obtaining positive excess returns if one invests in the strategy for a given time period (eg, three years) at any point during the complete history of the strategy. Third, we can of course assess live performance, even if it is relatively short, to get an idea of a strategy's behaviour in a real investment context on a post-launch basis.

Conditional performance

Figure 2 provides a conditional performance assessment using 10 years of data.

It is clear that the multi-strategy multi-factor indices are much more robust to factor conditions. They tend to deliver outperformance without much dependence on individual factor returns.

The picture is different for Smart Beta 1.0 strategies, which provide implicit exposure to only one or perhaps two factors, thus leading to high sensitivity of performance to factor regimes for individual factors.

Probability of outperformance

Since the performance of smart beta varies over time, the analytics reported over long horizons, for example excess returns over 10 years, have limited information because of averaging over time periods. Probability of outperformance is a measure that overcomes this limitation. The probability of outperformance is defined as the empirical frequency of outperforming the cap-weighted reference index over a given investment horizon. It is an intuitive and relevant measure which shows how often and consistently the strategy would be able to outperform the cap-weighted reference index in the past for all possible entry points. It comes in handy to differentiate between two strategies which have similar long-term performance, although one has small but consistent outperformance while the other benefits from a few periods of high gains combined with long runs of losses. In this example, the former strategy is more robust in an absolute sense and the performance of the latter is disrupted and accompanied by risk.

2. Conditional performance assessment

Developed world 31 Dec 2004–31 Dec 2014	SciBeta Developed MBMS EW	SciBeta Developed MBMS ERC	FTSE RAFI Developed	MSCI World Equal Weighted	MSCI World Minimum Volatility
Bull markets					
Annual relative returns	0.76%	0.99%	2.77%	2.45%	–6.65%
Annual tracking error	2.11%	1.82%	3.96%	3.07%	7.31%
Information ratio	0.36	0.54	0.70	0.80	–0.91
Bear markets					
Annual relative returns	3.24%	2.68%	–3.15%	–2.26%	11.72%
Annual tracking error	3.53%	3.31%	5.03%	4.89%	9.17%
Information ratio	0.92	0.81	–0.63	–0.46	1.28

Conditional performance (over the broad CW benchmark) of the FTSE RAFI 1000 Developed index, MSCI World Equal Weighted index, MSCI World Minimum Volatility index, Scientific Beta Multi-Beta Multi-Strategy (EW) and Scientific Beta Multi-Beta Multi-Strategy (ERC). The analysis is based on daily total return data in US dollars from 31 December 2004 to 31 December 2014 (10 years). All statistics are annualised. The benchmark is the cap-weighted portfolio of all stocks in the investable universe. Data source: Bloomberg and www.scientificbeta.com.

3. Conditional excess returns

Annual excess returns (over CW)	Top 25 quarters by *market* factor returns	Bottom 25 quarters by *market* factor returns
SciBeta Dev Multi-Beta Multi-Strategy (EW)	–1.49%	3.29%
SciBeta Dev Multi-Beta Multi-Strategy (ERC)	–0.56%	2.70%
FTSE RAFI Developed 1000 index	10.78%	–3.86%
MSCI World Equal Weighted index	8.70%	–2.08%
MSCI World Minimum Volatility index	–14.46%	12.34%
Annual excess returns (over CW)	**Top 25 quarters by *SMB* factor returns**	**Bottom 25 quarters by *SMB* factor returns**
SciBeta Dev Multi-Beta Multi-Strategy (EW)	4.48%	0.06%
SciBeta Dev Multi-Beta Multi-Strategy (ERC)	4.56%	–0.58%
FTSE RAFI Developed 1000 index	5.83%	–3.60%
MSCI World Equal Weighted index	9.44%	–4.92%
MSCI World Minimum Volatility index	–2.14%	6.73%
Annual excess returns (over CW)	**Top 25 quarters by *HML* factor returns**	**Bottom 25 quarters by *HML* factor returns**
SciBeta Dev Multi-Beta Multi-Strategy (EW)	1.85%	3.66%
SciBeta Dev Multi-Beta Multi-Strategy (ERC)	3.09%	2.54%
FTSE RAFI Developed 1000 index	13.75%	–6.56%
MSCI World Equal Weighted index	6.09%	–3.51%
MSCI World Minimum Volatility index	–1.89%	11.73%
Annual excess returns (over CW)	**Top 25 quarters by *low volatility* factor returns**	**Bottom 25 quarters by *low volatility* factor returns**
SciBeta Dev Multi-Beta Multi-Strategy (EW)	4.98%	–3.80%
SciBeta Dev Multi-Beta Multi-Strategy (ERC)	3.98%	–2.78%
FTSE RAFI Developed 1000 index	–2.20%	6.59%
MSCI World Equal Weighted index	–1.92%	6.91%
MSCI World Minimum Volatility index	17.44%	–18.89%

Conditional excess returns (over the broad CW benchmark) of the FTSE RAFI 1000 Developed index, MSCI World Equal Weighted index, MSCI World Minimum Volatility index, Scientific Beta Multi-Beta Multi-Strategy (EW) and Scientific Beta Multi-Beta Multi-Strategy (ERC). The quarters are divided into top and bottom 25 percentiles based on returns of the market, HML, SMB and low volatility factors. The SMB factor is the daily return series of a cap-weighted portfolio that is long small-cap stocks and short the 30% largest market cap stocks in the investable universe. The HML factor is the daily return series of a cap-weighted portfolio that is long the 30% highest and short the 30% lowest B/M ratio stocks in the investable universe. The low volatility factor is the daily return series of a cap-weighted portfolio that is long the 30% lowest and short the 30% highest 104-week returns volatility stocks in the investable universe. The analysis is based on daily total return data in US dollars from 31 December 2004 to 31 December 2014 (10 years). All statistics are annualised. The benchmark is the cap-weighted portfolio of all stocks in the investable universe. Data source: Bloomberg and www.scientificbeta.com.

Figure 4 presents the one-year, three-year and five-year probabilities of outperformance of the FTSE RAFI 1000 Developed index, MSCI World Equal Weighted index, MSCI World Minimum Volatility index, Scientific Beta Multi-Beta Multi-Strategy (EW) and Scientific Beta Multi-Beta Multi-Strategy (ERC) for the past 10 years. We can clearly see that the SciBeta MBMS EW and ERC strategies have higher outperformance probabilities than single factor strategies and are thus more robust in delivering consistent outperformance.

4. Outperformance probability

	Outperformance probability 1Y	Outperformance probability 3Y	Outperformance probability 5Y
SciBeta Dev Multi-Beta Multi-Strategy (EW)	77.66%	97.27%	100.00%
SciBeta Dev Multi-Beta Multi-Strategy (ERC)	80.43%%	99.73%	100.00%
FTSE RAFI Developed 1000 index	52.55%	60.38%	67.18%
MSCI World Equal Weighted index	59.15%	48.63%	90.46%
MSCI World Minimum Volatility index	42.55%	77.05%	79.01%

Outperformance probability (over the broad CW benchmark) of the FTSE RAFI 1000 Developed index, MSCI World Equal Weighted index, MSCI World Minimum Volatility index, Scientific Beta Multi-Beta Multi-Strategy (EW) and Scientific Beta Multi-Beta Multi-Strategy (ERC). The analysis is based on daily total return data in US dollars from 12 December 2004 to 12 December 2014 (10 years). It is computed using a rolling window analysis with window length corresponding to the investment horizon (1/3/5 years) and one-week step size. The benchmark is the cap-weighted portfolio of all stocks in the investable universe. Data source: Bloomberg and www.scientificbeta.com.

To conclude, what about live performance?
Many investors consider that smart beta is often sold as a substitute for an active manager, so it seems relevant to look at the indices' live track records too. While it appears difficult to find long-duration live track records for the new generations of smart factors and multi-factors or multi-smart-factor indices, it is possible to appraise the robustness of the first generations of smart beta indices. In figure 5, we present the live performances of four popular smart beta strategies, namely FTSE EDHEC-Risk Efficient US, FTSE RAFI US, S&P EW and MSCI Minimum Volatility US.

The considerable difference in live performance is in our view testimony to the attention paid by the designer of the methodology to offering robust weighting schemes over and above the simulated performance.

5. Performance analysis – FTSE EDHEC-Risk Efficient USA index and its competitors

USA 23 Nov 2009–31 Dec 2014	Broad CW	FTSE EDHEC Risk Efficient	FTSE RAFI	MSCI Min Vol	S&P 500 EW
Annual returns	15.22%	18.43%	16.20%	15.93%	17.79%
Annual volatility	15.82%	16.01%	16.60%	11.92%	17.54%
Sharpe ratio	0.96	1.15	0.97	1.33	1.01
Maximum drawdown	18.58%	19.11%	21.08%	13.98%	22.71%
Annual relative returns	–	3.21%	0.97%	0.71%	2.56%
Tracking error	–	2.64%	2.20%	5.46%	2.85%
Information ratio	–	1.21	0.44	0.13	0.90
95% tracking error	–	3.44%	2.44%	7.74%	3.82%
Maximum relative drawdown	–	4.22%	4.92%	12.04%	6.94%

The table shows the return and risk performance of the FTSE EDHEC-Risk Efficient US index and its competitors: FTSE RAFI US 1000 index, MSCI Minimum Volatility index and S&P 500 Equal Weight index. All statistics are annualised and daily total returns from 23 November 2009 to 31 December 2014 are used. Returns are in US dollars. The Secondary Market US Treasury Bills (3M) is the risk-free rate in US dollars for US. The cap-weighted benchmark is the SciBeta USA CW index. FTSE® is a registered trade mark of the London Stock Exchange Plc and The Financial Times Limited. RAFI® is a registered trademark of Research Affiliates, LLC. MSCI® is a registered trademark of MSCI Inc. S&P® and S&P 500® are registered trademarks of Standard & Poor's Financial Services LLC (S&P), a subsidiary of The McGraw-Hill Companies, Inc. Source: scientificbeta.com.

References

Amenc, N., F. Goltz and V. Le Sourd (2008). The Performance of Characteristics-based Indices. *European Financial Management* 15(2): 241–278.

Amenc, N., F. Goltz, A. Lodh and L. Martellini (2012). Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance, *Journal of Portfolio Management*.

Asness, C. (1992). Changing Equity Risk Premia and Changing Betas over the Business Cycle and January. University of Chicago working paper.

Blitz, D. and L. Swinkels (2008). Fundamental Indexation: An Active Value Strategy in Disguise. *Journal of Asset Management* 9(4): 264–269.

Chan, L. K. C., J. Karceski and J. Lakonishok (1999). On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model. *Review of Financial Studies* 12(5): 937–974.

Cohen, R. B., C. Polk and T. Vuolteenaho (2003). The Value Spread. *Journal of Finance* 58(2): 609–642.

DeMiguel, V., L. Garlappi, J. Nogales and R. Uppal (2009). A Generalized Approach to Portfolio Optimization: Improving Performance By Constraining Portfolio Norms. *Management Science* 55(5): 798–812.

Harvey, C. R. (1989). Time-Varying Conditional Covariances in Tests of Asset Pricing Models. *Journal of Financial Economics* 24: 289–317.

Harvey, C., Y. Liu, and H. Zhu (2013). … and the Cross-Section of Expected Returns. Duke University working paper.

Jagannathan, R. and T. Ma (2003). Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *Journal of Finance* 58(4): 1651–1684.

Kan, R. and G. Zhou (2007). Optimal Portfolio Choice with Parameter Uncertainty. *Journal of Financial and Quantitative Analysis* 42(3): 621–656.

Kogan, L., and M. Tian (2013). Firm Characteristics and Empirical Factor Models: A Data-Mining Experiment. MIT working paper.

Ledoit, O. and M. Wolf (2003). Improved Estimation of the Covariance Matrix of Stock Returns with an Application to Portfolio Selection. *Journal of Empirical Finance* 10: 603–621.

Plerous, V., P. Gopikrishnan, B. Rosenow, L. Amaral, T. Guhr and H. Stanley (2002). Random Matrix Approach to Cross Correlations in Financial Data. *Physical Review* E65(6): 066126.

Scherer, B. (2011). A Note on the Returns from Minimum Variance Investing. *Journal of Empirical Finance* 18(4): 652–660.

Timmermann, A. (2006). Forecast Combinations. In Elliott, G., C.W.J. Granger and A. Timmermann (eds), *Handbook of Economic Forecasting*, 135–196. Amsterdam: North Holland.

Tu, J. and G. Zhou (2011). Markowitz meets Talmud: A Combination of Sophisticated and Naive Diversification Strategies. *Journal of Financial Economics* 99(1): 204–215.

Back to basics: why diversification matters when constructing factor indices

Noël Amenc, Professor of Finance, EDHEC-Risk Institute, CEO, ERI Scientific Beta; Felix Goltz, Head of Applied Research, EDHEC-Risk Institute, Research Director, ERI Scientific Beta; Sivagaminathan Sivasubramanian, Quantitative Analyst, ERI Scientific Beta

Factor index construction: considering alternative approaches

With recent developments in risk-factor-based investing, many index providers, and more generally investment product providers, offer strategies that help investors to gain exposure to various identified risk factors, such as value, momentum, and size, amongst others. While there is a consensus on the factors that are rewarded over the long term, it must be acknowledged that the implementation of factor investing, notably in the long-only universe, is not subject to the same consensus.

Figure 1 on page 12 summarises the alternative implementation approaches, namely concentrated or well-diversified factor indices, also termed smart factor indices (see Amenc et al [2014]). Concentrated factor indices identify stocks that have a pronounced factor tilt for a given factor and aim to obtain strong exposure to this factor through a stock selection that is often restrictive, resulting in relatively few securities in the portfolio in terms of the nominal number of stocks. Moreover, the weighting scheme applied to the stock selection is either market cap-weighting or score-based weighting, resulting in a very uneven distribution of weights. Therefore, the effective number of stocks in the portfolio will also be low. The idea behind this approach is to maximise over the long term the return associated with the strongest exposure possible to the rewarded risk factor. ►

Smart factor indices implement a relatively mild stock selection, where stocks with above average exposure for a given factor are retained. In a second step, these stocks are weighted by a combination of diversification-based methods which aim to create a well-balanced portfolio in terms of weights and risks. The idea behind this approach is to reconcile the exposure to the right factor with avoidance of excessive portfolio concentration. Poor portfolio diversification exposes the investment to risks of excessive volatility over the short and medium term.

The objective of this article is to compare the results of smart factor indices with several stylised examples of concentrated factor indices. Before turning to the empirical comparison, a number of conceptual considerations are in order.

Products that aim to capture explicit risk-factor tilts through concentrated portfolios effectively neglect adequate diversification. This is a serious issue because diversification has been described as the only 'free lunch' in finance. It allows a given exposure to be captured with the lowest level of risk required. In contrast, gaining factor exposures exposes investors to risk factors, and therefore, such exposures do not constitute a 'free lunch'. They instead constitute compensation for risk in the form of systematic factor exposures. Such capturing of risk premia associated with systematic factors is attractive for investors who can accept the systematic risk exposure in return for commensurate compensation.

However, factor-tilted strategies, when they are very concentrated, may also take on other, non-rewarded, risks. Non-rewarded risks come in the form of idiosyncratic or firm-level risk, as well as potential risk for sector concentration. Financial theory does not provide any reason why such risk should be rewarded. Therefore, a sensible approach to factor investing should not only look to obtain a factor tilt, but also at achieving proper diversification within that factor tilt. To illustrate this point, we focus on the value factor as an example below, but the discussion carries over to other factors too.

In fact, if the objective was to obtain the most pronounced value tilt, for example, the only unleveraged long-only strategy that corresponds to this objective is to hold 100% in a single stock, the one with the largest value tilt, as measured for example by its estimated sensitivity to the value factor or its book-to-market ratio. This thought experiment clearly shows that the objective of maximising the strength of a factor tilt is not reasonable.

Moreover, this extreme case of a strong factor tilt indicates what the potential issues with highly concentrated factor indices are. First, such an extreme strategy will allow the highest possible amount of return to be captured from the value premium, but it will necessarily come with a large amount of idiosyncratic risk, which is not rewarded and therefore should not be expected to lead to an attractive risk-adjusted return. Second, it is not likely that the same stock will persistently have the highest value exposure within a given investment universe. Therefore, a periodically-rebalanced factor index with such an extreme level of concentration is likely to generate 100% one-way turnover at each rebalancing date, as the stock held previously in the strategy is replaced with a new stock that displays the highest current value exposure at the rebalancing date. While practical implementations of concentrated factor-tilted indices will be less extreme than this example, we can expect problems with high levels of idiosyncratic risk and high levels of turnover whenever index construction focuses too much on concentration and pays too little attention to diversification.

Interestingly, the importance of diversification for a given factor tilt was outlined more than 40 years ago in Benjamin Graham's famous book on value investing: "In the investor's list of common stocks there are bound to be some that prove disappointing ... But the diversified list itself, based on the above principles of selection [...] should perform well enough across the years. At least, long experience tells us so." Aiming at a highly-concentrated value portfolio would be completely inconsistent not only with financial theory, but also with the principles put forth by the early advocates of value investing.

"Non-rewarded risks come in the form of idiosyncratic or firm-level risk, as well as potential risk for sector concentration. Financial theory does not provide any reason why such risk should be rewarded"

Cap-weighted portfolios of value stock selections may at first seem to be more neutral implementations than score-weighted portfolios. However, it is well known that cap-weighting has a tendency to lead to very high concentration given the heavy tailed nature of the distribution of market cap across stocks in the same universe. It is well documented in the academic literature that simple cap-weighted value-tilted portfolios have not led to attractive performance. In fact across different studies (see eg, Fama and French [2012] among others), empirical results show that a value strategy needs to be well-diversified to deliver a significant premium. For example, the standard Fama and French value factor includes a broad selection of stocks, and uses a two-tiered weighting approach to obtain better diversification. In particular, the value factor is an equal-weighted combination of sub-portfolios for different market cap ranges, effectively overweighting smaller size stocks and increasing the effective number of stocks. The fact that the most widely cited research documenting the relevance of the value factor does not use simple cap-weighted factors, but rather constructs more balanced portfolios, shows the lack of support for industry practices using simple cap-weighted factor indices. For completeness, we may add that the literature does not use any score-weighted approaches either.

Overall, it thus appears that neither of the approaches that propose to construct concentrated factor indices is supported by the academic literature, or for that matter, by common sense. However, how severe the challenges for concentrated factor index approaches are in practice, is an empirical question which we address below.

Data and methodology: construction of factor-tilted portfolios

We construct a total of 30 portfolios, representing five different tilt-design approaches for each of the six factor tilts – mid cap, momentum, low volatility, value, investment and profitability. We construct four different proxies for concentrated portfolios and compare these portfolios to well-diversified indices that combine the diversified multi-strategy weighting scheme with a 50% stock selection, namely the Scientific Beta smart factor indices for the same six factors. Both for the diversification strategy indices and for the concentrated test portfolios, the factor scores are updated annually.[1] We test these competing approaches based on US long-term data for a 40-year time period from January 1975 to December 2014, where the stock universe consists of the top 500 stocks by market cap. Figure 2 provides an overview of the different strategies we test.

Turnover

As discussed in our thought experiment in the introduction, it is clear that high levels of concentration potentially lead to severe turnover. Turnover will be high especially when the stock selection criteria move fast, leading to pronounced changes in eligible stocks for a given factor tilt from one rebalancing date to the next. Of course, intuition suggests that the turnover will depend on the severity of the stock selection screen.

Figure 3 reports the relative increase in turnover for concentrated portfolios compared to the well-diversified multi-strategy indices for the same factor. The relative increase is calculated on the basis of annualised one-way turnover for the different diversified and concentrated strategies that we assess. One-way annual turnover is defined as

$$\text{1-way ann turnover} = \frac{1}{2}\sum_{i=1}^{n} \text{abs}\left(W_i^t - \widehat{W}_i^{t-1}\right)$$

It is clear from the results in figure 3 that turnover tends to increase dramatically with higher concentration. For example, turnover for the value strategies reaches levels of the order of 40% for both score-weighted and cap-weighted portfolios with a 20% selection screen,

1. Concentrated vs diversified (smart) factor indices

Concentrated factor tilts	● **Do not consider any diversification objective.** Ad hoc weighting schemes such as market cap-weighting or score-weighting are used. ● Often select a very **narrow universe** of stocks with the highest exposure.
Diversified factor tilts	● Use a **smart weighting scheme to ensure sufficient diversification** while respecting liquidity and turnover constraints. ● Use a **reasonably broad universe** of stocks that have above average exposure to the relevant factor.

1 The only exception is the Scientific Beta Momentum Diversified Multi-Strategy index, where the momentum score is updated semi-annually.

2. Construction of test portfolios

	SciBeta diversified multi-strategies	Cap-weighted 50% stock selection	Cap-weighted 20% stock selection	Score-weighted 50% stock selection	Score-weighted 20% stock selection
Universe	US long-term track records				
Number of securities in the universe	500				
Factors analysed	Mid-cap, high momentum, low volatility, value, low investment and high probability				
Benchmark	Broad cap-weighted				
Reselection frequency (rescoring frequency)	Annual				
Reweighting frequency	Quarterly conditional upon turnover rules (on average the rebalancing frequency is 11–13 months)	Systematic annual rebalancing			
Stock selection criteria	Top 50% stocks based on corresponding factor values	Top 50% stocks based on corresponding factor values	Top quintile (20%) stocks based on corresponding factor values	Top 50% stocks based on corresponding factor values	Top quintile (20%) stocks based on corresponding factor values
Weighting scheme	Diversified multi-strategy*	Cap-weighted		Score-weighted *zscore* of the values of the factor proxy variable is taken every year and *sscore* by taking the standard normal cumulative distribution value corresponding to the *zscore*. The *sscore* is used for stock selection and weighting.	

* Note that diversified multi-strategy is an equal weighted combination of five different weighting schemes: maximum deconcentration, maximum decorrelation, maximum Sharpe ratio, minimum volatility and diversified risk-weighted.

while turnover is close to 20% for the strategies based on broader selections. We also observe a tendency for score weighting to lead to higher turnover than cap-weighting for a similar stock selection approach.

These results provide strong support for the problems outlined on a conceptual level above. Narrowing down the stock universe to obtain strong factor tilts severely increases turnover, thus leading to strategies which are more difficult and costly to implement than strategies based on broader selections.

Idiosyncratic risk

Increasing concentration is also expected to lead to an increase in unrewarded, idiosyncratic risk. We assess this issue by regressing returns of the factor strategies onto the standard Carhart (1997) factors including the market, size, value and momentum factor. The idiosyncratic risk is then measured as the volatility of the residual return relative to the systematic return component which results from the factor exposures.

"The evidence suggesting increases in idiosyncratic risk for concentrated indices casts some doubt on their capacity to truly increase risk-adjusted returns even on a before-cost basis"

Figure 4 provides an overview of the estimated idiosyncratic volatility for the different index construction approaches across the six factor tilts.

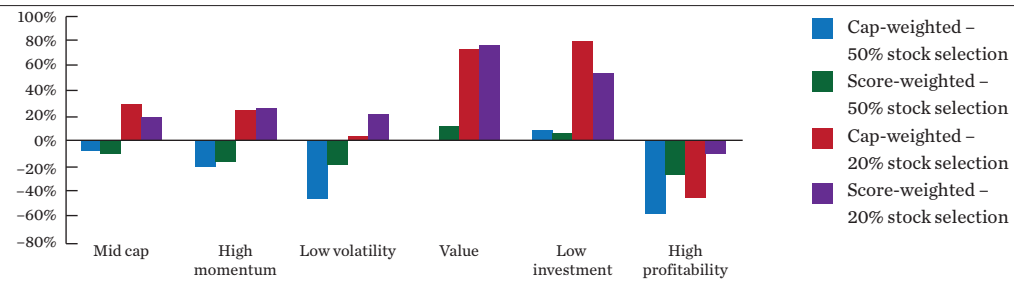
Figure 4 provides strong evidence that idiosyncratic volatility increases with higher concentration. Note that financial common sense suggests that idiosyncratic risk should be diversified away! This confirms that many concentrated indices are often exposed to risks which are not only unrelated to the factor tilts that they are intended to capture but also likely to be unrewarded over the long term.

Performance

The results discussed thus far show that increasing concentration comes with important challenges in terms of higher turnover. Given this additional implementation challenge and potential drag in terms of transaction costs on highly concentrated factor indices, concentrated factor indices would need to lead to marked improvements in performance. An open question would then be whether such performance improvements would lead to any practical improvements on a net-of-cost basis, after taking into account high turnover and ensuing transaction costs.

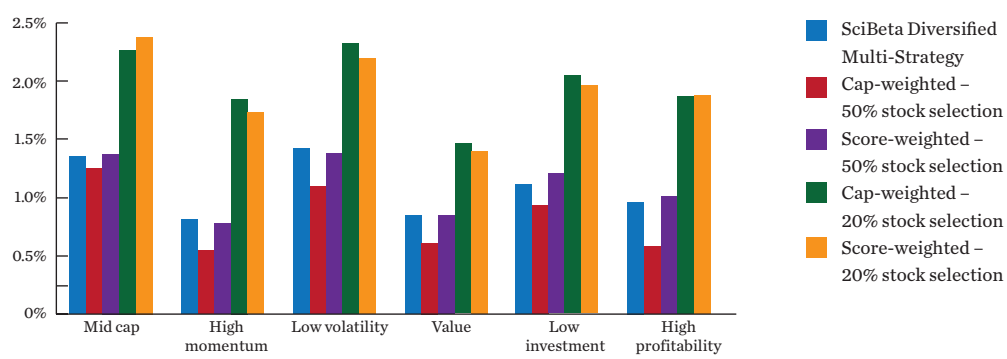
However, the evidence suggesting increases in idiosyncratic risk for concentrated indices casts some doubt on their capacity to truly increase risk-adjusted returns even on a before-cost basis. To verify what the ►

3. Increase in turnover for concentrated factor indices, relative to well-diversified indices



The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). Mid cap, high momentum, low volatility, value, low investment and high profitability selections represent 50%/20% of stocks with such characteristics in a US universe of 500 stocks. The full names of the US indices used are: SciBeta United States Mid-Cap Diversified Multi-Strategy, SciBeta United States High-Momentum Diversified Multi-Strategy, SciBeta United States Low-Volatility Diversified Multi-Strategy, SciBeta United States Value Diversified Multi-Strategy, SciBeta United States Low Investment Diversified Multi-Strategy and SciBeta United States High Profitability Diversified Multi-Strategy. Source: www.scientificbeta.com

4. Idiosyncratic volatility for concentrated and diversified factor indices



The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). Mid cap, high momentum, low volatility, value, low investment and high profitability selections represent 50%/20% of stocks with such characteristics in a US universe of 500 stocks. In order to compute Carhart four-factor exposures and the volatility attribution to these factors – market, size, value and momentum factors for the US universe available online at Kenneth French data library is used. The full names of the US indices used are: SciBeta United States Mid-Cap Diversified Multi-Strategy, SciBeta United States High-Momentum Diversified Multi-Strategy, SciBeta United States Low-Volatility Diversified Multi-Strategy, SciBeta United States Value Diversified Multi-Strategy, SciBeta United States Low Investment Diversified Multi-Strategy and SciBeta United States High Profitability Diversified Multi-Strategy. Source: www.scientificbeta.com.

comparative performance features of the various indices are, we provide an overview on risk-adjusted performance in terms of the Sharpe ratio in figure 5. The results in figure 5 clearly suggest that concentrated indices do not lead to higher Sharpe ratios than multi-strategy indices.

In fact, across the six factor tilts, the performance and risk statistics suggest that – relative to multi-strategy factor indices – highly concentrated factor-tilted approaches (eg, 20% stock selection with cap-weighted or score-weighted), lead to higher volatility, especially for the score-weighted approach (the only exception is the low-volatility tilt). Concentration in the top quintile portfolios does not lead to any consistent improvement in Sharpe ratio either. Multi-strategy indices, on the other hand, owing to better diversification, minimise the unrewarded risks and thus have better Sharpe ratios. On average across the six factors, the multi-strategy indices have a Sharpe ratio of 0.7 compared to, eg, an average Sharpe ratio of 0.56 for the cap-weighted top quintile indices. Therefore the increase in implementation challenges does not lead to any meaningful performance improvement (even before considering implementation costs).

Investability

To further assess the implementation aspects of concentrated and diversified factor indices, we provide an overview of the days to trade at an average rebalancing date for a $1bn investment. In particular, we report as 'days to trade' the number of days necessary to trade the total stock positions, assuming $1bn AUM and that 100% of the average daily dollar traded volume can be traded every day. Due to data availability, the period for this analysis is restricted to the last 10 years of the sample for the Scientific Beta US indices.

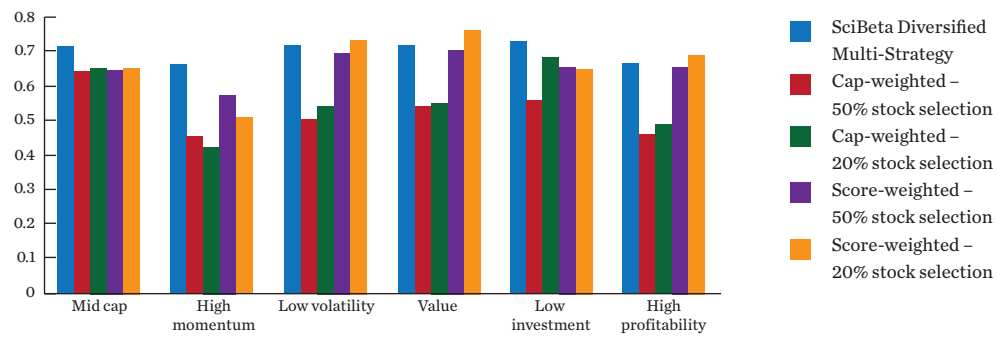
Figure 6 shows days to trade for each of the design approaches across six different factor tilts.

It is clear from figure 6 that 'days to trade' increases tremendously with a rise in concentration. Indeed, it is perhaps not surprising that rebalancing of very concentrated portfolios leads to strong implementation hurdles.

This finding confirms the implementation hurdles that were apparent from the analysis of turnover, but also shows that when considering the available trading volume in stocks whose rebalancing generates this turnover, the implementation problems of concentrated approaches compared to well-diversified approaches actually increase on a relative basis.

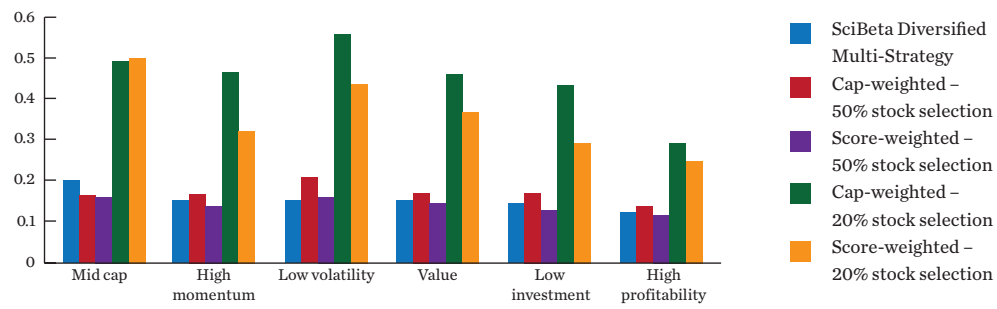
In fact, relative to the multi-strategy indices, and on average across the six factors, the score-weighted top 20% indices have on average more than 30% higher turnover, and more than 100% higher average days to trade. Clearly, concentrated factor tilts pose real challenges of investability.

5. Sharpe ratios for concentrated and diversified factor indices



The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). Mid cap, high momentum, low volatility, value, low investment and high profitability selections represent 50%/20% of stocks with such characteristics in a US universe of 500 stocks. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The full names of the US indices used are: SciBeta United States Mid-Cap Diversified Multi-Strategy, SciBeta United States High-Momentum Diversified Multi-Strategy, SciBeta United States Low-Volatility Diversified Multi-Strategy, SciBeta United States Value Diversified Multi-Strategy, SciBeta United States Low Investment Diversified Multi-Strategy and SciBeta United States High Profitability Diversified Multi-Strategy. Source: www.scientificbeta.com.

6. Days to trade for a $1bn investment for concentrated and diversified factor indices



The analysis is based on weights of total return indices on every rebalancing date for the time period 31 December 2004 to 31 December 2014 (10 years). Mid cap, high momentum, low volatility, value, low investment and high profitability selections represent 50%/20% of stocks with such characteristics in a US universe of 500 stocks. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The broad cap-weighted index based on the 500 largest stocks in the US universe is used as the benchmark. The full names of the US indices used are: SciBeta United States Mid-Cap Diversified Multi-Strategy, SciBeta United States High-Momentum Diversified Multi-Strategy, SciBeta United States Low-Volatility Diversified Multi-Strategy, SciBeta United States Value Diversified Multi-Strategy, SciBeta United States Low Investment Diversified Multi-Strategy and SciBeta United States High Profitability Diversified Multi-Strategy. Source: www.scientificbeta.com. 'Days to trade' is the number of days necessary to trade the total stock positions, assuming $1bn AUM and that 100% of the average daily dollar traded volume can be traded every day. Due to data availability, the period is restricted to the last 10 years of the sample for the Scientific Beta US indices.

Conclusions

Factor indices are a potentially value-adding tool. Investors can expect benefits from relying on indices which tilt towards well-documented factors, which carry sizeable and repeatable return benefits over long investment horizons. However, when aiming to implement the insight from empirical finance that certain factors lead to premia, one should not forget a perhaps even more fundamental insight from financial theory: the idiosyncratic risk of over-concentrating a portfolio is not rewarded. Our results suggest that index construction approaches which build diversified portfolios for a given factor tilt are exposed to less unrewarded risk. Considering these two aspects, factor tilts and diversification, should be an integral part of a sensible factor index design methodology. Moreover, factor indices are indices, and thus should be implementable with ease and low turnover. Our results suggest that increasing concentration leads to high turnover levels and real investability hurdles which are not compensated by any performance advantages.

References

Amenc, N., F. Goltz, A. Lodh and L. Martellini (2014). Towards Smart Equity Factor Indices: Harvesting Risk Premia without Taking Unrewarded Risks. *Journal of Portfolio Management* 40(4): 106–122.

Carhart, M. (1997). On Persistence in Mutual Fund Performance. *Journal of Finance* 52(1): 57–82.

Fama, E. F. and K. R. French (2012). Size, Value, and Momentum in International Stock Returns. *Journal of Financial Economics* 105: 457–472.

What does academic research teach us about factor investing?

Felix Goltz, Head of Applied Research, EDHEC-Risk Institute, Research Director, ERI Scientific Beta

The venerable 'academic grounding'

Equity index products that claim to provide exposure to factors which have been well documented in academic research, such as value and momentum, among others, have been proliferating in recent years. Interestingly, providers across the board put strong emphasis on the academic grounding of their factor indices.[1] It therefore appears useful to analyse what academic research has to say on equity factors to understand what we can learn from such research on designing or evaluating factor indices. When analysing academic publications on equity factor investing, three important lessons emerge, which are addressed in the sections below.

Lesson one: 'Be serious with data'

When establishing which factors carry a reward by way of empirical analysis, it is important to understand that this is an almost daunting task. In fact, since Merton (1980) it is well known that we struggle to estimate expected returns reliably, simply because we rely on very few data points to estimate long-term expected returns: the starting price level and the end date price level. Of course, this is also true for factor returns.

Given this difficulty, when testing whether a factor carries a positive premium, academic research conducts a thorough assessment, including the analysis of very long-term data (covering time spans of at least 40 years), analysis across different regions and asset classes, and various corrections for possible data-mining biases. Importantly, these studies are open to criticism. Numerous papers are written to question previous empirical results (see for example the debate on the 'low volatility puzzle'). For these reasons, academic research is much more capable of providing meaningful conclusions than a product backtest for a given factor index product. Even if a backtest is conducted very thoroughly by a product provider, it is hard to believe that the provider is able to conduct as thorough an analysis of the whole academic community, whose members have strong incentives not only to publish their own results but also to challenge the results of others by way of replicated tests. Therefore, factors which have undergone academic 'validation' constitute a much stronger empirical justification than a mere product backtest.

The first important characteristic of empirical evidence on factor premia, as mentioned above, is that this evidence is derived based on tests applied to long-term data. In fact, studies on US equity data typically span at least 40 years of data, and in many cases, data goes as far back as the 1920s. For the purpose of illustration, figure 1 provides an overview of results obtained on key factors with long-term US data.

A second important characteristic of empirical research on factor premia is the assessment across different regions and asset classes. In fact, merely deriving a result from US data, even if it holds in long-term data, does not allow the findings to be generalised to other geographic or investment contexts. From the standpoint of generalisation, it is therefore interesting if results can be confirmed on equity markets for other geographies or even in entirely different asset classes. Research has made considerable progress in this direction over the past decade, with surprisingly strong confirmation of the US equity results in other investment universes.

A third important precaution empirical research takes before jumping to conclusions on the premium for a given factor is to adjust for data-mining or so-called 'multiple testing'. In fact, standard statistical tests are only valid when we test a given single hypothesis, such as that high book-to-market stocks carry a premium over low book-to-market stocks. However, in practice researchers may run several tests, for example trying out a large number of metrics until they find one that leads to significant results. This is also known as data-snooping or data-mining. To consider why such multiple testing may lead to false inference, consider a simple example. Assume you simulate data for 100 variables (potential 'factors') that have zero mean. You would then expect to find about five variables with mean ('premium') ►

1 For example, consider the following quotes from marketing material of index providers: "MSCI currently identifies six equity risk premia factors... They are grounded in academic research.."; "In developing the Russell High Efficiency Factor Index series ... we ensured that all of our factor specifications were consistent with academic research findings," "The FTSE Global Factor Index Series is ... designed to represent ... factor characteristics for which there is a broad academic consensus"; ERI Scientific Beta: "factor indices are meant to be investable proxies for rewarded factors that have been analysed in the academic literature".

1. US evidence on equity factor premia

Factor	Factor definition	Period	Premium	t-stat	Source
Market	Excess returns of cap-weighted equity index	1926–2008	7.72% (annual)	3.47	Ang et al (2009)
Low risk	Stocks with low versus high risk (beta, volatility or idiosyncratic volatility)	1926–2012	0.70% (monthly)	7.12	Frazzini-Pedersen (2014)
Size	Stocks with low versus high market cap	1926–2008	2.28% (annual)	1.62	Ang et al (2009)
Value	Stocks with high versus low book-to-market	1926–2008	6.87% (annual)	3.27	Ang et al (2009)
Momentum	Stocks with high vs low returns over past 12 months (omitting last month)	1926–2008	9.34% (annual)	5.71	Ang et al (2009)
Profitability	Stocks with high vs low profitability (eg return on equity or gross profitability)	1963–2013	0.17% (monthly)	2.79	Fama-French (2014)
Investment	Stocks with low vs high investment (change in total assets)	1963–2013	0.22% (monthly)	3.72	Fama-French (2014)

2. Empirical evidence for selected factor premia

Factor	US equities	International equities	FCC
Value	Basu (1977); Rosenberg, Reid, Lahnstein (1985); Fama and French (1993)	Fama and French (2012)	Asness, Moskowitz, Pedersen (2013)
Momentum	Jegadeesh and Titman (1993); Carhart (1997)	Rouwenhorst (1998)	Asness, Moskowitz, Pedersen (2013)
Low risk	Ang, Hodrick, Xing, Zhang (2006); Frazzini and Pedersen (2014)	Ang, Hodrick, Xing, Zhang (2009); Frazzini and Pedersen (2014)	Frazzini and Pedersen (2014)
Size	Banz (1981); Fama and French (1993)	Heston, Rouwenhorst, Wessels (1999); Fama and French (2012)	na
Profitability	Novy-Marx (2013); Hou, Zhang, Xue (2014); Fama and French (2014)	Ammann, Odoni, Oesch (2012)	na
Investment	Cooper, Gulen, Schill (2008); Hou, Zhang, Xue (2014); Fama and French (2014)	Watanabe, Xu, Yao, Yu (2013)	na

significantly different from zero. This means that, even though the true mean ('premium') on all of the variables ('factors') is zero in the simulation, the statistical inference will tell you that some of the means are significantly positive, as long as you run enough tests.

In order to adjust for this problem, researchers have come up with tighter requirements for significance levels to take into account the possibilities of multiple testing. For example, Harvey, Liu and Zhu (2015) adjust t-ratios that are used for evaluating the significance of factor premia to take into account the fact that researchers have run many tests across hundreds of factors to document their premia. Interestingly, when applying these methods to standard equity risk factors, researchers find that the main factors, such as value and momentum among others, remain statistically significant.

Despite the thorough evidence supporting the existence of premia for the main factors, there is continuous debate over the set of relevant equity factors. In fact, research often debates whether a factor has disappeared or a new factor has appeared. While questioning the baseline results and discussing relevant actors is obviously useful, investors in practice should be prudent before making abrupt changes to their set of factors or the associated investment beliefs. As mentioned before, the measurement of a risk premium is highly sensitive to the chosen sample (Merton [1980]), and estimates of factor premia are subject to considerable uncertainty. Therefore, any conclusions based on empirical evidence should only be drawn from studying very long time periods, and conducting tests across different datasets. Moreover, any arguments in favour of the disappearance of standard factors or the appearance of new factors should not be investigated based on empirical evidence alone, but should also consider the underlying economic mechanisms, an issue we turn to in the next section.

Lesson two: 'Being serious with data is not enough'

In addition to convincing empirical evidence, the existence of a factor premium should be supported by a compelling economic rationale. Kogan and Tian (2013) make this point prominently when they write: "We should place less weight on the data the models are able to match, and instead closely scrutinize the theoretical plausibility and empirical evidence in favor of or against their main economic mechanisms."

To illustrate why the existence of an economic rationale is an important requirement for considering a factor to be rewarded, it is useful to take the equity market risk premium as an example. From an empirical perspective, the equity risk premium can be statistically indistinguishable from zero even for relatively long sample periods. However, economic reasoning suggests that stocks should have higher reward than bonds. Clearly, even if the premium for holding equity is well documented empirically, investors are reluctant to hold too much equity due to its risks. Similar reasoning can be applied to additional equity risk factors. Instead of focusing only on the empirical evidence, investors' due diligence should look at why there should be a risk premium for a given factor in the first place. In other words, investors should ask what the economic rationale for a factor premium is, to form an opinion on its existence and persistence.

The existence of factor premia can be explained in two different ways – a risk-based explanation and a behavioural-bias explanation. The risk-based explanation premises that the risk premium is compensation to investors who are willing to take additional risk by being exposed to a particular factor. Additional risk exists when assets that correspond to a given factor tilt tend to provide poor payoffs in bad times, thus exposing investors to a risk of losses in times when their economic situation is already poor, their consumption is low, and marginal utility of consumption is high. The behavioural explanation conceives that the factor premia exist because investors make systematic errors due to behavioral biases such as over-reaction or under-reaction to news on a stock.

Whether such behavioural biases can persistently affect asset prices is a point of contention given the presence of smart market participants who do not suffer from these biases. For behavioural explanations to be relevant, it is necessary to assume that – in addition to biases – there are so called 'limits to arbitrage', ie, some market characteristics, such as short-sales constraints and funding-liquidity constraints, which prevent smart investors from fully exploiting the opportunities arising from the irrational behaviour of other investors.

If the risk premium can only be explained by behavioural reasoning, it is expected to disappear in the absence of limits to arbitrage. On the other hand, a risk factor with a strong rational or risk-based explanation is more likely to continue to have a premium in the future. Therefore, it is perhaps more reassuring for an investor to have a risk-based explanation.

We refer to figure 3 for a brief list of risk-based and behavioural explanations of each factor.

Lesson three: 'Be practical'

A common criticism of academic research on factor premia is the supposed impracticality of academic factor definitions, simply because most results in academic research abstract from transaction costs and other implementation issues such as turnover. It is indeed the case that many academic studies do not necessarily aim to consider implementation issues. In fact, product providers often justify deviations from academic factors with implementation needs. But while early studies indeed abstract away from implementation issues, recent academic research addresses this shortcoming. In particular, recent research examines whether the premia to common equity risk factors survive net of transaction costs. Moreover, it assesses whether we can use mitigation strategies to ease implementation when harvesting these premia.

Novy-Marx and Velikov (2014) assess turnover and estimate transaction costs for common factor strategies. They find that the net-of-cost factor premia mostly remain significant. Figure 4 provides a summary of their findings.

In addition to assessing whether the returns to simple strategies are robust to transaction costs, research has tested adjusted implementations of factor premium strategies that try to ease implementation. Novy-Marx and Velikov (2014) test several such mitigation strategies and find that such approaches can substantially ease implementation while sustaining most of the return benefits, which often results in improvements in net of cost factor premia.

Frazzini, Israel and Moskowitz (2012) conduct a similar analysis and find that after taking into account realistic transaction costs, factor premia remain significant, especially when making adjustments to ease implementation: "We measure the real-world transaction costs and price impact function ... and apply them to size, value, momentum, and short-term reversal strategies. [...] Strategies designed to reduce transaction costs can increase net returns and capacity substantially, without incurring significant style drift. We conclude that the main anomalies ... are robust, implementable and sizeable."

Moreover, Amenc et al (2012) provide a clear implementation framework for factor-tilted indices in a long-only context with an aim of providing factor-tilted indices which are not only implementable, but also well-diversified.

3. Economic mechanisms behind main factors

Factor	Risk-based explanation	Behavioural explanation
Value	Costly reversibility of assets in place: high sensitivity to economic shocks in bad times	Overreaction to bad news and extrapolation of the recent past leads to under-pricing
Momentum	High-expected-growth firms are more sensitive to shocks to expected growth	Investor overconfidence and self-attribution bias leads to returns continuation in the short term
Low risk	Liquidity-constrained investors have to sell leveraged positions in low-risk assets in bad times when liquidity constraints become binding	Disagreement of investors about high-risk stocks leads to overpricing due to short-sales constraints
Size	Low liquidity, high distress and downside risk is compensated by higher returns	Limited investor attention to smaller cap stocks
Profitability	Firms facing high cost of capital will invest only in the most profitable projects	Investors do not discern high and low profitability in growth firms
Investment	Low investment reflects firms' limited scope for projects given high cost of capital	Investors under-price low investment firms due to expectation errors

4. Net-of-cost factor premia, as reported by Novy-Marx and Velikov (2014)

(Monthly)	Gross premium		Turnover	T-costs	Net premium	
	Average	[t-stat]			Average	[t-stat]
Size	0.33%	[1.66]	1.23%	0.04%	0.28%	[1.44]
Profitability	0.40%	[2.94]	1.96%	0.03%	0.51%	[3.77]
Value	0.47%	[2.68]	2.91%	0.05%	0.42%	[2.39]
Investment	0.56%	[4.44]	6.40%	0.10%	0.46%	[3.60]
Low volatility	0.63%	[2.13]	24.59%	0.52%	0.11%	[0.37]
Momentum	1.33%	[4.80]	34.52%	0.65%	0.68%	[2.45]

Extracted from Novy-Marx and Velikov (2014). See their table 3. All values are monthly. Factors based on cap-weighted decile portfolios. Portfolios are rebalanced annually for most factors but monthly for low idiosyncratic volatility and momentum. Factors are return differences between two extreme decile portfolios (cap-weighted). Time period is July 1963 to December 2013.

Practical implementations of such well-diversified indices leads to risk/return improvements over simple cap-weighted quintile portfolios[2], as well as considerable investability improvements through lower turnover and fewer average days to trade at rebalancing (Amenc et al [2015]).

In summary, while much of the early evidence did not consider practical implementation issues, more recent research confirms that the standard factors lead to rewards even net of implementation considerations. Moreover, straightforward adjustments to strategy design that ease implementation lead to even more pronounced premia net of transaction costs. Therefore, there is a strong case that academically-grounded factors can be used to design implementable strategies. Given this evidence, when considering deviating from academic factor definitions, investors should be careful to not throw out the baby (academic grounding) with the bathwater (unrealistic assumptions on implementation issues).

Conclusion: What 'academic grounding' does not mean

The fact of the matter is that many factor-investing strategies and indices offered by product providers create a considerable mismatch with academic definitions. Figure 5 provides an overview of factor definitions retained in several commercially-available factor indices and contrasts them with the Fama and French (2012, 2014) factor definitions, which are widely used in academic research that either tests the empirical evidence on these factors or assesses their economic rationale.

The mismatch between the provider definitions and the standard academic definitions is striking. While the Fama and French definitions rely on straightforward variables and make a choice of selecting one key metric to come up with a factor score for each stock in a transparent and simple way, the proprietary definitions from providers use different sets of variables, as well as various adjustments and often consist of complex combinations of several variables. For example, some factor scores are calculated relative to the industry or regional groups a stock belongs to. Some providers use such industry or region adjustments for certain variables within a given factor score while not using it for other variables making up the same factor score. Moreover, providers often use variables which are quite far removed from the original factor definition, such as the change in sales over total assets or the leverage in quality scores, as compared to the simple use of a profitability measure by Fama and French. Overall, the different index providers are in stark disagreement with how academic research defines these factors.

In general, such proprietary definitions increase the amount of flexibility providers have in testing many variations of factors and thus pose a risk of data-mining, and all the more so in that it remains unclear why these adjustments are made and in particular whether there are any fundamental economic reasons for using some of these variables and adjustments for a given factor. In fact, it appears that providers sometimes explicitly aim at selecting ad-hoc factor definitions which have performed well over short-term backtests. As an illustration, consider the following statements from white papers that select factor definitions for factor indices based on backtesting various combinations of variables on a particular dataset spanning a time period of about 13 years:[3]

> "For each composite value index, factors are selected on the basis of the most significant t-stat values"
> "Our preferred measure of momentum is the Residual Sharpe Ratio, which displays relatively high risk-adjusted performance outcomes, and relatively low levels of volatility"

Moreover, some providers have launched 'enhanced' factor indices which replace the factor definitions in their standard factor indices with new and improved recipes.

Of course, selecting proprietary combinations or making proprietary tweaks to variable definitions offers the possibility of improving the performance of a factor index in a backtest. The question is whether the improvement of the 'enhanced' factor definition will also hold going forward, especially if there is no solid economic foundation for it. There is clearly a risk that one ends up with what academics have termed 'lucky factors'. Harvey and Liu (2015) show that by snooping through data on a large number of candidate factors and retaining those with the highest t-stat, one takes the risk of uncovering flukes, which will not repeat out of sample. Perhaps even more importantly, it is unclear what – if anything – factors with extensive proprietary tweaks still have in common with the factors from academic research. Therefore, the empirical evidence in favour of the academic factors and their economic grounding cannot be transposed to such new proprietary factors.

In the absence of a clear relation with academic standard factors, such proprietary factor strategies are merely ad-hoc constructs resulting from product backtests. In fact, to find out whether any of these new proprietary factors are indeed related to the well-documented academic factors one would first need to assess how they align empirically with standard factors. This point was also made clear by Eugene Fama in a recent interview, when on the topic of value factor and more proprietary versions of this factor he states: "Now everybody talks about value ... Some stuff is fly-by-night. There are like 45 versions of that and every guy has their own marketing ploy. The acid test is you put it in the three factor model and it says it is a value portfolio."

In the end, a minimum requirement for good practice in factor investing is to avoid creating a mismatch with academic factors. This can be achieved easily by referring to indicators for which academic research has provided thorough tests and economic explanations, and by refraining from proprietary 'tweaks'.

Alternatively, when using novel or proprietary factors, one needs to make sure that they are thoroughly tested (ie, tested in very long term data, across asset classes, for robustness to data-mining and to transaction costs) as well as linked to economic mechanisms. Of course it seems like a heroic objective for a product provider to aim to replicate the work that the whole academic community has been doing on standard factors, only to assess the robustness of his own proprietary factor. Therefore, one can make a reasonable case that proprietary factors may never be able to reach the amount of thorough testing that their standard academic counterparts benefit from.

Given the strong emphasis providers put on the 'academic grounding' of their factor strategies, it is indeed surprising that they then chose to implement products which represent a gross mismatch with academic factor definitions and do not respect the key academic principle of parsimony. Instead of paying lip service to an 'academic grounding' and coming up with a marketing innovation of tweaked factors, perhaps it is time that product providers actually used academic research in their product development. Moreover, investors should hold providers to high standards and conduct thorough due diligence on the soundness of particular implementations of factor investing.

It is also worth emphasising that a key idea behind the use of simple standard factors is to obtain robustness through parsimony. Parsimony refers to the idea that one can explain 'a lot' with 'a little'. While proprietary factor definitions may be able to explain more in sample, they also pose a risk of picking up noise, which one can avoid with more parsimonious factor definitions such as the standard factors from the literature. The statistician George E P Box famously argued in favour of parsimony by writing that "over-elaboration and over-parameterisation is often the mark of mediocrity". Indeed, the parsimony of standard academic equity factor definitions may be preferable to

5. Mismatch with academic factor definitions: examples

Provider	Value	Momentum	Quality
Fama-French (2012, 2014)	Price to book	Past 12 months return (omitting last month)	ROE (operating profits divided by book equity)
Goldman Sachs Equity Factor Index World	Value score from proprietary risk model (Axioma) relative to stock's regional industry group	Residuals from cross-sectional regression of 12-month return (omitting last month) on stock volatility	Composite based on asset turnover, liquidity, ROA, operating CF to assets, accruals, gross margin, leverage
MSCI Multi Factor indexes	Sector-relative composite based on enterprise value/operating CF, forward P/E, price to book	Composite score based on excess return divided by annual volatility over past 12 months and past six months	Composite based on return on equity, standard deviation of earnings, debt-to-equity
FTSE Global Factor Index Series	Composite based on cash flow to price, net income to price and country-relative sales to price	Mean/standard deviation of 'average residual' from 11 rolling window regressions of past 36 months' returns on country and industry index	Composite based on operating CF to debt, net income to assets annual change in (sales over assets), accruals
Deutsche Bank Equity Factor indices	Composite based on inverse of enterprise value to EBITDA and dividend yield	12-month return (omitting last month) minus risk adjustment time idiosyncratic volatility score	Composite based on return on invested capital and net operating assets growth

2 On average across six well-documented factors, diversified multi-strategy indices have a Sharpe ratio of 0.7 compared to an average Sharpe ratio of 0.56 for cap-weighted quintile portfolios.

3 As reported in the papers, Factor Exposure Indices – Value Factor and Factor Exposure Indices – Momentum Factor, recovered on 1 July 2015 at www.ftse.com/products/downloads/FTSE_Value_Factor_Paper.pdf and www.ftse.com/products/downloads/FTSE_Momentum_Factor_Paper.pdf.

over-elaboration and over-parameterisation of tweaked proprietary factors that are sometimes proposed by product providers.

References

Amenc, N., F. Goltz and S. Sivasubramanian (2015). Concentrated versus Diversified. Alternatives for Implementing Factor Indices. Work in progress.

Amenc, N., F Goltz and A. Lodh (2012). Choose Your Betas: Benchmarking Alternative Equity Index Strategies. *Journal of Portfolio Management* 39(1): 88–111.

Ammann, M., S. Odoni and D. Oesch (2012). An Alternative Three-Factor Model for International Markets: Evidence from the European Monetary Union. *Journal of Banking & Finance* 36(7): 1857–1864.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The Cross-Section of Volatility and Expected Returns. *Journal of Finance* 61(1): 259–99.

Ang, A., R. J. Hodrick, Y. Xing and X. Zhang (2009). High Idiosyncratic Volatility and Low Returns: International and Further US Evidence. *Journal of Financial Economics* 61(1): 259–299.

Asness, C., T. Moskowitz and L. H. Pedersen (2013). Value and Momentum Everywhere. *Journal of Finance* 68(3): 929–985.

Banz, R. W. (1981). The Relationship between Return and Market Value of Common Stocks. *Journal of Financial Economics* 6: 3–18.

Basu, S. (1977). Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis. *Journal of Finance* 12: 129–156.

Carhart, M. (1997). On Persistence in Mutual Fund Performance. *Journal of Finance* 52(1): 57–82.

Cooper, M. J., H. Gulen and M. J. Schill (2008). Asset Growth and the Cross Section of Stock Returns. *Journal of Finance* 63(4): 1609–1651.

Fama, E. F. and K. R. French (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33: 3–56.

Fama, E. F. and K. R. French (2012). Size, Value, and Momentum in International Stock Returns. *Journal of Financial Economics* 105: 457–472.

Fama, E. F. and K. R. French (2014). A Five-Factor Asset Pricing Model. Working paper.

Frazzini, A., R. Israel and T. Moskowitz (2012). Trading Costs of Asset Pricing Anomalies. Working paper.

Frazzini, A., and L. H. Pedersen (2014). Betting Against Beta. *Journal of Financial Economics* 111(1): 1–25.

Harvey, C. R., Y. Liu, and H. Zhu (2015). ... and the Cross-Section of Expected Returns. Working paper.

Harvey, C. R, and Y. Liu (2015). Lucky Factors. Working paper.

Heston, S. L., G. K. Rouwenhorst and R. Wessels (1999). The Role of Beta and Size in the Cross-Section of European Stock Returns. *European Financial Management*.

Hou, K., L. Zhang and C. Xue (2014). Digesting Anomalies: An Investment Approach, *Review of Financial Studies* forthcoming.

Jegadeesh, N. and S. Titman (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48(1): 65–91.

Kogan, L., and M. Tian (2015). Firm Characteristics and Empirical Factor Models: a Model-Mining Experiment. Working paper.

Merton, R. C. (1980). On Estimating the Expected Return on the Market: an Exploratory Investigation. *Journal of Financial Economics* 8(4): 323–361.

Novy-Marx, R. (2013). The Quality Dimension of Value Investing. Simon Graduate School of Business working paper.

Novy-Marx, R., and M. Z. Velikov (2014). A Taxonomy of Anomalies and their Trading Costs. Working paper.

Rosenberg B., K. Reid and R. Lanstein (1985). Persuasive Evidence of Market Inefficiency. *Journal of Portfolio Management* 11: 9–17.

Rouwenhorst, G. K. (1998). International Momentum Strategies. *Journal of Finance* 53.

Watanabe, A., Y. Xu, T. Yao and T. Yu (2013). The Asset Growth Effect: Insights from International Equity Markets, *Journal of Financial Economics* 108(2): 529–563.

EDHEC Risk smart allocation offerings: general principles

Noël Amenc, Professor of Finance, EDHEC-Risk Institute, CEO, ERI Scientific Beta; Felix Goltz, Head of Applied Research, EDHEC-Risk Institute, Research Director, ERI Scientific Beta; Ashish Lodh, Deputy Research Director, ERI Scientific Beta; Sivagaminathan Sivasubramanian, Quantitative Analyst, ERI Scientific Beta

Smart beta product offerings have proliferated over the past decade, offering investors an ample choice of different factors and different weighting schemes to select from for a relevant smart beta index. However, in addition to the question of selecting a suitable index as a stand-alone investment, the question of combining different smart beta strategies naturally arises in the context of an extensive range of smart beta offerings. This article addresses the issue of combining several smart beta strategies, and clarifies the conceptual underpinnings and relevant questions arising when considering smart beta index combinations.

We first look at the design of efficient and investable proxies for risk premia, and then assess simple combinations of smart beta strategies through naïve diversification, and finally discuss additional potential for value added inherent in customised smart beta allocations.

Designing efficient and investable proxies for risk premia

Current smart beta investment approaches only provide a partial answer to the main shortcomings of capitalisation-weighted (cap-weighted) indices. We discuss a new approach to equity investing referred to as smart factor investing. It provides an assessment of the benefits of simultaneously addressing the two main shortcomings of cap-weighted indices, namely their undesirable factor exposures and their heavy concentration, by constructing factor indices that explicitly seek exposures to rewarded risk factors while diversifying away unrewarded risks. Addressing these two points simultaneously is made possible through the Smart Beta 2.0 approach, which combines a stock selection step (to select stocks with the desired factor tilt or characteristics) with a diversification-based weighting scheme. This weighting scheme is applied to the relevant stock selection to obtain a well-diversified portfolio within a given factor tilt. Our results suggest that such smart factor indices lead to considerable improvements in risk-adjusted performance.

The results in figure 1 confirm that the combination of relevant security selection and appropriate weighting schemes in a two-step process leads to substantial improvements in risk-adjusted performance with respect to the use of a standard cap-weighted index, which typically implies an inefficient set of factor exposures and an excess of unrewarded risk.

On the one hand, starting with a focus on the systematic risk exposure, we find that a higher Sharpe ratio can be achieved with the same weighting scheme, here a cap-weighting scheme, for stocks selected on the basis of their loadings on the value, size, momentum, low volatility, low investment and high profitability factors, compared to the case where the full universe is held in the form of a cap-weighted portfolio.

This finding underlines that these factors carry a long-term premium, and in this sense constitute rewarded risk. In fact, financial researchers have argued that stocks that provide these tilts tend to be exposed to different sources of systematic risk than a broad market index, implying that investors are exposed to a risk of poor returns in bad times when following such strategies. For example, value stocks have been shown to have increasing market betas during recessionary shocks, smaller-size companies have been argued to be exposed to liquidity and distress risk, momentum stocks have been argued to be heavily exposed to negative shocks in expected economic growth, low investment and high profitability stocks reflect a high discount rate for investments and even low volatility stocks have been argued to carry additional risks, in the sense that they may suffer during times of liquidity stress. For a comprehensive review of these explanations of the different premia, we refer the reader to the article in this supplement that discusses portfolios combining six rewarded risk factors.

The results we obtain, reported in figure 1, show that while the Sharpe ratio of the broad cap-weighted index is 0.41 on the sample period, it is considerably improved by a cap-weighted strategy using a stock selection to tilt towards rewarded factors.[1] These results suggest that a systematic attempt to harvest equity risk premia

1 The cap-weighted tilted strategies are implemented by selecting on a quarterly basis the top 50% of stocks in the reference universe by the relevant factor score (ie, the 50% of stocks with respectively the lowest market cap, highest book-to-market, highest past returns, or the lowest volatility) and weighting them in proportion to their free-float adjusted market cap.

1. Diversifying away unrewarded risks: performance comparison of US cap-weighted factor indices and US multi-strategy factor indices

US long term (Dec 1974–Dec 2014)		Mid cap		High momentum		Low volatility		Value		Low investment		High profitability	
	Broad CW	CW	Diversified multi-strategy	CW	Diversified multi-strategy	CW	Diversified multi-strategy	CW	Diversified multi-strategy	CW	Diversified multi-strategy	CW	Diversified multi-strategy
Annualised returns	12.16%	15.49%	16.75%	13.10%	15.65%	12.40%	15.03%	13.66%	16.70%	13.96%	16.05%	12.63%	15.49%
Annualised volatility	17.12%	17.59%	16.57%	17.30%	16.12%	15.50%	14.16%	17.83%	16.37%	15.96%	15.34%	17.06%	15.95%
Sharpe ratio	0.41	0.59	0.7	0.46	0.65	0.47	0.7	0.48	0.71	0.55	0.71	0.44	0.65
Maximum drawdown	54.53%	60.13%	58.11%	48.91%	49.00%	50.50%	50.13%	61.20%	58.41%	53.38%	53.20%	52.29%	48.28%
Annualised excess returns	–	3.33%	4.59%	0.94%	3.49%	0.24%	2.87%	1.51%	4.54%	1.80%	3.89%	0.47%	3.33%
Annualised tracking error	–	5.75%	6.38%	3.50%	4.72%	4.47%	6.04%	4.53%	5.56%	3.85%	5.44%	3.34%	4.39%
95% tracking error	–	9.39%	11.42%	6.84%	8.58%	9.20%	11.53%	8.72%	10.14%	6.89%	10.06%	6.75%	7.58%
Information ratio	–	0.58	0.72	0.27	0.74	0.05	0.48	0.33	0.82	0.47	0.72	0.14	0.76
Outperformance probability (1Y)	–	61.69%	67.78%	62.23%	67.24%	49.36%	66.06%	60.27%	70.83%	61.54%	71.86%	51.23%	70.58%
Outperformance probability (3Y)	–	69.25%	74.38%	78.47%	83.13%	52.85%	76.04%	66.25%	78.73%	75.21%	81.16%	58.59%	82.35%

The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). The benchmark used for the relative analytics is the SciBeta CW US 500 index. Mid cap, high momentum, low volatility, value, low investment and high profitability selections all represent 50% of stocks with such characteristics in a US universe of 500 stocks. The risk-free rate is the return of the 3-month US Treasury Bill. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1 (or 3) years at any point during the history of the strategy. A rolling window of length 1 (or 3) years and a step size of 1 week is used. The full names of the US indices used are: SciBeta United States Mid-Cap Diversified Multi-Strategy, SciBeta United States High-Momentum Diversified Multi-Strategy, SciBeta United States Low-Volatility Diversified Multi-Strategy, SciBeta United States Value Diversified Multi-Strategy, SciBeta United States Low-Investment Diversified Multi-Strategy and SciBeta United States High-Profitability Diversified Multi-Strategy. Source: www.scientificbeta.com.

above and beyond broad market exposure leads to additional risk-adjusted performance.

On the other hand, shifting to the management of specific risk exposures, we find that even higher levels of Sharpe ratio can be achieved for each selected factor exposure through the use of a well-diversified weighting scheme, which we take to be an equally-weighted combination of five popular smart weighting schemes[2] that enable the unrewarded or specific risk of each smart factor index to be reduced.

The category of specific risks corresponds to all the risks that are unrewarded in the long run, and therefore not ultimately desired by the investor, but that can have a strong influence on either the volatility and the maximum absolute drawdown of the index, or the tracking error or maximum relative drawdown of the index. Specific risks can correspond to important financial risk factors that do not explain, over the long term, the value of the risk premium associated with the index. There are many of these unrewarded financial risk factors. The academic literature considers, for example, that commodity, currency or sector risks do not have a positive long-term premium. These risks can have a strong influence on the volatility, tracking error, maximum drawdown or maximum relative drawdown over a particular period, which might sometimes be greater than that of systematically-rewarded risk factors (eg, exposure to the financial sector during the 2008 crisis or to sovereign risk in 2011).

In line with portfolio theory, among the unrewarded financial risks, we also find specific financial risks (also called idiosyncratic stock risks) which correspond to the risks that are specific to the company itself (its management, the risk of the poor quality of its products, the failure of its sales team, the relevance of its R&D and innovation, etc). It is this type of risk that asset managers are supposed to be the best at knowing, evaluating and choosing in order to create alpha, but portfolio theory considers it to be neither predictable nor rewarded, so it is better to avoid it by investing in a well-diversified portfolio. A globally effective diversification weighting scheme reduces the quantity of unrewarded risk, whether it involves unrewarded risk factors or unrewarded specific financial risks. However, like any model, it is imperfect and can itself lead to non-negligible residual exposures to certain unrewarded risks. This imperfection stems from the fact that the methodologies used seldom lead to an optimal and unique maximum Sharpe ratio portfolio as in Modern Portfolio Theory, whether it is a question of having accepted ex ante not to seek it (optimality risk), or to establish ex post the distance for a portfolio that would not be subject to parameter estimation errors. For example, minimum volatility portfolios, which are robust proxies for efficient portfolios, and therefore well diversified, are nonetheless not optimal portfolios ex ante since they do not target the maximum Sharpe ratio except if one considers that all stocks have the same return. De facto, efficient minimum volatility portfolios are often exposed to significant sector biases. Naturally, ERI Scientific Beta always tries to implement diversification models that are the least exposed possible to these unrewarded risks. For example, the use of norm constraints is a good compromise between the desire to fully utilise the potential to reduce the volatility in an efficient way procured by a minimum-volatility-type weighting scheme, while avoiding over-concentration in a small number of low-volatility stocks.

Specific or unrewarded risks can also correspond to operational or non-financial risks that are specific to the implementation of the diversification model. As such, for example, a maximum decorrelation scheme depends on a good estimation of the correlation matrix for the robustness of the diversification proposed. As part of the quality assurance for these indices, ERI Scientific Beta attaches a high price to the technical quality of the models used and their implementation to reduce this type of specific risk (for example, our research on the estimation of correlation matrices is part of this approach). In spite of all the attention paid to the quality of model selection and the implementation methods for these models, this specific operational risk, like the unrewarded financial risks described above, remains present nonetheless and it therefore seems interesting to be able to reduce even further the exposures that each weighting scheme, even if it is smart, is not able to diversify. This is the objective of the diversified multi-strategy approach.

Thus, the Sharpe ratio of the diversified multi-strategy indices reaches even higher levels of risk-adjusted return (Sharpe ratio) than cap-weighted tilted indices for the same factors.

These results suggest that multi-strategy factor-tilted indices obtain the desired factor tilts without undue concentration, which provides an explanation for their superior risk-adjusted performance with respect to the cap-weighted combination of the same selection of stocks.

Overall, it appears that the combined effects of a rewarded factor exposure ensured by a dedicated proper security selection process and an efficient harvesting of the associated premium through improved portfolio diversification leads to considerable Sharpe ratio improvements compared to the broad cap-weighted index.

In a nutshell, an improved weighting scheme which focuses on diversification such as diversified multi-strategy weighting allows unrewarded risks to be diversified away. This reduction of unrewarded risk through diversification is at the heart of the Smart Beta 2.0 approach advocated by Scientific Beta.

Obtaining a well-diversified index within each factor tilt is at the core of the improved performance of these indices. However, one may expect further benefits by allocating across different factor premia rather than focusing on a single factor tilt, notably because the academic literature and empirical research show that there is a good level of decorrelation for the risk premia associated with these factors. This allocation across different rewarded factors is at the heart of multi-smart-beta-allocation approaches, which we turn to below.

Combining multiple factors

Below, we look at Scientific Beta Multi-Beta Multi-Strategy (Equal-Weighted) indices as an example of combining different factor indices. ►

2 Diversified multi-strategy weighting is an equal-weighted combination of the following five weighting schemes – maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. Maximum deconcentration consists of maximising the effective number of stocks subject to turnover and liquidity constraints and thus corresponds to an adjusted version of equal-weighting. Diversified risk weighted attributes stocks weights inversely proportional to their volatility. Maximum decorrelation constructs a portfolio of stocks that behave differently over time, which is achieved by minimising portfolio volatility subject to the assumption that volatility is identical across stocks. Efficient minimum volatility consists of a volatility minimisation subject to norm constraints. Efficient maximum Sharpe ratio maximises the Sharpe ratio of the portfolio given the assumption that expected returns are proportional to the median semi-deviation of stocks in the same decile resulting from a sort on stock-level semi-deviation. The three latter strategies require a covariance matrix as an input to the optimisation problem. The covariance matrix is estimated using a robust estimation procedure employing a statistical factor model based on principal component analysis where the number of components is selected using a criterion from random matrix theory. For more details on the weighting schemes and the derivation of required input parameters, see www.scientificbeta.com.

This index provides simple access to smart beta allocation by simply combining the different factor-tilted indices in equal proportions. For a long-term US track record (1974–2014) this index produces annual outperformance over a broad cap-weighted index of 3.95%. The index has been live since 20 December 2013 and has confirmed this performance with live annual outperformance of 1.47% as of 30 June 2015.[3]

The index draws on the diversified multi-strategy indices for four factor tilts presented in figure 1 above, namely the value, low volatility, small size and momentum tilts. These four factor-tilted indices represent access to different rewarded risk factors. Combining exposures to these four factors provides access to the associated rewards, but the simple equal weighted allocation does not allow for accounting for any particular objective in terms of management of absolute or relative risk objectives and in this sense constitutes a naive form of factor allocation. In particular, equal-weighted allocation does not account for differences in correlations across the different pairs of factor indices that are being combined, nor does it consider differences in volatility across the different component indices. This naive diversification across factors is nevertheless a starting point for the use of the relative return correlations documented in figure 2, which logically allows the risk to be reduced for a level of return that is the average of the returns of the various smart factor indices, and therefore improves the information ratio compared to the average information ratio of the indices.

Naturally, a less naive form of diversification across smart factor indices, taking account of the matrix of excess return correlations or of their relative contributions to tracking error risks, would have led to much better results in terms of relative risks: that is the objective of the relative risk management of smart beta allocations.

Moreover, by using the diversified multi-strategy weighting scheme, these indices provide simple diversification of unrewarded risks but do not explicitly account for the correlation across different weighting strategies, which, for the same factor tilt, can be considerably below 1 and therefore corresponds to a potential improvement in risk-adjusted performance when it is taken into account. Figure 3 illustrates this point and shows that it is possible to be able to benefit from an attractive average pairwise correlation that justifies the use of different indices corresponding to different diversification strategies for the same factor tilt. Naturally, approaches that take account of extreme correlations will be able to benefit from maximising the diversification between these smart factor indices.

Figure 4 provides performance and risk results for the simple multi-beta multi-strategy index. The performance and risk of the combination of four factor-tilted multi-strategy indices is compared to the stand-alone performance and risk of each multi-strategy factor-tilted index.

It is of particular interest to compare the risk-adjusted relative performance (information ratio) of the combination to the stand-alone results obtained by each single factor tilt. In fact, while the single factor-tilted indices all generate positive information ratios, the results display considerable differences across factor tilts with an information ratio (IR) of 0.48 for the low-volatility index to an IR of 0.82 for the value index. Interestingly, the multi-beta multi-strategy equal weight index obtains an IR which is almost identical to the best result obtained among all the single factor tilts. The IR of the multi-factor combination is indeed higher than the average information ratio of the four factor-tilted indices which make up its components. In fact, the IR of the MBMS index of 0.79 compared to the average IR of the component indices of 0.69 corresponds to a 14.5% increase in IR. This clearly shows the allocation effect of diversifying across different factor tilts, which elevates risk-adjusted performance relative to the average result for component indices. In a nutshell, the results in figure 4 provide evidence that choosing good factor tilts generates attractive risk-adjusted performance, and that combining them allows the relative risk-adjusted return to be improved.

3 Annual relative return of the US MBMS EW index over the period from 20 December 2013 to 30 June 2015, using the Scientific Beta broad CW index as the reference index.

2. Correlation of excess returns across factor tilts

SciBeta US long-term track records (Dec 1974–Dec 2014)	Diversified multi-strategy				
Diversified multi-strategy	Momentum	Low volatility	Value	Low investment	High profitability
Mid cap	0.67	0.63	0.86	0.85	0.74
Momentum	–	0.61	0.64	0.74	0.65
Low volatility	–	–	0.70	0.82	0.60
Value	–	–	–	0.84	0.51
Low investment	–	–	–	–	0.69

All statistics are annualised and daily total returns from 31 December 1974 to 31 December 2014 are used for the US Long-Term universe. The universe contains 500 stocks. The full names of the indices used are: SciBeta United States LTTR Mid-Cap Diversified Multi-Strategy, SciBeta United States LTTR High-Momentum Diversified Multi-Strategy, SciBeta United States LTTR Low-Volatility Diversified Multi-Strategy, SciBeta United States LTTR Value Diversified Multi-Strategy, SciBeta United States LTTR Low Investment Diversified Multi-Strategy and SciBeta United States LTTR High Profitability Diversified Multi-Strategy. Source: www.scientificbeta.com.

Tailored risk allocation with smart factor indices

The standard multi-beta multi-strategy indices provide simple access to the combination benefits of several smart beta strategies. The standard indices provide equal allocations both to the weighting schemes, and to the factor tilts. They thus provide a first attempt at diversifying away weighting-scheme-specific risk, as well as allocating across multiple sources of rewarded risk (factor tilts). However, it is entirely possible to conceive of improved allocation schemes which account for the risk properties of the different weighting schemes and factor tilts. Such risk allocation approaches can provide less naive ways of allocating based on specific objectives.

In an attempt to identify, and analyse the benefits of, the possible approaches to efficient risk allocation across the various smart factor indices, we identify four main dimensions that can be taken into consideration when design-

3. Average pairwise correlations of excess returns across five weighting schemes

SciBeta US long-term track records (Dec 1974–Dec 2014)	Momentum	Low volatility	Value	Size	Low investment	High profitability
Average correlation across five weighting schemes	0.87	0.96	0.87	0.89	0.89	0.85
Maximum correlation across five weighting schemes	0.96	0.99	0.97	0.97	0.97	0.96
Minimum correlation across five weighting schemes	0.71	0.91	0.73	0.74	0.77	0.64

The analysis is based on daily total returns of US long-term track records from 31 December 1974 to 31 December 2014. The average, minimum and maximum pairwise correlations across the five weighting schemes – maximum deconcentration, maximum decorrelation, maximum Sharpe ratio, minimum volatility and diversified risk weighted for the six factors – momentum, low volatility, value, size, low investment and low profitability are provided. Source: www.scientificbeta.com

4. Performance benefits of US multi-beta multi-strategy

SciBeta US long-term track records (Dec 1974–Dec 2014)	SciBeta US Broad CW	Mid cap	Momentum	Low volatility	Value	Average across four tilts	Multi-beta multi-strategy EW
Annualised returns	12.16%	16.75%	15.65%	15.03%	16.70%	16.03%	16.11%
Annualised volatility	17.12%	16.57%	16.12%	14.16%	16.37%	15.81%	15.58%
Sharpe ratio	0.41	0.70	0.65	0.70	0.71	0.69	0.71
Maximum drawdown	54.53%	58.11%	49.00%	50.13%	58.41%	53.91%	53.86%
Excess returns	–	4.59%	3.49%	2.87%	4.54%	3.87%	3.95%
Tracking error	–	6.38%	4.72%	6.04%	5.56%	5.68%	4.98%
95% tracking error	–	11.42%	8.58%	11.53%	10.14%	10.42%	8.95%
Information ratio	–	0.72	0.74	0.48	0.82	0.69	0.79
Outperformance probability (3Y)	–	74.38%	83.13%	76.04%	78.73%	78.07%	80.38%

The table compares the performance and risk of the SciBeta Diversified Multi-Strategy index. The Multi-Beta Multi-Strategy EW (ERC) index is the equal-weighted (equal relative risk contribution) combination of the four diversified multi-strategy indices with stock selection based on mid cap, momentum, low volatility, and value respectively. All statistics are annualised and daily total returns from 31 December 1974 to 31 December 2014 are used for the analysis. The SciBeta CW US-500 index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The full names of the US indices used are: SciBeta United States Mid-Cap Diversified Multi-Strategy, SciBeta United States High-Momentum Diversified Multi-Strategy, SciBeta United States Low-Volatility Diversified Multi-Strategy, SciBeta United States Value Diversified Multi-Strategy, SciBeta United States Multi-Beta Multi-Strategy EW and SciBeta United States Multi-Beta Multi-Strategy ERC. Source: www.scientificbeta.com.

ing a sophisticated allocation methodology (see figure 5).

The first, and arguably most important, dimension relates to whether risk is defined by the investor from an absolute perspective in the absence of a benchmark, or whether it is instead defined in relative terms with respect to an existing benchmark, which is more often than not a cap-weighted index. In the former situation, one would use volatility as a relevant risk measure, while tracking error with respect to the cap-weighted index would instead be used in the latter case.

In the case of absolute risk allocation, a commonly important case is to define a benchmark with the best risk-adjusted return characteristics. This improvement will come for example from the reduction in benchmark risk through a minimum volatility allocation or with the constraint of a volatility budget relative to that of the cap-weighted index to benefit from the asymmetry of volatility in bull and bear markets. It is this case that we will present in our low risk benchmark construction exercises.

The relative risk approach can give rise to the application of numerous techniques. EDHEC-Risk Institute has carried out cases of equalising the contribution to tracking error risk and even creating multi-factor portfolios under the constraint of a market beta equal to that of the reference cap-weighted index to minimise the extreme tracking error risk relating for example to an overly defensive exposure of the smart beta portfolio, as is often the case. This approach, which we present in a contribution to this supplement, enables the tracking error risk to be limited while preserving the smart beta portfolio's strong exposure to the risks that are rewarded over the long term. In addition, good diversification of the idiosyncratic relative risk will ultimately optimise the relative risk-adjusted return, which will only depend on a deliberate choice of well-rewarded factors.

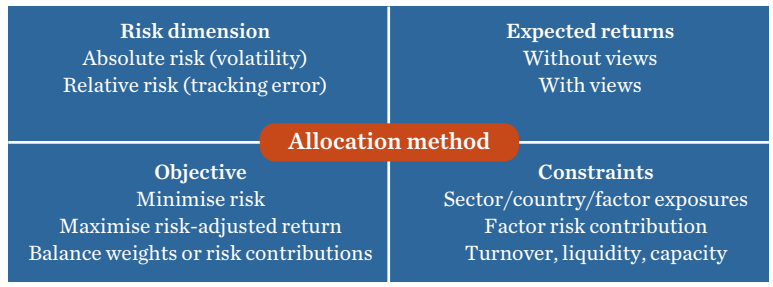
Relative risk objectives may also be defined in an asset-liability-management framework, rather than an asset-management-only framework. Moreover, it is feasible, and may more often than not be potentially desirable, to improve the relevance of portfolios and the resulting investment outcomes by designing highly-customised efficient multi-factor equity portfolio solutions that are optimised from an asset-liability management perspective that reflects the investor's specific investment context. For example, a mature pension fund facing a stream of bond-like pension obligations may find it useful to select stocks that show an above-average degree of 'liability-friendliness', which can be measured for example in terms of their correlation or tracking error with respect to a liability proxy and/or their ability to pay a high and predictable stream of dividends. Once these stocks are selected, a dedicated efficient factor index can be designed, and used as an additional building block in allocation exercises dedicated to achieving the optimal trade-off between liability-hedging benefits and performance benefits. This approach, which was presented by EDHEC-Risk Institute as part of research conducted by Coqueret, Deguest, Martellini, and Milhau (2014)[4] allows investment in a portfolio that not only has a good Sharpe ratio but also better correlation with the liabilities, which, for a given level of funding ratio volatility, enables the investment in the performance-seeking portfolio to be increased.

The second dimension concerns whether one would like to incorporate views regarding factor returns in the optimisation process. While additional benefits can be obtained from the introduction of views on factor returns at various points of the business cycle, we focus in what follows only on approaches that are solely based on risk parameters, which are notoriously easier to estimate with a sufficient degree of robustness and accuracy (Merton [1980]).

The third dimension is related to the objective of the allocation procedure. Indeed, there are several possible targets for the design of a well-diversified portfolio of factor exposure, depending upon whether one would like to use naive approaches (equal dollar allocation or equal risk allocation) or scientific approaches based on minimising portfolio risk (volatility in the absolute return context or tracking error in the relative return context). The fourth and last dimension related to the presence of various forms of constraints such as minimum/maximum weight constraints, turnover constraints, or factor exposure constraints, which are obviously highly relevant in the context of risk factor allocation.

In practice, investors may thus select among various ways of combining smart factor indices in order to account for their investment beliefs, objectives and constraints. Further articles in this supplement provide illustrations where multi-smart-beta allocations are crafted in order to accommodate investors' particular objectives and investment context. While possibilities for adding value through smart beta allocation are manifold, the robust performance improvements obtained through simple equal-weighted allocations to the five weighting schemes and the main consensual factors displayed above in this article, provide evidence that the benefits of multi-factor allocations are sizable. Investors and asset managers may be well advised to further explore the potential of multi-factor allocations in a variety of investment contexts.

5. The various dimensions of allocation methodologies across assets or risk factors



4 Coqueret G., R. Deguest, L. Martellini and V. Milhau (December 2014). Equity Portfolios with Improved Liability-Hedging Benefits, EDHEC-Risk Institute working paper.

References

Coqueret G., R. Deguest, L. Martellini and V. Milhau (2014). Equity Portfolios with Improved Liability-Hedging Benefits. EDHEC-Risk Institute working paper.

Merton, R. (1980). On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics* 8(4): 323–361.

Absolute risk allocation with smart factor indices

Noël Amenc, Professor of Finance, EDHEC-Risk Institute, CEO, ERI Scientific Beta; Ashish Lodh, Deputy Research Director, ERI Scientific Beta

Traditional approaches to defensive smart beta strategies

There are two popular ways to implement defensive smart beta strategies – one based on minimum variance portfolios and another relying on the selection of low-volatility stocks.

The first approach refers to the traditional mean-variance framework from Modern Portfolio Theory. The global minimum variance (GMV) portfolio is a portfolio on the efficient frontier in the sense that there is no portfolio that has a better return for the same level of risk. It is not the optimal portfolio (ie, the maximum Sharpe ratio) but the advantage is that it is not necessary to estimate expected returns in order to construct the portfolio. This parsimonious parameter estimation is actually a popular construction technique among managers and investors. The defensive nature of the portfolio, since it is the efficient portfolio with the lowest level of risk, has allowed these portfolios to exhibit very good performance relative to cap-weighted indices in highly volatile markets and their track record has benefited from the recent crises of 2008 and 2011.

The second approach is a two-step process where one selects stocks that exhibit low volatility and then weights them either using ►

a well-diversified weighting scheme, or cap-weighting, or an ad-hoc weighting scheme such as score-weighted. The low-volatility selection has received considerable interest, notably since the work of Ang et al (2006), who showed that low-volatility stocks did not necessarily produce lower returns than high-volatility stocks, and indeed produced higher returns. Haugen and Heins (1972, 1975) analyse pitfalls in commonly-used cross-sectional tests of the risk-return relationship, and express doubts regarding the existence and significance of the risk premia implied by standard asset pricing models. Blitz and Van Vliet (2007) show that portfolios of low-volatility stocks have higher returns than portfolios of high-volatility stocks because investors overpay for volatility, possibly because of leverage restrictions. Similarly, Baker, Bradley and Wurgler (2011) find that portfolios formed by sorting stocks by past volatility display higher returns for the low-volatility quintile over the subsequent month than for the high-volatility quintile. Baker, Bradley and Wurgler (2011) explain the low-volatility premium by the lottery preferences of investors and Hong and Sraer (2012) show that in the presence of short-sale constraints, the disagreement among investors on the future cash flow of firms leads to overpricing of stocks. As disagreement increases with a stock's beta, high-beta stocks are more likely to be overpriced.

More recently, it has been shown that there was a low-volatility risk premium that can be explained by economic reasoning. For example, Frazzini and Pedersen (2014) argue that liquidity-constrained investors are able to invest in leveraged positions of low-beta assets but are forced to liquidate these assets in bad times when their liquidity constraints mean they can no longer sustain the leverage, thus exposing themselves to the risk of liquidity shocks. This rational explanation means that low-volatility stocks are considered to be representative of a risk factor that is rewarded over the long term.

The objective of the Scientific Beta efficient minimum volatility strategy is to minimise the overall portfolio volatility by using the information on pair-wise correlations and volatilities of stocks. The aim is thus to provide a good proxy for the least risky portfolio in the MPT framework.[1] In order to avoid the problem of concentration in low-volatility stocks in the resulting portfolio, flexible de-concentration constraints are also imposed.[2] Post-optimisation, the long-only adjustment follows:

$$w^* = argmin\left\{\sqrt{w^T * \Sigma * w}\right\} \quad \begin{cases} \sum_{i=1}^{N} w_i = 1 \\ \frac{1}{w^T * w} \geq \frac{N}{3} \end{cases} \qquad (1)$$

With regard to the second approach of making an explicit or implicit choice of exposure to the low-volatility factor, Scientific Beta low volatility multi-strategy addresses this problem by constructing a smart factor index on the low-volatility factor. The low volatility multi-strategy portfolio is constructed using the Smart Beta 2.0 approach, a two-step process (Amenc et al [2013]). The idea is to construct a factor-tilted portfolio to extract the low-volatility factor premia most efficiently and is based on two pillars: 1) explicitly selecting low-volatility stocks – the stocks with the lowest past-two-year volatility and 2) using a diversification-based weighting scheme known as diversified multi-strategy.

Stock-specific risk can be reduced through the use of a suitable diversification strategy such as maximum Sharpe ratio or minimum volatility. However, due to imperfections in the diversification model used, residual exposures to unrewarded strategy-specific risks remain. Furthermore, in spite of all the attention paid to the quality of model selection and the implementation methods for these models, the specific operational risk remains present to some extent. The diversified multi-strategy approach, which combines the five different weighting schemes in equal proportion,[3] is based on this specific risk diversification principle (Kan and Zhou [2007]) and it enables the non-rewarded risks associated with each of the weighting schemes to be diversified away.

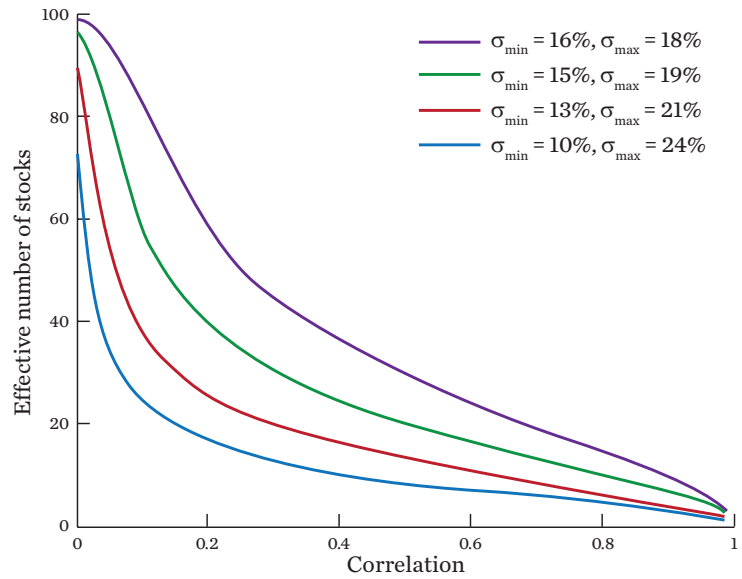
The similarity between these two approaches is that both lead to the overweighting of low-volatility stocks. While the low volatility multi-strategy approach does it explicitly by discarding the 50% of stocks with the highest volatility, the efficient minimum volatility strategy does it implicitly. Although the minimum-volatility optimiser takes into account both the volatility and correlations of stocks, it is a well-documented fact that the minimum-volatility optimiser overweights stocks that have low volatility. In other words, when the objective is minimisation of total portfolio volatility, the volatility characteristic of a stock plays a more important role than its correlation with other stocks. Figure 1 shows that the more diverse the volatilities in the universe are, the more concentrated the GMV is in the lower volatility assets. Despite having extremely low correlations, the problem of high concentration occurs when the dispersion in volatility across stocks is high.

In both cases, the resulting portfolios concentrate on low-volatility stocks. Therefore, the choice of constraints that tackle the problem of high concentration plays an important role in determining the performance and risk of minimum-volatility portfolios. The high concentration problem of minimum-volatility strategies has been long documented (Chan et al [1999], Clarke et al [2011], DeMiguel et al [2009]) and consequently there exists a large body of academic research that aims at dealing with this specific problem.

The most straightforward way to avoid this problem is by using rigid constraints on stock weights, sector weights, or country weights (in the case of multi-country regions). For example, the MSCI USA Minimum Volatility index uses rigid constraints in its optimiser. Individual stocks are subject to a fixed lower bound and an upper bound determined by their market capitalisation. The sector weights are also bounded between levels determined by parent index sector weights.[4] This approach is very popular due to its simple nature but it has two drawbacks. First, there is a potential for 'ex-post optimisation' of such ad-hoc constraints, meaning that the constrained portfolio is sub-optimal by definition. The second problem is that these kinds of constraints do not allow the risk reduction potential of the covariance structure of returns to be fully utilised.

DeMiguel et al (2009) go beyond considering rigid constraints at the individual stock level and introduce flexible constraints on overall portfolio concentration (so-called 'norm constraints'). They show that such flexible concentration constraints, instead of rigid upper and lower bounds on individual stock weights, allows for a better use of the correlation structure and therefore leads to better out-of-sample risk and return properties for minimum-volatility portfolios. The results in figure 2 show that the norm-constrained portfolio makes more efficient use of the risk budget compared to the

1. Role of stock volatilities and stock correlation in GMV optimisation



The figure is obtained from Amenc et al (2011 – an ERI research paper entitled A Post-Crisis Perspective on Diversification for Risk Management). The authors consider a hypothetical universe of 100 stocks, the annualised volatilities of which are equally spaced in the following ranges: [16%, 18%], [15%, 19%], [13%, 21%], and [10%, 24%]. In the three cases, the average volatility is one and the same; the only difference is the dispersion of the volatilities around the average. On these assumptions, they numerically calculate the GMV portfolios for degrees of correlation ranging from 0 to 0.99 in a constant correlation model.

1 For a complete description of how the strategy is implemented we refer the reader to the strategy construction rules of the Scientific Beta Efficient Minimum Volatility indices available at www.scientificbeta.com.

2 These flexible deconcentration or 'norm' constraints are discussed in more detail in the upcoming sections of this article.

3 Diversified multi-strategy weighting is an equal-weighted combination of the following five weighting schemes – maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio.

4 The weight constraints can be relaxed to handle infeasible optimisations. Other constraints relating to risk factor exposure and turnover are also used. The methodological details are obtained from the following web link: https://www.msci.com/resources/pdfs/MSCI_Minimum_Volatility_Indexes_Investor_Insight.pdf.

rigidly-constrained portfolio. The Sharpe ratio of the Scientific Beta US efficient minimum volatility strategy is superior to that of the MSCI US minimum volatility.

Benefits of factor diversification and weighting schemes

Although norm constraints prove to be an excellent way to deconcentrate at the stock level, they do not guarantee diversification of risks at the risk-factor or weighting-scheme level. The norm constraint approach is applied at the stock level and not at the risk-factor level. It will therefore be possible to have a portfolio that is not very concentrated but remains highly concentrated at the factor level – ie, that is globally a portfolio that is exposed to stocks with lower volatility than the average in the universe. In this sense, the exposure to low-volatility risk is not very different between the Scientific Beta low volatility multi-strategy indices and the Scientific Beta efficient minimum volatility indices.

In view of this observation, it seems reasonable to envisage constructing benchmarks that take account of the limitations of the above-mentioned approaches in terms of factor diversification. That is the sense of the approach designed by EDHEC Risk within the framework of smart allocation between smart beta indices, which provides factor diversification and weighting scheme diversification benefits. This smart allocation is based on the idea of conducting risk allocation that relies on indices that are representative of differentiated exposure to both risk factors and weighting schemes. The ingredients in this allocation (ie, the indices used) are the same as those that are contained in Scientific Beta's multi-beta multi-strategy offering.

For the implementation of this smart allocation, 30 factor indices representing a choice of six systematic risk factors associated with five weighting schemes are used. The six factors are mid cap, value, momentum, low volatility, low investment and high profitability.[5] The five weighting schemes are maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. The GMV allocation under 1/3 norm constraints is performed on these ingredients.

The allocation problem can be written mathematically as:

$$w^* = argmin\left\{\sqrt{w^T * \Sigma * w}\right\} \quad \begin{cases} \sum_{i=1}^{N} w_i = 1 \\ w_i \geq 0 \,\forall\, i \\ \dfrac{1}{w^T * w} \geq \dfrac{N}{3} \end{cases} \qquad (2)$$

W_i represents the weight of the i-th constituent. N is the number of constituents. Σ is the covariance matrix of total returns. Weekly total returns over the past 104 weeks are used to estimate the covariance matrix and benchmark volatility.

5 The following selection rules are applied to select stocks for each tilt: mid cap: bottom 50% free-float-adjusted market-cap stocks are selected; value: top 50% stocks are selected by book-to-market (B/M) ratio, B/M is defined as the ratio of available book value of shareholders' equity to company market cap; high momentum: top 50% stocks are selected by returns over past 52 weeks, minus the last four weeks; low volatility: bottom 50% stocks are selected by their standard deviation of weekly stock returns over the past 104 weeks; high profitability: top 50% stocks with highest gross profit/total asset ratio are selected; low investment: bottom 50% stocks with least two-year total asset growth rate are selected. This score-based selection is done twice a year (June and December) for momentum and once a year (June) for the other three factors.

2. Flexible norm constraints and rigid constraint minimum volatility portfolios

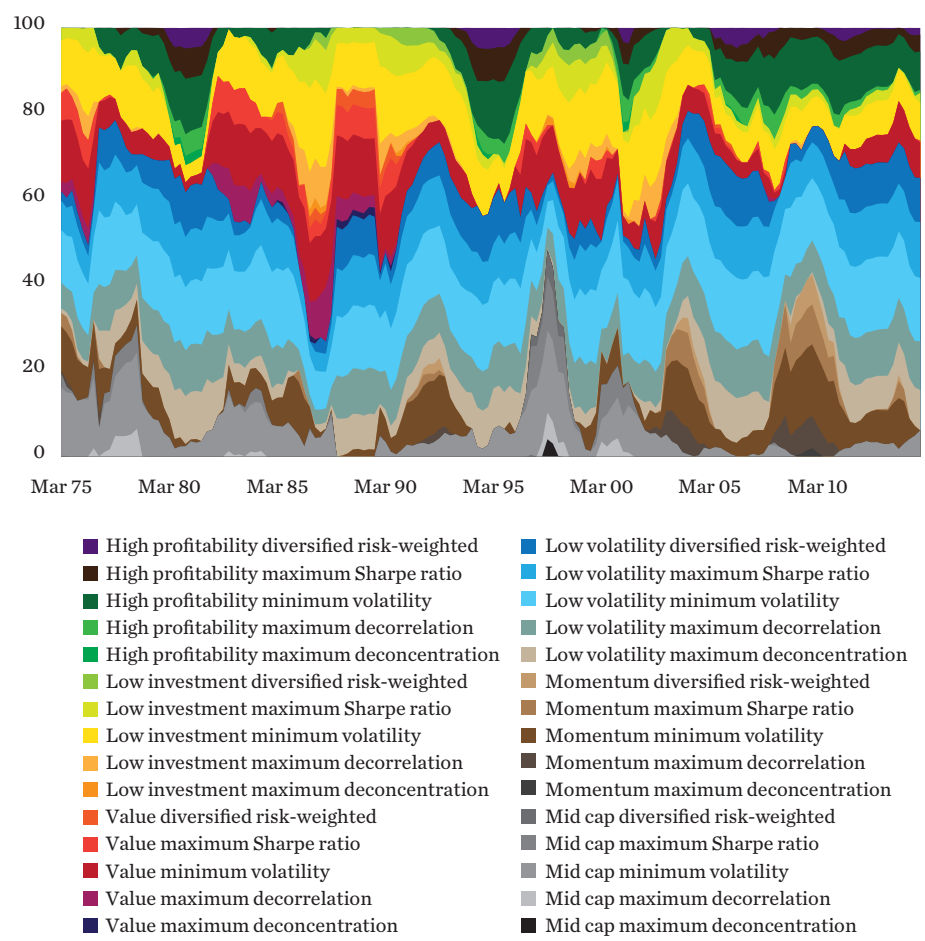
31 Dec 2004–31 Dec 2014 (10 years)	Scientific Beta US CW	Scientific Beta US efficient minimum volatility	MSCI USA minimum volatility
Annualised returns	7.90%	10.34%	8.96%
Annualised volatility	20.26%	17.51%	16.91%
Sharpe ratio	0.32	0.51	0.45
Maximum drawdown	54.63%	47.33%	46.61%
Annualised relative returns	–	2.44%	1.06%
Tracking error	–	4.14%	5.20%
Information ratio	–	0.59	0.20
Outperformance probability (1Y)	–	66.60%	44.26%
Average relative returns	–	1.66%	0.03%
Average of positive relative returns	–	3.46%	5.12%
Outperformance probability (3Y)	–	93.44%	80.05%
Average relative returns	–	2.32%	1.61%
Average of positive relative returns	–	2.52%	2.39%
Outperformance probability (5Y)	–	100.00%	79.39%
Maximum relative drawdown	–	7.51%	13.26%

The analysis is based on daily total returns in US dollars in the period 31 December 2004 to 31 December 2014 (10 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Probability of outperformance is the probability of obtaining positive excess returns if one invests in the strategy for a period of 1 (or 3 or 5) years at any point during the history of the strategy. Rolling window of length 1 (or 3 or 5) years and a step size of 1 week is used. The corresponding average relative returns is the average of relative returns across all rolling windows and the corresponding average of positive relative returns is the average across rolling windows where the relative returns are positive. The Scientific Beta US universe contains 500 stocks. Source: scientificbeta.com and Bloomberg.

Figure 3 shows the fractional weights of the 30 smart factor indices in the GMV allocation. It is interesting to see that, unlike what is observed with traditional GMV or low volatility indices, smart allocation with different factor indices allows the benchmark's exposure to risk factors to be genuinely diversified. Figure 3 clearly shows significant exposure over time to factors other than the low volatility factor; which means that the factor risk is well diversified.

The results of this GMV allocation with 30 factor indices are consistent with this good level of risk diversification. Figure 4 on page 26 shows that the norm-constrained GMV allocation across smart factor indices fulfils the objective of overall volatility reduction. It achieves lower volatility than the first two approaches – efficient minimum volatility and low volatility multi-strategy. Its performance remains high in comparison to these two approaches because it benefits from explicit exposure to the six long-term rewarded factors. In this sense, it is similar to the performance of a multi-beta multi-strategy benchmark that would contain, in equal proportions, exposure to six well-diversified multi-strategy factor indices representative of the value, mid cap, momentum, high profitability, low investment and low volatility factors, while having ►

3. Constrained GMV allocation weights



The figure shows the evolution of weights across the 30 smart factor indices.

lower volatility and therefore a smaller maximum drawdown. Due to diversification of risk factors, the maximum relative drawdown also falls to 48.36%, compared to 54.53% for that of the benchmark.

The drawback of this approach is that its relative performance is highly dependent on market conditions. In particular, the strategy outperforms by large margins in bear markets but its outperformance in bullish markets is quite poor, notably in periods of extreme bull markets. This problem is common to the other two defensive strategies as well. This kind of allocation is attractive for an investor who wants to protect the portfolio value (or reduce the losses) in bear markets. Therefore, it is complementary to most actively-managed portfolios, which are known to have high market beta.

Dissymmetric defensive smart allocation

To respond to the problem posed by the underperformance of defensive strategies in the event of bull markets, EDHEC Risk has designed a smart allocation methodology that is no longer based on the absolute risk allocation objective that corresponds to an absolute reduction (and therefore a constant reduction in volatility budget), but instead to a relative reduction in the volatility budget. In this case, it involves reducing the volatility of the defensive benchmark in proportion to the observed volatility of the cap-weighted index. This reduction in volatility therefore varies depending on the volatility of the cap-weighted index.

This approach enables us to perform dissymmetric defensive allocations. This risk allocation method allows the level of defensive nature to be adjusted based on the state of market using the asymmetrical property of market volatility in bull and bear markets. In general, low-volatility markets are correlated with bull markets, which are not a favourable regime for defensive portfolios.

We perform a maximum deconcentration allocation with a constraint of 90% of market volatility. This approach aims to create asymmetry by reducing the defensive character of the portfolio when the cap-weighted volatility is decreasing. The allocation problem can be written mathematically as:

$$w^* = argmax\left\{1/\left(w^T * w\right)\right\} \quad \begin{cases} \sum_{i=1}^{N} w_i = 1 \\ w_i \geq 0 \,\forall\, i \\ \sqrt{w^T * \Sigma * w} \leq 90\% * Volatility_{CW} \end{cases} \qquad (3)$$

W_i represents the weight of the i-th constituent. N is the number of constituents. Σ is the covariance matrix of total returns. Weekly total returns over the past 104 weeks are used to estimate the covariance matrix and benchmark volatility.

Figure 6 shows that the volatility-constrained maximum deconcentration allocation achieves low levels of volatility (14.66%) compared to that of a broad CW index (17.12%) and a simple equal-weighted allocation – the multi-beta multi-strategy six-factor EW (15.52%). Due to well diversified exposure to the six rewarded factors, it shows strong outperformance of 3.68%. Since this allocation is not "defensive" at all times, its overall tracking error is also improved compared to the other three defensive approaches. As a result, it delivers a high information ratio of 0.73 over a 40-year period.

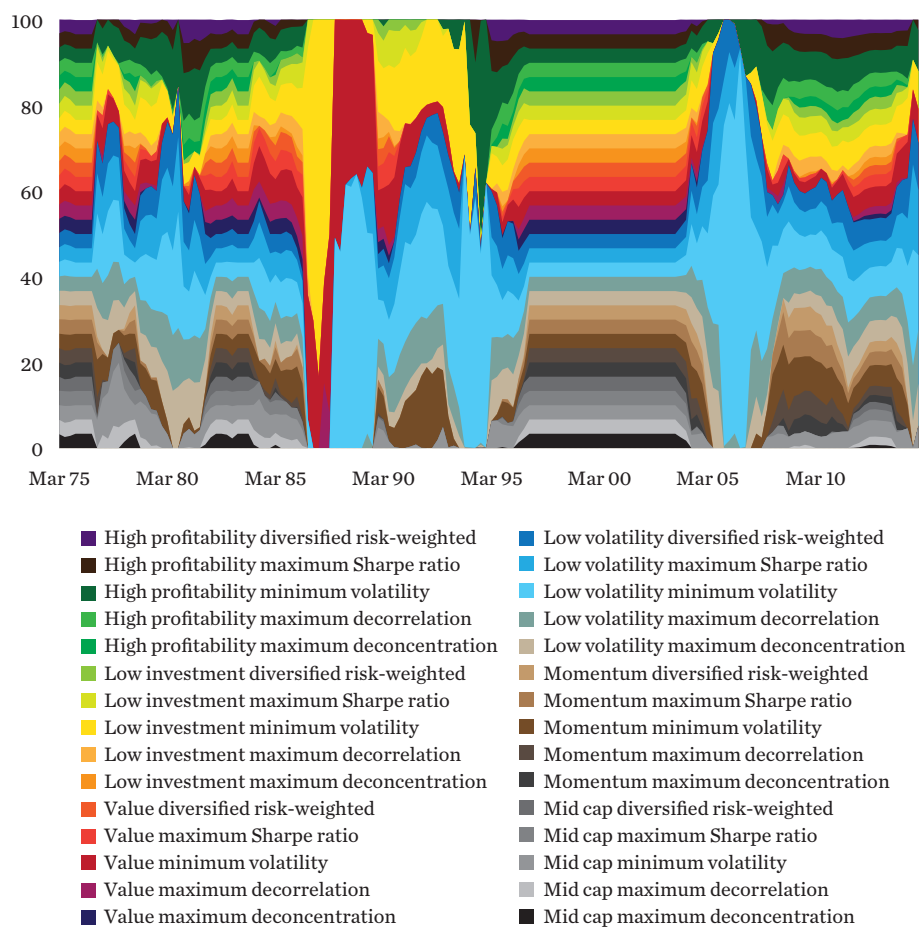
More importantly, its conditional performance is more symmetrical. While the norm-constrained GMV's outperformance is –3.94% and 7.79% in extreme bull and extreme bear markets respectively, the volatility-constrained maximum deconcentration allocation's conditional performances are 0.34% and 5.95% respec-

4. Norm-constrained GMV allocation across smart factor indices compared to traditional approaches

31 Dec 1974–31 Dec 2014 (40 years)	Broad CW index	GMV allocation (Norm 1/3)	Multi-beta multi-strategy six-factor (EW)	SciBeta USA LTTR efficient minimum volatility	SciBeta USA LTTR low volatility multi-strategy
Annualised returns	12.16%	15.51%	16.01%	14.65%	15.03%
Annualised volatility	17.12%	14.14%	15.52%	14.50%	14.16%
Sharpe ratio	0.41	0.74	0.70	0.66	0.70
Maximum drawdown	54.53%	48.36%	52.83%	50.03%	50.13%
Annualised relative returns	–	3.35%	3.86%	2.50%	2.87%
Tracking error	–	5.78%	4.73%	5.09%	6.04%
Information ratio	–	0.58	0.81	0.49	0.48
Outperformance probability (1Y)	–	70.19%	74.61%	71.66%	66.06%
Average relative returns	–	2.96%	3.61%	2.19%	2.52%
Average of positive relative returns	–	6.55%	6.50%	5.16%	6.30%
Outperformance probability (3Y)	–	80.23%	81.16%	79.35%	76.04%
Average relative returns	–	3.00%	3.48%	2.22%	2.64%
Average of positive relative returns	–	5.07%	5.29%	4.12%	4.69%
Outperformance probability (5Y)	–	86.98%	89.99%	80.42%	85.39%
Maximum relative drawdown	–	41.49%	32.88%	40.10%	43.46%
Relative returns bull markets	–	0.25%	2.98%	–0.07%	–0.85%
Relative returns bear markets	–	7.82%	4.86%	6.16%	8.35%
Relative returns 25% bull markets	–	–3.94%	3.42%	–3.84%	–5.90%
Relative returns 25% bear markets	–	7.79%	4.59%	6.02%	8.50%
One-way annual turnover	2.7%	34.2%	25.0%	30.3%	25.8%

The analysis is based on daily total returns in US dollars in the period 31 December 1974 to 31 December 2014 (40 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 160 rebalancings in the 40-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Probability of outperformance is the probability of obtaining positive excess returns if one invests in the strategy for a period of 1 (or 3 or 5) years at any point during the history of the strategy. Rolling window of length 1 (or 3 or 5) years and a step size of 1 week is used. Corresponding average relative returns is the average of relative returns across all rolling windows and corresponding average of positive relative returns is the average across rolling windows where the relative returns are positive. Quarters with positive benchmark index returns are bull quarters and remaining are bear quarters. Top 25% quarters with best benchmark index returns are extreme bull quarters and bottom 25% quarters with worst benchmark index returns are extreme bear quarters. The Scientific Beta USA LTTR universe contains 500 stocks. The multi-beta multi-strategy is based on 30 underlying strategies which are combinations of six factor tilts (mid cap, value, high momentum, low volatility, high profitability and low investment) and five diversification-based weighting schemes (max deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio and diversifed risk-weighted). The GMV allocation under 1/3 Norm Constraint is also based on these 30 underlying strategies. Source: scientificbeta.com/Scientific Beta USA Long-Term Track Records.

5. Volatility-constrained maximum deconcentration allocation weights



The figure shows the evolution of weights across the 30 smart factor indices.

6. Volatility-constrained maximum deconcentration allocation compared to defensive approaches

31 Dec 1974–31 Dec 2014 (40 years)	Broad CW index	GMV allocation (Norm 1/3)	Maximum deconcentration (90% BM volatility)	Multi-beta multi-strategy six-factor (EW)	SciBeta US LTTR efficient minimum volatility	SciBeta US LTTR low volatility multi-strategy
Annualised returns	12.16%	15.51%	15.84%	16.01%	14.65%	15.03%
Annualised volatility	17.12%	14.14%	14.66%	15.52%	14.50%	14.16%
Sharpe ratio	0.41	0.74	0.73	0.70	0.66	0.70
Maximum drawdown	54.53%	48.36%	48.99%	52.83%	50.03%	50.13%
Annualised relative returns	–	3.35%	3.68%	3.86%	2.50%	2.87%
Tracking error	–	5.78%	5.01%	4.73%	5.09%	6.04%
Information ratio	–	0.58	0.73	0.81	0.49	0.48
Outperformance probability (1Y)	–	70.19%	70.97%	74.61%	71.66%	66.06%
Average relative returns	–	2.96%	3.28%	3.61%	2.19%	2.52%
Average of positive relative returns	–	6.55%	6.40%	6.50%	5.16%	6.30%
Outperformance probability (3Y)	–	80.23%	80.18%	81.16%	79.35%	76.04%
Average relative returns	–	3.00%	3.25%	3.48%	2.22%	2.64%
Average of positive relative returns	–	5.07%	5.10%	5.29%	4.12%	4.69%
Outperformance probability (5Y)	–	86.98%	86.98%	89.99%	80.42%	85.39%
Maximum relative drawdown	–	41.49%	33.29%	32.88%	40.10%	43.46%
Three-year rolling volatility mean	16.55%	13.62%	14.11%	14.93%	13.93%	13.63%
Three-year rolling volatility std dev	5.49%	4.62%	4.78%	5.15%	4.88%	4.70%
Three-year rolling volatility 95%ile	29.34%	24.78%	25.47%	27.89%	26.21%	25.17%
Relative returns bull markets	–	0.25%	1.90%	2.98%	–0.07%	–0.85%
Relative returns bear markets	–	7.82%	6.06%	4.86%	6.16%	8.35%
Relative returns 25% bull markets	–	–3.94%	0.34%	3.42%	–3.84%	–5.90%
Relative returns 25% bear markets	–	7.79%	5.95%	4.59%	6.02%	8.50%
One-way annual turnover	2.7%	34.2%	37.6%	25.0%	30.3%	25.8%

The analysis is based on daily total returns in US dollars in the period 31 December 1974 to 31 December 2014 (40 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 160 rebalancings in the 40-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Probability of outperformance is the probability of obtaining positive excess returns if one invests in the strategy for a period of 1 (or 3 or 5) years at any point during the history of the strategy. Rolling window of length 1 (or 3 or 5) years and a step size of 1 week is used. Corresponding average relative returns is the average of relative returns across all rolling windows and corresponding average of positive relative returns is the average across rolling windows where the relative returns are positive. Quarters with positive benchmark index returns are bull quarters and the remaining are bear quarters. Top 25% quarters with best benchmark index returns are extreme bull quarters and bottom 25% quarters with worst benchmark index returns are extreme bear quarters. The Scientific Beta US LTTR universe contains 500 stocks. The multi-beta multi-strategy is based on 30 underlying strategies which are combinations of six factor tilts (mid cap, value, high momentum, low volatility, high profitability and low investment) and five diversification-based weighting schemes (maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio and diversifed risk-weighted). The GMV allocation under 1/3 norm constraint is also based on these 30 underlying strategies. Source: scientificbeta.com/Scientific Beta US Long-Term Track Records.

tively. Even though most of the performance of this dissymmetric allocation comes from bear markets, we observe that even in extreme bull markets the investor has similar performance to the cap-weighted index.

Conclusion: effectiveness of smart allocation solutions

The profusion of smart beta indices, which is often stimulated by new index construction ideas, leads to a risk of model mining or factor fishing. Many index providers multiply innovations in order to distract from the poor out-of-sample performance of methodologies that were designed to perform in-sample, notably because they were well exposed to a factor that was highly rewarded over the period that corresponded to the simulated track record (Amenc et al [2015]).

The success of GMV with many smart beta investors is explained by the confusion maintained with the low-volatility anomaly, which does not necessarily lead to the adoption of a minimum-volatility index with the minimum-volatility performance observed during the recent financial crises, but rather to a low-volatility index.

The objective of this article is not therefore to propose a new index that is constructed to optimise performance over a recent period, but to show that on the basis of existing smart factor indices, allocation between these indices can allow an investor who wishes to implement a defensive strategy to avoid concentration in a single factor and above all to benefit from the particular properties of volatility and its dissymmetric nature with respect to market conditions, and thereby adjust the portfolio's defensive bias to market conditions.

Figure 7 shows that smart allocation solutions on a set of smart factor indices always give a better result than the traditional minimum- ►

7. Comparison of performance over last 10 years

31 Dec 2004–31 Dec 2014 (10 years)	SciBeta US CW	GMV allocation (Norm 1/3)	Maximum deconcentration (90% BM volatility)	SciBeta US efficient minimum volatility	SciBeta US low volatility multi-strategy	MSCI US minimum volatility
Annualised returns	7.90%	10.52%	10.66%	10.34%	10.07%	8.96%
Annualised volatility	20.26%	17.36%	17.90%	17.51%	17.00%	16.91%
Sharpe ratio	0.32	0.52	0.52	0.51	0.51	0.45
Maximum drawdown	54.63%	48.36%	48.99%	47.33%	48.33%	46.61%
Annualised relative returns	–	2.61%	2.76%	2.44%	2.17%	1.06%
Tracking error	–	4.66%	4.22%	4.14%	5.12%	5.20%
Information ratio	–	0.56	0.65	0.59	0.42	0.20
Outperformance probability (1Y)	–	73.62%	78.72%	66.60%	61.49%	44.26%
Average relative returns	–	1.96%	2.25%	1.66%	1.40%	0.03%
Average of positive relative returns	–	3.59%	3.61%	3.46%	4.12%	5.12%
Outperformance probability (3Y)	–	95.63%	94.81%	93.44%	92.35%	80.05%
Average relative returns	–	2.51%	2.67%	2.32%	2.24%	1.61%
Average of positive relative returns	–	2.64%	2.85%	2.52%	2.48%	2.39%
Outperformance probability (5Y)	–	100.00%	100.00%	100.00%	100.00%	79.39%
Maximum relative drawdown	–	7.87%	7.34%	7.51%	8.59%	13.26%
Relative returns bull markets	–	–0.74%	0.11%	–1.17%	–2.01%	–4.64%
Relative returns bear markets	–	6.82%	5.92%	7.19%	7.97%	9.49%
One-way annual turnover	4.3%	31.6%	36.7%	30.5%	28.7%	–NC

The analysis is based on daily total returns in US dollars in the period 31 December 2004 to 31 December 2014 (10 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across all rebalancings in the 10-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Probability of outperformance is the probability of obtaining positive excess returns if one invests in the strategy for a period of 1 (or 3 or 5) years at any point during the history of the strategy. Rolling window of length 1 (or 3 or 5) years and a step size of 1 week is used. Corresponding average relative returns is the average of relative returns across all rolling windows and corresponding average of positive relative returns is the average across rolling windows where the relative returns are positive. Quarters with positive benchmark index returns are bull quarters and the remaining are bear quarters. Top 25% quarters with best benchmark index returns are extreme bull quarters and bottom 25% quarters with worst benchmark index returns are extreme bear quarters. Scientific Beta US universe contains 500 stocks. The GMV allocation under 1/3 norm constraint is based on the 30 underlying strategies which are combinations of six factor tilts (mid-cap, value, high momentum, low volatility, high profitability and low investment) and five diversification-based weighting schemes ((maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio and diversifed risk-weighted). Source: scientificbeta.com/Scientific Beta US Long-Term Track Records and Bloomberg.

EDHEC-RISK Institute

15-16 March 2016, The Brewery, London

EDHEC-Risk Days 2016

Bringing Research Insights to Institutional Investment Professionals

Passive Investment and Smart Beta Conference

The Development of Passive Investment in Europe
Smart Beta Risk Allocation Solutions
Smart Beta & ALM
Factor-Based Investment Strategies
Active Manager Substitution with Smart Beta
Robustness and Live Performance of Smart Beta

Institutional Money Management Conference

New Frontiers in Retirement Solutions
Multi-Dimensional Risk and Performance Analysis
Multi-Asset Allocation Solutions
Active Allocation and Smart Beta
Hedge Fund Investing

Infrastructure Forum

Latest research results on infrastructure investing from the EDHEC Infrastructure Investment Institute
How do Investors Invest in Infrastructure?

conference

To register: e-mail: eridays2016@edhec-risk.com or +33 **493 187 837**

volatility approaches, which are either mono-factor dependent or else, due to rigid constraints defined in-sample, give disappointing out-of-sample results. Specifically, we observe that a norm-constraint GMV weighting applied to a set of indices representative of factors that are well rewarded over the long term gives much better risk-adjusted performance and above all, much better conditional performance, when comparing for example MSCI minimum volatility and maximum deconcentration with an ex-ante relative volatility constraint of 90%. Naturally, these better conditional performances procure much better information ratios and, above all, much better outperformance probabilities.

References

Amenc, N., F. Goltz, S. Sivasubramanian and A. Lodh (2015). Robustness of Smart Beta Strategies. *Journal of Index Investing* 6(1): 17–38.

Amenc, N., and F. Goltz (2013). Smart Beta 2.0. *Journal of Index Investing* 4(3): 15–23.

Amenc, N., F. Goltz and S. Stoyanov (2011). *A Post-Crisis Perspective on Diversification for Risk Management*. EDHEC Risk-Institute publication (May).

Ang, A., R. J. Hodrick, Y. Xing and X. Zhang (2006). The Cross-Section of Volatility and Expected Returns. *Journal of Finance* 61(1): 259–99.

Baker, M. P., B. Bradley and J. Wurgler (2011). Benchmarks as Limits to Arbitrage: Understanding the Low Volatility Anomaly. *Financial Analysts Journal* 67(1): 1–15.

Blitz, D. C., and P. Van Vliet (2007). The Volatility Effect: Lower Risk without Lower Return. *Journal of Portfolio Management* 34(1): 102–13.

Chan, K. C., J. Karceski and J. Lakonishok (1999). On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model. *Review of Financial Studies* 12(5): 937–974.

Clarke, R., H. De Silva and S. Thorley (2011). Minimum Variance Portfolio Composition. *Journal of Portfolio Management* 37(2): 31–45.

DeMiguel, V., L. Garlappi, J. Nogales and R. Uppal. 2009. A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms. *Management Science* 55(5): 798–812.

Frazzini, A., and L. H. Pedersen (2014). Betting Against Beta. *Journal of Financial Economics* 111(1): 1–25.

Haugen, R. A., and A. J. Heins (1975). Risk and the Rate of Return on Financial Assets: Some Old Wine in New Bottles. *Journal of Financial and Quantitative Analysis* 10(5): 775–784.

Haugen, R. A., and A. J. Heins (1972). On the Evidence Supporting the Existence of Risk Premiums in the Capital Markets. Wisconsin working paper.

Hong, H., and D. Sraer (2012). Speculative Betas. NBER working paper no 18548.

Kan, R. and G. Zhou (2007). Optimal Portfolio Choice with Parameter Uncertainty. *Journal of Financial and Quantitative Analysis* 42(3): 621–656.

Relative risk allocation with smart factor indices

Noël Amenc, Professor of Finance, EDHEC-Risk Institute, CEO, ERI Scientific Beta;
Ashish Lodh, Deputy Research Director, ERI Scientific Beta

Introduction

Smart beta strategies can have two broad usages in the investment process – as a substitute for cap-weighted benchmarks and as a substitute for active managers. All surveys and studies on smart beta investing show that while many investors have adopted smart beta, not so many think that these smart beta strategies, even when they are in the form of indices, can replace cap-weighted indices as a reference for the asset allocation policy.[1] This is not surprising, as the long-standing monopoly and popularity of cap-weighted indices as benchmarks, owing to their simplicity, are not easy to replace. Smart beta techniques find rather broader application as a complement to cap-weighted indices and as a substitute for active managers. Ultimately, smart beta is often perceived as a means of improving investment performance in an asset class through diversification, or more recently through factor investing.

When used in its latter role, the comparison between smart beta strategies and active managers becomes unfair because smart beta strategies usually exhibit high levels of tracking error, extreme tracking error and relative drawdown, while active managers operate under strict, explicit target-tracking-error constraints. Active manager performance appraisal typically pays a great deal of attention to the relative risk budgets that have been used to achieve outperformance. Therefore, in order to truly replace active managers, we require smart beta strategies that are able to respect strict relative-risk targets.

Panel A of figure 1 shows that the relative risk of common smart beta strategies is much higher than that of a typical benchmarked manager. The extreme tracking error is of the order of 8–11% and maximum relative drawdown over a 40-year period can be as high as 30–40%. Panel B of figure 1, which displays the relative risk of smart factor indices over the very long term, shows that the choice of factor exposure, even when accompanied by good diversification of the index that represents it, which is the case with multi-strategy weighting[2], can also lead to higher tracking error and maximum relative drawdown.

1 See for example Amenc, N., F. Goltz, V. Le Sourd and A. Lodh, July 2015. *Alternative Equity Beta Investing: a Survey*. EDHEC-Risk Institute Publication (p. 137), produced with the support of SGCIB (Newedge).

2 Multi-strategy draws on the following smart beta weighting schemes: maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient maximum Sharpe ratio and efficient minimum volatility. For more information, please refer to the Scientific Beta Diversified Multi-Strategy Index white paper.

1. Overview of relative risk

Panel A

31 Dec 1974–31 Dec 2014 (40 years)	Scientific Beta US LTTR				
	Maximum deconcentration	Diversified risk weighted	Maximum decorrelation	Efficient minimum volatility	Efficient maximum Sharpe ratio
Annualised relative returns	2.56%	2.57%	2.60%	2.50%	2.87%
Tracking error	4.12%	4.06%	4.14%	5.09%	4.33%
Maximum relative drawdown	30.07%	34.10%	30.00%	40.10%	30.66%
Three-year rolling TE mean	3.94%	3.74%	4.01%	4.75%	4.08%
Three-year rolling TE std dev	1.37%	1.71%	1.21%	2.06%	1.67%
Three-year rolling TE 95th percentile	7.02%	8.30%	6.99%	10.42%	8.67%

Panel B

31 Dec 1974–31 Dec 2014 (40 years)	Scientific Beta US LTTR Diversified Multi-Strategy					
	Mid cap	Momentum	Low volatility	Value	Low investment	High profitability
Annualised relative returns	4.59%	3.49%	2.87%	4.54%	3.89%	3.33%
Tracking error	6.38%	4.72%	6.04%	5.56%	5.44%	4.39%
Maximum relative drawdown	42.06%	17.28%	43.46%	32.68%	38.49%	25.21%
Three-year rolling TE mean	6.13%	4.53%	5.40%	5.19%	5.07%	4.27%
Three-year rolling TE std dev	2.03%	1.52%	2.97%	2.18%	2.20%	1.29%
Three-year rolling TE 95%ile	11.00%	7.74%	13.87%	11.20%	11.03%	6.86%

The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). The benchmark used is the Scientific Beta USA LTTR CW index. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The Scientific Beta USA LTTR universe contains 500 stocks. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Source: scientificbeta.com/Scientific Beta USA Long-Term Track Records.

Multi-factor diversification to manage tracking error

In the case of this article, we take six highly liquid Scientific Beta smart factor indices – mid cap, value, momentum, low volatility, low investment and high profitability diversified multi-strategy.[3] The choice of liquidity is guided by the concern to avail of smart indices that facilitate dynamic risk allocation. Figure 2 shows that the systematic risks to which the Scientific Beta smart factor indices provide consistent exposure are not synchronised, which suggests potential to smooth investment risk by holding a portfolio of single-factor indices. Low correlations between the relative returns of these indices suggest that multi-factor solutions will achieve tracking error reduction.

The first multi-factor approach to reduce tracking error is to use a relative equal risk contribution (rel-ERC) allocation. The objective is to equalise ex-ante tracking error risk from each of the underlying components. Thirty highly-liquid factor indices representing the six systematic risk factors discussed above and five weighting schemes are used for this approach. Weighting scheme diversification is achieved by using five weighting schemes – maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. The optimisation problem can be specified as follows:

$$TE(w)=\sqrt{w^T*\Omega*w} \quad RC_i = w_i.\frac{(\Omega*w)_i}{\sqrt{w^T*\Omega*w}} \qquad (1)$$

$$RC_i = RC_{ij}\ \forall\ i,j \quad \begin{cases} \sum_{i=1}^{N} w_i = 1 \\ w_i \geq 0 \forall\ i \end{cases} \qquad (2)$$

W_i represents the weight of the *i-th* constituent. RC_i is the contribution of the *i-th* constituent to portfolio tracking error. N is the number of constituents. Ω is the covariance matrix of total excess returns over the benchmark and is estimated using weekly total returns over the past 104 weeks.

Figure 3 shows that the tracking error of a relative-ERC allocation over the entire analysis period is a mere 3.22% and the improved information ratio is 0.86. This approach provides much better results in terms of relative risk (tracking error) management compared to the flagship Scientific Beta Multi-Beta Multi-Strategy EW index, which equalises the weights of the available indices. In order to analyse the time variation of tracking error, we also look at tracking error over three-year rolling windows. Although this approach is successful in bringing down the average level of tracking error, it does not guarantee explicit control of tracking error. The 95% worst three-year tracking error of relative-ERC allocation is 5.89% while the standard deviation is 1.09%. The maximum relative drawdown (with respect to a broad cap-weighted index) of this strategy is brought down to 14.82%. This is far below the level of maximum relative drawdown of traditional smart beta strategies, but still counts as sizeable underperformance for a benchmarked manager.

3 The following selection rules are applied to select stocks for each tilt: mid cap: bottom 50% free-float-adjusted market-cap stocks are selected; value: top 50% stocks are selected by book-to-market (B/M) ratio, B/M is defined as the ratio of available book value of shareholders' equity to company market cap; high momentum: top 50% stocks are selected by returns over past 52 weeks, minus the last four weeks; low volatility: bottom 50% stocks are selected by their standard deviation of weekly stock returns over the past 104 weeks; high profitability: top 50% stocks with highest gross profit/total asset ratio are selected; low investment: bottom 50% stocks with least two-year total asset growth rate are selected. This score-based selection is done twice a year (June and December) for momentum and once a year (June) for the other three factors. The 'highly liquid' version of the indices picks the 60% of stocks with the highest liquidity score amongst the stocks resulting from factor selection.

2. Correlation of excess returns

Panel A: Unconditional correlation

31 Dec 1974–31 Dec 2014 (40 years)	Scientific Beta Highly Liquid Diversified Multi-Strategies				
	High momentum	Low volatility	Value	Low investment	High profitability
Mid cap	0.35	0.31	0.71	0.62	0.41
High momentum		0.32	0.40	0.50	0.42
Low volatility			0.52	0.74	0.29
Value				0.71	0.15
Low investment					0.39

Panel B: Conditional correlation - bull markets

31 Dec 1974–31 Dec 2014 (40 years)	Scientific Beta Highly Liquid Diversified Multi-Strategies				
	High momentum	Low volatility	Value	Low investment	High profitability
Mid cap	0.37	0.28	0.70	0.62	0.45
High momentum		0.30	0.41	0.47	0.38
Low volatility			0.48	0.68	0.23
Value				0.70	0.16
Low investment					0.34

Panel C: Conditional correlation - bear markets

31 Dec 1974–31 Dec 2014 (40 years)	Scientific Beta Highly Liquid Diversified Multi-Strategies				
	High momentum	Low volatility	Value	Low investment	High profitability
Mid cap	0.33	0.33	0.73	0.61	0.37
High momentum		0.35	0.39	0.53	0.48
Low volatility			0.57	0.81	0.35
Value				0.71	0.14
Low investment					0.46

The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). The benchmark used is the Scientific Beta USA LTTR CW index. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The Scientific Beta US LTTR universe contains 500 stocks. Source: scientificbeta.com/Scientific Beta US Long-Term Track Records.

3. Relative ERC allocation performance

31 Dec 2004–31 Dec 2014 (10 years)	Broad CW index	Scientific Beta US LTTR High Liquidity MBMS EW	Scientific Beta US LTTR High Liquidity Rel-ERC
Annualised returns	12.16%	15.36%	14.93%
Annualised volatility	17.12%	16.01%	16.17%
Sharpe ratio	0.41	0.64	0.61
Maximum drawdown	54.53%	52.85%	52.77%
Annualised relative returns	–	3.21%	2.77%
Tracking error	–	3.84%	3.22%
Information ratio	–	0.83	0.86
Maximum relative drawdown	–	23.82%	14.82%
Three-year rolling TE mean	–	3.56%	3.07%
Three-year rolling TE standard deviation	–	1.58%	1.09%
Three-year rolling TE 95th percentile	–	8.04%	5.89%
Relative return bull markets	–	2.78%	2.72%
Relative return bear markets	–	3.58%	2.63%
Relative return 25% bull markets	–	2.90%	3.24%
Relative return 25% bear markets	–	3.48%	2.42%
One-way annual turnover	2.7%	28.3%	29.9%

The analysis is based on daily total returns in US dollars in the period 31 December 1974 to 31 December 2014 (40 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 160 rebalancings in the 40-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Quarters with positive benchmark index returns are bull quarters and the remaining are bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The Scientific Beta US LTTR universe contains 500 stocks. The MBMS EW and Rel-ERC strategies are based on 30 underlying strategies which are combinations of six factor tilts (mid cap, value, high momentum, low volatility, high profitability and low investment) and five diversification-based weighting schemes (maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio and diversified risk weighted). Post stock selection, we apply a high-liquidity filter which selects the most liquid constituents (60% most liquid stocks) among the stocks that belong to one of the available stock selections. Source: scientificbeta.com/Scientific Beta US Long-Term Track Records.

Core-satellite approach

As discussed before, the tracking error budgets of benchmarked active managers are smaller and therefore the multi-factor diversification solution is not a viable approach in its present form. However, in order to manage tighter tracking budgets, one must impose explicit tracking-error constraints by using the core-satellite approach. In this approach, we combine an optimised satellite portfolio whose tracking error is well behaved – the relative ERC allocation with the core portfolio, the broad cap-weighted index (Amenc et al [2012]).

In this example, we construct four portfolios with target tracking errors of 2.0%, 1.5%, 1.0%, and 0.50% respectively. The ratio in which core and satellite are combined quarterly is deter-

mined by the two-year ex-ante tracking error of the satellite and a buffer tracking error.

$$Ratio_{Satellite} = \frac{TE_{target}}{TE_{2-Y\,ex\,ante} + TE_{buffer}} \quad (3)$$

$$TE_{2-Y\,ex\,ante} = \sqrt{w^T * \Omega * w} \quad (4)$$

W is the optimised weights of constituent indices. Ω is the covariance matrix of total excess returns over the benchmark and is estimated using weekly total returns over the past 104 weeks. The buffer tracking error is a fixed long-term parameter that is calibrated only once. We calibrate it over the time period of the first 10 years (31 December 1974 to 31 December 1984). The buffer tracking error is the average three-year rolling tracking error observed over this period.

It should be noted that the allocation between core and satellite portfolios is dynamic in nature and allocates more weight to the satellite when its ex-ante tracking error goes down, thereby making efficient use of the relative risk budget. Figure 4 shows that the core-satellite approach is indeed successful in respecting the target tracking error ex-post. For example, the '1% target' portfolio's three-year rolling TE averages at 0.91% with a standard deviation of 0.20%. Despite a low tracking-error budget, the strategy delivers a strong probability of outperformance – 84% for a three-year investment horizon.

Improved management of tracking error

The approach described above, due to the relative importance of the variation in tracking error (the average value of tracking error of the relative ERC satellite over a period of three years is 3.07%, but the standard deviation of the tracking error over the same period is 1.09%, so the tracking error could be higher), led necessarily to an underexposure to the satellite ex-ante in order to take account of the volatility of the tracking error ex-post. In addition, even when underexposing to the satellite, the ex-post tracking error could be quite high, as the measure of extreme tracking error shows.

The approach proposed here aims to control the volatility of the satellite's tracking error better in order to improve the core-satellite allocation, by reducing the ex-ante weighting of the satellite compared to what it would have been if the ex-ante tracking error were representative of the ex-post tracking error, and to reduce the risk of extreme tracking error.

In order to manage tracking error in an efficient manner, one must understand that the tracking error (TE) risk is made up of two components – systematic TE and idiosyncratic TE. In order to outperform the cap-weighted (CW) benchmark, the portfolio has to seek risk factors that are different from those of the benchmark. This excess exposure to risk factors that are systematically rewarded in the long term becomes a source of tracking error. This variety of tracking error is termed systematic TE and since it is rewarded in nature it is desired tracking error.

The tracking error that cannot be explained by the exposure to systematic risk factors is unrewarded and therefore is undesired. This unrewarded or idiosyncratic TE can of course be reduced by good diversification of the satellite or its components. For example, the smart factor index approach proposed by Scientific Beta enables this idiosyncratic risk to be reduced substantially in comparison with traditional factor indices, which are often highly concentrated. Moreover, the multi-smart-factor allocation also allows this TE to be reduced.

Figure 5 compares the idiosyncratic TE of an array of factor-tilted indices in panel A and that of single-factor indices and multi-factor indices in panel B. Panel A shows that the idiosyncratic TE increases as concentration increases, since factor-tilted portfolios constructed on 20% of stocks have more idiosyncratic TE than those constructed on 50% of stocks. Panel B shows that the idiosyncratic TE of the equal-weighted multi-factor index is far below the average idiosyncratic TE of its components.

However we must recognise that even if we can reduce the idiosyncratic TE, there is also a choice of difference in systematic risk that is implicit and does not necessarily correspond to the choice of rewarded factors. It is that of the systematic market factor. In fact the majority of smart beta strategies have a market beta of less than 1. This difference in market beta has a strong influence on the tracking error of the strategy. ►

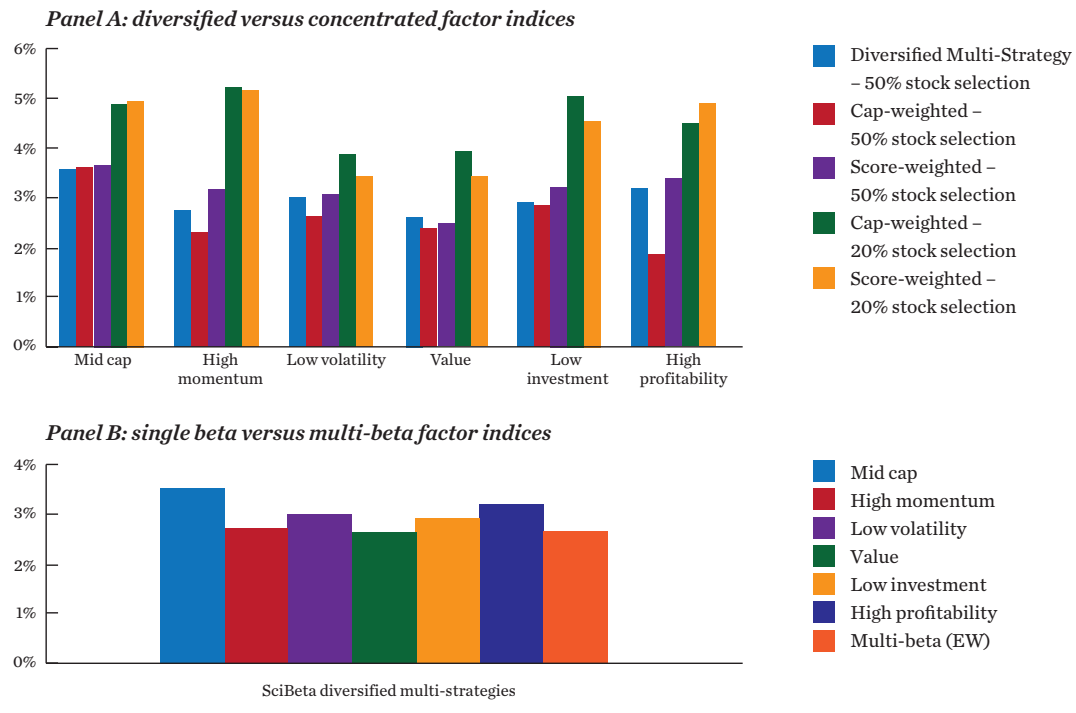
4. Core-satellite portfolio performance

31 Dec 1974–31 Dec 2014 (40 years)

	Broad CW index	0.5% target TE	1.0% target TE	1.5% target TE	2.0% target TE	Scientific Beta US LTTR HiLiq Rel-ERC
Annualised returns	12.16%	12.51%	12.86%	13.21%	13.56%	14.93%
Annualised volatility	17.12%	16.95%	16.80%	16.65%	16.52%	16.17%
Sharpe ratio	0.41	0.44	0.46	0.49	0.51	0.61
Maximum drawdown	54.53%	54.20%	53.86%	53.52%	53.18%	52.77%
Annualised relative returns	–	0.35%	0.70%	1.05%	1.40%	2.77%
Tracking error	–	0.46%	0.92%	1.38%	1.84%	3.22%
Information ratio	–	0.77	0.77	0.76	0.76	0.86
Outperformance probability (1Y)	–	76.33%	76.33%	76.28%	76.28%	76.03%
Average relative returns	–	0.33%	0.66%	0.99%	1.32%	2.62%
Average of positive relative returns	–	0.56%	1.13%	1.70%	2.27%	4.19%
Outperformance probability (3Y)	–	83.70%	83.70%	83.64%	83.64%	84.11%
Average relative returns	–	0.32%	0.64%	0.96%	1.28%	2.56%
Average of positive relative returns	–	0.46%	0.92%	1.38%	1.83%	3.45%
Outperformance probability (5Y)	–	89.00%	88.89%	88.84%	88.79%	90.97%
Maximum relative drawdown	–	2.64%	5.21%	7.73%	10.19%	14.82%
Three-year rolling TE mean	–	0.46%	0.91%	1.37%	1.83%	3.07%
Three-year rolling TE standard deviation	–	0.10%	0.20%	0.30%	0.40%	1.09%
Three-year rolling TE 95th percentile	–	0.61%	1.21%	1.82%	2.42%	5.89%
One-way annual turnover	2.7%	7.9%	12.0%	16.4%	21.0%	29.9%

The table presents the results of a core-satellite approach wherein the core is represented by SciBeta Long-Term United States Cap-Weighted and the satellite is represented by SciBeta Long-Term United States HLiq Rel-ERC. The analysis is based on daily total returns in US dollars in the period 31 December 1974 to 31 December 2014 (40 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 160 rebalancings in the 40-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Probability of outperformance is the probability of obtaining positive excess returns if one invests in the strategy for a period of 1 (or 3 or 5) years at any point during the history of the strategy. Rolling window of length 1 (or 3 or 5) years and a step size of 1 week is used. Corresponding average relative returns is the average of relative returns across all rolling windows and corresponding average of positive relative returns is the average across rolling windows where the relative returns are positive. Quarters with positive benchmark index returns are bull quarters and the remaining are bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The Scientific Beta US LTTR universe contains 500 stocks. Source: scientificbeta.com/Scientific Beta US Long-Term Track Records.

5. Idiosyncratic tracking error comparison



The analysis is based on daily total return data from 31 December 1974 to 31 December 2014 (40 years). The mid cap, high momentum, low volatility, value, low investment and high profitability selections represent the 50%/20% of stocks with such characteristics in a US universe of 500 stocks. The Carhart four-factor model is used. The market, size, value and momentum factors for the US universe available online in Kenneth French's data library are used. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The Scientific Beta US LTTR universe contains 500 stocks. Source: scientificbeta.com/Scientific Beta USA Long-Term Track Records.

◄ In order to deal with the problem of low market beta, we propose another multi-factor approach that neutralises beta bias – maximum deconcentration with a constraint of unitary market beta. The optimisation problem can be written as:

$$w^* = argmax\left\{1/\left(w^T * w\right)\right\} \quad \begin{cases} \sum_{i=1}^{N} w_i = 1 \\ w_i \geq 0 \,\forall\, i \\ beta = \dfrac{Cov\left[\left(R * w\right), R_{CW}\right]}{Var\left(R_{CW}\right)} = 1 \end{cases} \qquad (5)$$

W_i represents the weight of the i-th constituent. R_{CW} is benchmark returns and R is component returns. N is the number of constituents. Beta is estimated using weekly total returns over the past 104 weeks.

Figure 6 shows that beta-constrained maximum deconcentration achieves low tracking error with an improved information ratio (0.91). Compared to the flagship Scientific Beta US LTTR MBMS EW, the improvement in information ratio is 9.64%, with a reduction in TE from 3.84% to 3.26%. The most striking feature of this allocation is its maximum relative drawdown, which is 7.30% compared to 23.82% for the equal-weighted allocation. Due to market beta constraints, the strategy does not fall short on performance when the markets are bullish like most smart beta strategies do. The standard deviation and 95th percentile of three-year rolling tracking error also show improvement, meaning that tracking error is more stable over time. Its 'well behaved' tracking error makes it a good candidate for a satellite in core-satellite allocations.

In conclusion, we find that value, in terms of risk-adjusted relative performance, can be added through allocation across smart factor indices, for investors with a tracking error budget. The favourable factor tilts generate outperformance and two-fold diversification, one across factors and another across weighting schemes, reducing tracking error. As a result, extremely substantial levels of relative risk-adjusted outperformance can be achieved. Implementation of an allocation that guarantees a level of market beta equivalent to that of a cap-weighted index allows the benefits of this relative risk diversification to be optimised. Figure 7 shows that this dynamic allocation between smart factor indices can be combined with a core-satellite approach to limit the tracking error to desired levels, as low as 0.5%, while maintaining high information ratios.

References

Amenc, N., F. Goltz, V. Le Sourd and A. Lodh (2015). *Alternative Equity Beta Investing: a Survey*. EDHEC-Risk Institute Publication produced with the support of SGCIB (Newedge).

Amenc, N., F. Goltz, A. Lodh and L. Martellini (2012). Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance. *Journal of Portfolio Management* 38(3): 72–88.

Amenc, N., F. Goltz, A. Lodh and L. Martellini (2014). Towards Smart Equity Factor Indices: Harvesting Risk Premia without Taking Unrewarded Risks. *Journal of Portfolio Management* 40(4).

Amenc, N., F. Goltz, A. Lodh, L. Martellini and S. Sivasubramanian (2015). *Scientific Beta Multi-Strategy Factor Indices: Combining Factor Tilts and Improved Diversification*. ERI Scientific Beta publication.

Lodh, A., and S. Sivasubramanian (2015). *Scientific Beta Diversified Multi-Strategy Index*. ERI Scientific Beta publication.

6. Maximum deconcentration (beta=1) allocation performance

31 Dec 1974–31 Dec 2014 (40 years)

	Broad CW index	High liquidity MBMS EW	High liquidity MBMS Rel-ERC	Scientific Beta US LTTR high liquidity maximum deconcentration (beta=1)
Annualised returns	12.16%	15.36%	14.93%	15.11%
Annualised volatility	17.12%	16.01%	16.17%	16.87%
Sharpe ratio	0.41	0.64	0.61	0.59
Maximum drawdown	54.53%	52.85%	52.77%	53.59%
Annualised relative returns	–	3.21%	2.77%	2.95%
Tracking error	–	3.84%	3.22%	3.26%
Information ratio	–	0.83	0.86	0.91
Maximum relative drawdown	–	23.82%	14.82%	7.30%
CAPM market beta	–	0.91	0.93	0.97
Carhart market beta	–	0.93	0.94	0.99
Three-year rolling TE mean	–	3.56%	3.07%	3.17%
Three-year rolling TE standard deviation	–	1.58%	1.09%	0.93%
Three-year rolling TE 95th percentile	–	8.04%	5.89%	5.20%
Relative return bull markets	–	2.78%	2.72%	4.15%
Relative return bear markets	–	3.58%	2.63%	1.00%
Relative return 25% bull markets	–	2.90%	3.24%	6.90%
Relative return 25% bear markets	–	3.48%	2.42%	0.64%
One-way annual turnover	2.7%	28.3%	29.9%	40.2%

The analysis is based on daily total returns in US dollars in the period 31 December 1974 to 31 December 2014 (40 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 160 rebalancings in the 40-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Quarters with positive benchmark index returns are bull quarters and the remaining are bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The Scientific Beta US LTTR universe contains 500 stocks. The MBMS EW and Rel-ERC strategies are based on 30 underlying strategies which are combinations of six factor tilts (mid cap, value, high momentum, low volatility, high profitability and low investment) and five diversification-based weighting schemes (maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio and diversified risk weighted). Post stock selection, we apply a high-liquidity filter which selects the most liquid constituents (60% most liquid stocks) among the stocks that belong to one of the available stock selections. The last column presents results of another approach that neutralises beta bias – maximum deconcentration with a constraint of unitary market beta. Source: scientificbeta.com/Scientific Beta US Long-Term Track Records.

7. Core-satellite portfolio performance summary

31 Dec 1974–31 Dec 2014 (40 years)

	Broad CW index	0.5% target TE	1.0% target TE	1.5% target TE	2.0% target TE	Scientific Beta US LTTR maximum deconcentration (beta=1)
Annualised returns	12.16%	12.52%	12.89%	13.25%	13.61%	15.11%
Annualised volatility	17.12%	17.05%	16.99%	16.95%	16.91%	16.87%
Sharpe ratio	0.41	0.43	0.46	0.48	0.50	0.59
Maximum drawdown	54.53%	54.36%	54.20%	54.03%	53.87%	53.59%
Annualised relative returns	–	0.36%	0.73%	1.09%	1.43%	2.95%
Tracking error	–	0.43%	0.86%	1.29%	1.72%	3.26%
Information ratio	–	0.85	0.85	0.85	0.84	0.91
Maximum relative drawdown	–	1.17%%	2.33%	3.48%	4.63%	7.30%
Three-year rolling TE mean	–	0.43%	0.85%	1.28%	1.71%	3.17%
Three-year rolling TE standard deviation	–	0.09%	0.18%	0.27%	0.36%	0.93%
Three-year rolling TE 95th percentile	–	0.59%	1.19%	1.78%	2.37%	5.20%
One-way annual turnover	2.7%	8.8%	13.9%	19.3%	24.8%	40.2%

The table presents the results of a core-satellite approach wherein the core is represented by SciBeta Long-Term US Cap-Weighted and the satellite is represented by an approach that neutralises beta bias – maximum deconcentration with a constraint of unitary market beta. The analysis is based on daily total returns in US dollars in the period 31 December 1974 to 31 December 2014 (40 years). All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 160 rebalancings in the 40-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of strategy index to the benchmark index. The Scientific Beta US LTTR universe contains 500 stocks. Source: scientificbeta.com/Scientific Beta US Long Term Track Records.

Scientific Beta
An EDHEC-Risk Institute Venture

EDHEC-Risk Institute
393 promenade des Anglais
BP 3116 – 06202 Nice Cedex 3
France
Tel: +33 (0)4 93 18 78 24

EDHEC Risk Institute–Europe
10 Fleet Place – Ludgate
London EC4M 7RB
United Kingdom
Tel: +44 207 871 6740

EDHEC Risk Institute–Asia
1 George Street
#07-02
Singapore 049145
Tel: +65 6438 0030

ERI Scientific Beta–North America
One Boston Place, 201 Washington Street
Suite 2608/2640
Boston – MA 02108 – USA
Tel: +1 857 239 8891

EDHEC Risk Institute—France
16-18 rue du 4 septembre
75002 Paris
France
Tel: +33 (0)1 53 32 76 30

ERI Scientific Beta–Japan
East Tower 4th Floor, Otemachi First Square
1-5-1 Otemachi, Chiyoda-ku
Tokyo 100-0004 – Japan
Tel: +81 352 191 418

www.edhec-risk.com