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Introduction
Noël Amenc

It is a great pleasure to introduce the latest Scientific Beta special issue of the Research Insights supplement to IPE.

We first clarify the conceptual underpinnings and the need for diversification in factor investing, discuss the benefits of combining various factor strategies, the evolution of multi-factor allocation in recent times and the key features that distinguish the various multi-factor offerings.

We show that it is possible to reconcile environmental and financial objectives using low carbon indices. While these indices achieve an environmental objective by excluding high carbon stocks, and thus putting pressure on high polluting companies to reform, they achieve a financial objective by retaining exposure to rewarded risk factors and by maintaining a high level of diversification.

We analyse the benefits of multi-smart factor indices for emerging markets. We make a case for avoiding country bets for emerging market indices, by highlighting the fact that applying country neutrality is better than taking high tracking error risk, by not being country neutral, when it is not rewarded.

On the subject of defensive solutions and indices, we present three separate articles. We start by looking at the concepts underlying low risk equity strategies. We then introduce alternative approaches to limiting concentration in minimum and low volatility strategies. Finally, we introduce solutions which rely on a risk-based allocation model to dynamically allocate to smart factor indices carrying long-term risk premia, with a view to delivering a dissymmetric defensive profile.

We look at the live performance of Scientific Beta Multi-Beta Multi-Strategy indices. Live performance does not benefit from hindsight in the way that back-tests potentially can, so the key question for investors is not back-tested performance, but the live performance they will ultimately experience when adopting indices.

Finally, we review the current offerings in the world of multi-factor indices and look at the conceptual considerations involved in designing the different approaches. The key issues that we discuss involve the robustness and consistency of the multi-factor indices as well as the (lack of) diversification among the various products.

We hope you will find the articles on smart beta in the supplement informative and relevant. We extend our warmest thanks to our friends at IPE for their partnership on the supplement.

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The evolution of multi-factor indices: smart factor indices, multi-beta indices and solutions

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Smart beta product offerings have proliferated over the past decade, offering investors an ample choice of different factors and different weighting schemes to select a relevant smart beta index from. Leading academic studies document that small/mid-cap, momentum, low volatility, value, low investment and high profitability stocks generate higher risk-adjusted returns than broad equity exposure in the long term. However, in addition to the question of selecting a suitable index as a stand-alone investment, the question of combining different smart beta strategies naturally arises in the context of an extensive range of smart beta offerings. This article clarifies the conceptual underpinnings and the need for diversification in factor investing, discusses the benefits of combining various factor strategies, the evolution of multi-factor allocation in recent times and the key features that distinguish the various multi-factor offerings.

Designing efficient and investable proxies for risk premia

Current smart beta investment approaches only provide a partial answer to the main shortcomings of capitalisation-weighted (cap-weighted) indices. We discuss a new approach to equity investing referred to as smart factor investing. It provides an assessment of the benefits of simultaneously addressing the two main shortcomings of cap-weighted indices, namely their undesirable factor exposures and their heavy concentration, by constructing factor indices that explicitly seek exposures to rewarded risk factors while diversifying away unrewarded risks. Addressing these two points simultaneously is made possible through the Smart Beta 2.0 approach, which combines a stock selection step (to select stocks with the desired factor tilt or characteristics) with a diversification-based weighting scheme. This weighting scheme is applied to the relevant stock selection to obtain a well-diversified portfolio within a given factor tilt.

Factor investing postulates that investors are rewarded for gaining exposure to the right risk factors. However, having a well-diversified portfolio is equally important to selecting the right factor tilts. Many factor index providers ignore the aspect of diversification and solely focus on maximising the factor exposures. For example, many providers follow either cap-weighting or score times market cap-weighting to construct factor indices. However, it is well known that these weighting schemes result in concentration in few stocks, which leads to excessive stock-level specific risks. Amenc et al (2016) explains in detail the benefits of well-diversified factor indices and the drawbacks of concentrated factor indices. Diversification is the only free lunch available in finance and investors ignore diversification at their peril. Thus, following the Smart Beta 2.0 approach of combining the factor-based stock selection with a diversification-based weighting scheme benefits the investor by minimising unrewarded risks while maintaining the desired factor exposure.

However, different diversification-based weighting schemes come with their own specific risks in addition to the stock-level specific risks. As per modern portfolio theory, each investor should optimally combine risky assets so as to achieve the maximum Sharpe ratio (MSR) portfolio. Implementing this objective, however, is a complex task because of the presence of estimation risk for the required parameters, namely expected returns and covariance parameters. In practice, the costs of estimation error may entirely offset the benefits of optimal portfolio diversification. Similarly, an investor may be better off, for example, investing in a proxy for the global minimum variance (GMV) portfolio or the equal risk contribution (ERC) portfolio, which only require estimates for covariance parameters, as opposed to trying to estimate the MSR portfolio, which also requires expected return estimates that are known to be noisier (Merton [1980]).

In other words, the choice in risk and return parameter estimation for efficient diversification is between 'trying', which has a cost related to estimation risk (ie, the risk of a substantial difference between the estimated parameter value and the true parameter value) or 'giving up', which has a cost related to optimality risk, that is the risk that the simplified benchmark (such as the EW or GMV portfolio) can be far from the optimal MSR benchmark. Different portfolios are intuitively expected to incur more estimation risk or more optimality risk. The parameter estimation risk of the maximum deconcentration weighting (which is a practical and flexible implementation of EW that reduces to EW in the absence of constraints) is zero, but it contains high optimality risk because this portfolio is optimal only in the very special case when the volatilities, expected returns and pair-wise correlations of all stocks are identical. Similarly, the efficient MSR portfolio does not have any optimality risk, but it does entail high parameter estimation risk as one needs a reasonable estimate of expected returns, volatilities and correlations for a robust MSR optimisation. Direct estimation of expected returns from past returns is not likely to lead to robust outcomes (see Merton [1980]). However, improved estimates of expected returns can be estimated indirectly by assuming a relationship between risk and return, in particular the downside risk. Chen et al (2009) document a positive relationship between expected return and downside risk.[1] Amenc et al (2011) draw upon this literature and propose a way to estimate expected return by assuming a relationship between downside risk and return. In particular, excess expected return over the risk-free rate is proxied by the median semi-deviation[2] of the decile group to which a stock belongs, where deciles are formed after sorting stocks on the basis of semi-deviation. This is a parsimonious approach, which does not rely on either direct estimation of expected return from past returns or from specifying an asset pricing model to derive expected return estimates, thus providing more robust MSR proxies.

With varying trade-offs between optimality and estimation risks different weighting schemes have different specific risks which are unrewarded. Ultimately, a weighting strategy diversification approach, like in the case of multi-management, allows these risks to be diversified, except that in active management the manager's risks are more difficult to identify, and in particular, behavioural biases mean that these managers often tend to behave in the

1 This is in contrast to the CAPM, which predicts that expected excess returns are proportional to betas.

2 The semi-deviation is a measure of the downside risk. Compared to volatility, the semi-deviation is a more meaningful definition of risk since it only takes into account deviations below the mean. Over the calibration period, we compute the semi-deviation of the returns of each constituent SEM_i with respect to the sample mean return $\hat{\mu}_i$ of the *i-th* stock as

$$SEM_i = \sqrt{E\left\{\min\left[r_{i,t} - \bar{\mu}_i\right], 0\right\}^2}$$

where E(.) denotes the expectation operator computed as the arithmetic average, min(x,y) denotes the minimum of x and y, and $r_{i,t}$ is the return of stock *i* in week t. Our aim is to estimate expected return indirectly rather than directly. It is clear, however, that any risk measure is typically defined as a deviation from the mean which means that the calculation of the mean is included in the risk measure. However, without loss of generality one could in principle replace the mean in such deviation measures with zero. We use the below mean semi-deviation rather than below zero semi-deviation in order to avoid acquiring any reversal effects. In fact, by looking at the below mean semi-deviation, we ignore differences in mean returns across stocks, whereas using below zero semi-deviation would – all else equal – lead to higher semi-deviations for stocks with low average returns. The idea behind using below mean semi-deviation is to take into account only the shape of the distribution around the mean return rather than the level of the mean return.

1. Diversifying away unrewarded risks: performance comparison of US cap-weighted factor indices, US score times cap-weighted factor indices and US multi-strategy factor indices

US long term (Dec 1970–Dec 2015)		Mid cap			High momentum			Low volatility		
	Broad CW	Score weighted	Cap weighted	Multi strategy	Score weighted	Cap weighted	Multi strategy	Score weighted	Cap weighted	Multi strategy
Annualised returns	10.45%	13.47%	12.97%	14.22%	12.47%	11.39%	13.39%	10.94%	10.76%	13.00%
Annualised volatility	16.88%	17.10%	17.09%	15.80%	18.06%	17.26%	16.01%	14.44%	15.39%	13.85%
Sharpe ratio	0.32	0.49	0.46	0.58	0.41	0.37	0.52	0.41	0.37	0.57
Maximum drawdown	54.63%	57.92%	57.09%	53.42%	52.30%	50.81%	53.25%	43.76%	51.10%	48.31%
Annualised excess returns	–	3.02%	2.52%	3.77%	2.02%	0.94%	2.95%	0.49%	0.32%	2.55%
Annualised tracking error	–	6.63%	5.72%	6.42%	5.34%	3.49%	4.84%	5.58%	4.27%	5.99%
One-year rolling TE 95%ile	–	11.69%	9.27%	11.54%	10.84%	6.24%	8.59%	12.14%	8.18%	11.38%
Information ratio	–	0.46	0.44	0.59	0.38	0.27	0.61	0.09	0.07	0.43
Maximum relative drawdown	–	47.99%	35.94%	42.06%	21.45%	14.44%	17.28%	37.72%	33.82%	43.46%

US long term (Dec 1970–Dec 2015)		Value			Low investment			High profitability		
	Broad CW	Score weighted	Cap weighted	Multi strategy	Score weighted	Cap weighted	Multi strategy	Score weighted	Cap weighted	Multi strategy
Annualised returns	10.45%	13.06%	11.90%	14.28%	12.63%	12.32%	14.01%	11.05%	10.79%	12.99%
Annualised volatility	16.88%	17.97%	17.17%	15.67%	15.93%	15.83%	14.93%	17.01%	17.02%	15.73%
Sharpe ratio	0.32	0.44	0.40	0.59	0.47	0.46	0.60	0.35	0.34	0.50
Maximum drawdown	54.63%	62.27%	60.01%	53.75%	52.05%	51.12%	50.82%	52.72%	52.29%	48.86%
Annualised excess returns	–	2.61%	1.45%	3.84%	2.19%	1.87%	3.57%	0.60%	0.34%	2.54%
Annualised tracking error	–	5.65%	4.42%	5.47%	4.27%	3.79%	5.42%	4.36%	3.22%	4.35%
One-year rolling TE 95%ile	–	9.95%	8.12%	10.03%	7.58%	6.58%	9.88%	7.44%	6.48%	7.20%
Information ratio	–	0.46	0.33	0.70	0.51	0.49	0.66	0.14	0.11	0.58
Maximum relative drawdown	–	20.28%	20.31%	32.68%	25.32%	26.47%	38.49%	35.91%	24.52%	25.21%

The analysis is based on daily total return data from 31 December 1970 to 31 December 2015 (45 years). The benchmark used for the relative analytics is the SciBeta CW US 500 index. Mid cap, high momentum, low volatility, value, low investment and high profitability selections all represent 50% of stocks with such characteristics in a US universe of 500 stocks. The 'Score weighted' columns represent the performance of corresponding factor score times market cap-weighted indices, the 'Cap weighted' columns represent the performance of corresponding factor cap-weighted indices and the 'Multi strategy' columns represent the performance of corresponding factor diversified multi-strategy indices. The risk-free rate is the return of the 3-month US Treasury Bill. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of one (or three) years at any point during the history of the strategy. A rolling window of length of one (or three) years and a step size of one week are used. Source: www.scientificbeta.com.

same way during extreme market phases when diversification of their management styles is most needed.

Scientific Beta follows a multi-strategy weighting scheme to construct its smart factor indices. The diversified multi-strategy weighting scheme is an equal-weighted combination of the following five weighting schemes – maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. Amenc et al (2014) elaborate the benefits of a diversified multi-strategy weighting scheme and suggest that the smart factor indices constructed based on diversified multi-strategy weighting lead to considerable improvements in risk-adjusted performance.

The empirical results in figure 1 confirm the above theoretical discussion. Two main observations can be made from figure 1:

- The combination of relevant security selection and appropriate weighting schemes in a two-step process leads to substantial improvements in risk-adjusted performance compared to the broad cap-weighted benchmark. The Sharpe ratio of the broad cap-weighted index is 0.32 on the long-term sample period (45 years) and the Sharpe ratio of all the factor-tilted indices regardless of the weighting scheme chosen is higher than that of the broad cap-weighted index. This finding reasserts that the six factors carry a long-term premium, and in this sense constitute rewarded risk.
- A well-diversified weighting scheme such as the diversified multi-strategy used by Scientific Beta further adds diversification benefits compared to the more concentrated weighting schemes such as market-cap weighting or score times market-cap weighting. The average Sharpe ratio across the six factor tilts based on diversified multi-strategy indices is 0.56 compared to the average Sharpe ratio of 0.43 in the case of score times market cap-weighted factor tilts. This represents an increase of 30.74% in the Sharpe ratio from using the well-diversified multi-strategy weighting schemes.

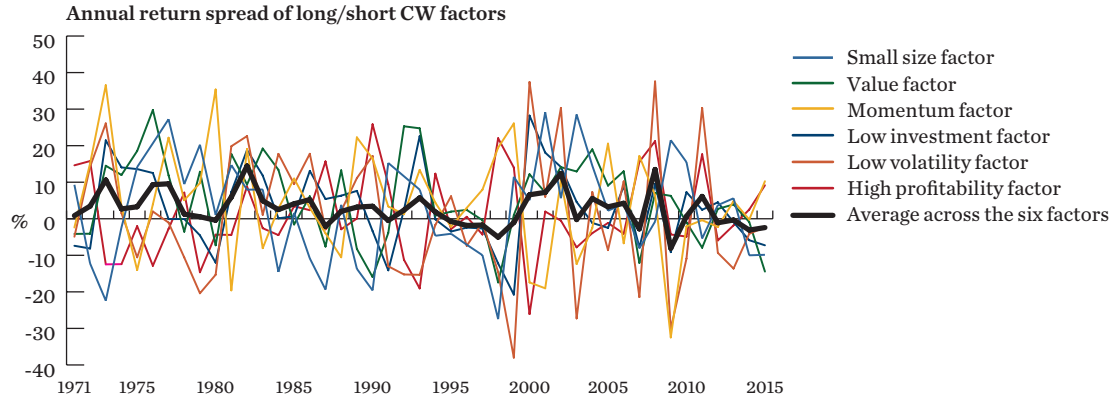
As we have seen above, the Scientific Beta Diversified Multi-Strategy factor indices present very good long-term performance. However, one may expect further benefits by allocating across different factor premia rather than focusing on a single factor tilt, notably because the academic literature and empirical research show that there is a good level of decorrelation for the risk premia associated with these factors. This allocation across different rewarded factors is at the heart of multi-smart-beta-allocation approaches, which we turn to below.

Benefits of multi-factor allocation

There is strong intuition suggesting that multi-factor allocations will tend to result in improved risk-adjusted performance. In fact, even if the factors towards which the factor indices are tilted are all rewarded over the long term, there is extensive evidence that they may each encounter prolonged periods of underperformance. More generally, the rewards for exposure to these factors have been shown to vary over time (Harvey [1989]; Asness [1992]; Cohen, Polk and Vuolteenaho [2003]). If this time variation in returns is not completely in sync for different factors (see figure 2 for an illustration of the different cyclicality of typical long/short factors), allocating across factors allows investors to diversify the sources of their outperformance and smooth their performance across market conditions (as different factors work at different times).

Furthermore as shown in figure 3, the ►

2. Cyclicality of the factors – US example



Calendar year returns of risk factors – Factors are obtained from the Scientific Beta US long-term track records. The analysis is based on daily total returns in US dollars from 31 December 1970 to 31 December 2015 (45 years). The size/momentum/volatility factors are long/short cap-weighted portfolios long the stocks in the bottom three size deciles/top three past 12M-1M stock return deciles/bottom three past 2Y volatility deciles and short the stocks in the top three size deciles/bottom three past 12M-1M stock return deciles/top three past 2Y volatility deciles. The valuation/investment /profitability factors are long/short cap-weighted portfolios long the stocks in the top three book-to-market deciles/bottom three 2Y total asset growth rate deciles/top three gross profit-to-total asset ratio deciles and short the stocks in the bottom three book-to-market deciles/top three 2Y total asset growth rate deciles/bottom three gross profit-to-total asset ratio deciles. The average across the six factors is the mean annual return in each year.

◄ pair-wise correlations between the relative returns of the six smart factor indices over the cap-weighted benchmark are not perfect (below 1). Importantly, this indicates that a combination of these indices would significantly lower the overall tracking error of the portfolio relative to the cap-weighted benchmark. This is consistent with research findings in asset allocation studies. For instance, Ilmanen and Kizer (2012) have shown that factor diversification was more effective than the traditional asset-class diversification method (and that the benefits of factor diversification were still very meaningful for long-only investors).

Figure 4 provides performance and risk results for six-factor multi-smart factor indices for both US long-term track records (43 years). It is of particular interest to compare the risk-adjusted relative performance (information ratio), relative risk and extreme relative risk of the combination to the stand-alone results obtained by each single factor tilt. The information ratio (IR) of the multi-factor combination is indeed higher than the average information ratio of the six factor-tilted indices that make up its components. This higher IR is observed because of the reduction in tracking error. The tracking error of the multi-smart factor indices is less than that of the average of the component single factor indices. The multi-smart indices reduce the tracking error on average by 15% compared to that of the average of the component single factor indices. The reduction in extreme relative risk is even more significant. The multi-smart indices reduce the extreme tracking error on average by 19% compared to that of the average of the component single factor indices. This clearly shows the allocation effect of diversifying across different factor tilts, which elevates risk-adjusted performance relative to the average result for component indices.

Evolution of multi-factor allocation

Size, momentum, volatility and value have been known as rewarded factors for a very long time and they have been well documented and empirically analysed in the literature. The live performance of these factors has been assessed by investors for a considerable length of time. However, more recently, two new factors associated with the 'quality' factor premium have been identified in the literature, namely investment and profitability. Consequently, two different variants of multi-smart factor allocations are available as part of Scientific Beta's flagship multi-factor offerings. The four-factor multi-smart factor, also known as the four-factor multi-beta multi-strategy index, provides exposure to the four well-known factors, namely size, value, momentum and low volatility, and the six-factor multi-smart factor, also termed six-factor multi-beta multi-strategy index, provides exposure to the four well-known factors and the two new 'quality' factors – investment and profitability. These two new factors have been subject to extensive discussion as to their introduction into the set of rewarded factors. It is always important for academic researchers to justify the introduction of new factors, not only because there is empirical evidence of their existence, but also because there is genuine economic justification for these factors. It was only when a consensus on the economic justification for these factors had been achieved that ERI Scientific Beta accepted to include them in the choice of smart factor indices[3]. Figure 5 presents an overview of the economic justifications both from the rational asset pricing viewpoint favoured by EDHEC-Risk Institute researchers, and from a behavioural viewpoint.

Also, as seen from figure 3, the two new factors are imperfectly correlated with each other and with the other four factors, thus making a strong case to include the two factors to construct the six factor multi-smart factor indices. Adding these two factors further adds to the diversification benefits by providing a smoothed outperformance. Thus, six-factor multi-smart factor indices were launched by Scientific Beta on 18 September 2015. Figure 6 compares the performance of six-factor and four-factor multi-smart factor indices over a 43-year period. As can be seen from figure 6, the six-factor multi-smart factor indices have similar levels of returns (13.72% for EW and 13.50% for ERC) compared to the four-factor multi-smart factor indices (13.83% for EW and

3 For a more detailed discussion of the investment and profitability factors, please refer to Amenc et al (2016b).

3. Correlation of excess returns across factor tilts

US Long Term (Dec 1970–Dec 2015)	Mid cap	High momentum	Low volatility	Value	Low investment	High profitability
	Diversified Multi-Strategy					
Mid cap	1.00	0.74	0.67	0.85	0.85	0.77
High momentum		1.00	0.58	0.66	0.71	0.65
Low volatility			1.00	0.75	0.84	0.61
Value				1.00	0.86	0.56
Low investment					1.00	0.70
High profitability						1.00

Daily total returns in US dollars from 31 December 1970 to 31 December2015 are used for the US long-term universe. Correlations among all factor-tilted multi-strategy indices. The universe contains 500 stocks. The full names of the multi-strategy indices used are: SciBeta United States LTTR Mid-Cap Diversified Multi-Strategy, SciBeta United States LTTR High-Momentum Diversified Multi-Strategy, SciBeta United States LTTR Low-Volatility Diversified Multi-Strategy, SciBeta United States LTTR Value Diversified Multi-Strategy, SciBeta United States LTTR Low Investment Diversified Multi-Strategy and SciBeta United States LTTR High Profitability Diversified Multi-Strategy. Source: www.scientificbeta.com.

4. Performance benefits of six-factor multi-beta multi-strategy indices

US Long-Term Track Records (Dec 1972–Dec 2015)	US Long-Term CW	Mid cap	Momentum	Low volatility	Value	Low investment	High profitability	Average	Multi-beta multi-strategy (six factors)	
		Scientific Beta Diversified Multi-Strategy								
									Equal weight	ERC
Annualised returns	10.16%	14.30%	13.32%	12.94%	14.47%	14.08%	12.79%	13.65%	13.72%	13.50%
Annualised volatility	17.15%	16.03%	16.27%	14.08%	15.90%	15.15%	15.98%	15.57%	15.34%	15.40%
Sharpe ratio	0.29	0.57	0.50	0.56	0.59	0.59	0.48	0.55	0.56	0.54
Maximum drawdown	54.63%	53.42%	53.25%	48.31%	53.75%	50.82%	48.72%	51.38%	50.93%	50.70%
Excess returns	–	4.14%	3.16%	2.77%	4.31%	3.92%	2.63%	3.49%	3.55%	3.34%
Tracking error	–	6.49%	4.90%	6.08%	5.53%	5.50%	4.40%	5.48%	4.83%	4.49%
95% tracking rrror	–	11.54%	8.64%	11.43%	10.07%	9.98%	7.31%	9.83%	8.43%	7.58%
Information ratio	–	0.64	0.64	0.46	0.78	0.71	0.60	0.64	0.74	0.74

The table compares the performance and risk of the SciBeta USA LTTR Diversified Multi-Strategy indices. The Multi-Beta Multi-Strategy EW (ERC) index is the equal-weighted (equal relative risk contribution) combination of the six Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, value, low investment and high profitability respectively. All statistics are annualised and daily total returns from 31 December 1972 to 31 December2015 are used for the analysis. The SciBeta CW US-500 index is used as the cap-weighted benchmark for US LTTR. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of one (or three) years at any point during the history of the strategy. A rolling window of length one (or three) years and a step size of one week are used. Source: www.scientificbeta.com. Scientific Beta US Long-Term Smart Factor indices have a 45-year track record, of which two years are used for calibration of parameters of MBMS ERC index. Consequently, performance is reported only for a 43-year period.

5. Economic rationale for factor investing

Factor	Risk-based explanation	Behavioural explanation
Value	Costly reversibility of assets in place: high sensitivity to economic shocks in bad times	Over-reaction to bad news and extrapolation of the recent past leads to under-pricing
Momentum	High-expected-growth firms are more sensitive to shocks to expected growth	Investor overconfidence and self-attribution bias leads to returns continuation in the short term
Low risk	Liquidity-constrained investors have to sell leveraged positions in low-risk assets in bad times when liquidity constraints become binding	Disagreement of investors about high-risk stocks leads to overpricing due to short-sales constraints
Size	Low liquidity, high distress and downside risk is compensated by higher returns	Limited investor attention to smaller cap stocks
Profitability	Firms facing high cost of capital will invest only in the most profitable projects	Investors do not discern high and low profitability in growth firms
Investment	Low investment reflects firms' limited scope for projects given high cost of capital	Investors under-price low investment firms due to expectation errors

13.64% ERC). Similarly the Sharpe ratios of the six-factor multi-smart factor indices (0.56 for EW and 0.54 for ERC) are similar to those of the four-factor multi-smart factor indices (0.57 for EW and 0.56 for ERC). However, the addition of two new factors reduces the tracking error and the extreme tracking error in the long term. The tracking errors of six-factor multi-smart factor indices are 4.83% for EW and 4.49% for ERC indices, compared to 5.10% for EW and 4.78% for the ERC version of four-factor multi-smart factor indices. Similar observations can be made in terms of extreme tracking error. The extreme tracking errors of six-factor multi-smart factor indices are 8.43%% for EW and 7.58% for ERC indices, compared to 8.82% for EW and 8.04% for the ERC version of four-factor multi-smart factor indices. On average the addition of the two quality-related factors reduces the tracking and extreme tracking error by 5.7% and 5.1% respectively.

If we look at the conditional performance of the six-factor and four-factor multi-smart factor indices, the smoothing of returns by six-factor indices can be observed. The six-factor multi-smart factor indices provide more balanced performance in both bull and bear markets compared to the four-factor multi-smart factor indices. The average information ratios of the two six-factor multi-smart factor indices are 0.67 in bull markets and 0.71 in bear markets, whereas the dispersion of information ratios is wider in the case of four-factor multi-smart factor indices. The average information ratios of the two four-factor multi-smart factor indices are 0.58 in bull markets and 0.77 in bear markets. The smoothing of performance is even more evident in the short term. Figure 7 shows the year-wise excess returns comparison of four-factor and six-factor Developed Multi-Beta Multi-Strategy indices. The best excess return in each year is highlighted in green and the worst excess return in each year is highlighted in red. It is clear that the six-factor MBMS indices avoid the worst performance in most years because the two new factors help in smoothing out the outperformance in the short term.

The four-factor multi-smart factor indices on the other hand have longer live track records and limit exposure to the four historically well-known factors – size, value, momentum and volatility. In addition, to the extent that the multi-smart factor index approach is based on an aggregation of single smart factor indices, it is clear that switching to six factors tends to reduce the importance of the first four factors, notably the low volatility factor, in the factor allocation, which explains the lower level of relative performance in bear markets of the Scientific Beta Multi-Beta Multi-Strategy six-factor index compared to the Scientific Beta Multi-Beta Multi-Strategy four-factor index.

To summarise, the two variants of multi-smart factor indices (four-factor and six-factor) provide very similar absolute performance – returns and Sharpe ratio. However, the inclusion of two additional factors adds diversification benefits and consequently, the six-factor multi-smart factor indices manage tracking error and extreme tracking error better and provide smoothed outperformance. These two multi-factor indices follow naïve allocation schemes such as equal weighting or equal relative risk contribution to allocate among the different single factor indices. However, further value can be added by following more scientific approaches to allocate among the various single factor indices by taking the investors' risk objectives and providing a tailored allocation among the various single-factor indices. The next section describes in detail how the flagship ►

6. Performance comparison of four-factor and six factor US LTTR Multi-Beta Multi-Strategy indices

US Long-Term Track Records (Dec 1972–Dec 2015)	US Long-Term Cap-Weighted	Scientific Beta Diversified Multi-Strategy: Multi-Beta Multi-Strategy (four factors)		Multi-Beta Multi-Strategy (six factors)	
		Equal weight	ERC	Equal weight	ERC
Annualised returns	10.16%	13.83%	13.64%	13.72%	13.50%
Annualised volatility	17.15%	15.34%	15.35%	15.34%	15.40%
Sharpe ratio	0.29	0.57	0.56	0.56	0.54
Maximum drawdown	54.63%	51.93%	51.45%	50.93%	50.70%
Excess returns	–	3.67%	3.48%	3.55%	3.34%
Tracking error	–	5.10%	4.78%	4.83%	4.49%
95% tracking error	–	8.82%	8.04%	8.43%	7.58%
Information ratio	–	0.72	0.73	0.74	0.74
Bull markets					
Excess returns	–	2.41%	2.38%	2.62%	2.66%
Tracking error	–	4.29%	4.05%	4.07%	3.80%
Information ratio	–	0.56	0.59	0.64	0.70
Bear markets					
Excess returns	–	5.10%	4.71%	4.54%	4.01%
Tracking error	–	6.59%	6.13%	6.23%	5.78%
Information ratio	–	0.77	0.77	0.73	0.69

The table compares the performance and risk of the SciBeta US LTTR Multi-Beta Multi-Strategy indices. The six-factor Multi-Beta Multi-Strategy EW (ERC) index is the equal-weighted (equal relative risk contribution) combination of the six Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, value, low investment and high profitability respectively. The four-factor Multi-Beta Multi-Strategy EW (ERC) index is the equal-weighted (equal relative risk contribution) combination of the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility and value respectively. All statistics are annualised and daily total returns from 31 December 1972 to 31 December 2015 are used for the analysis. The SciBeta CW US-500 index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Calendar quarters with positive benchmark returns comprise bull markets and the rest constitute bear markets. Source: www.scientificbeta.com. Scientific Beta US Long-Term Smart Factor indices have a 45-year track record, of which two years are used for calibration of parameters of MBMS ERC index. Consequently, performance is reported only for a 43-year period.

7. Year-wise excess returns of four-factor and six-factor Developed Multi-Beta Multi-Strategy indices

SciBeta Developed (Dec 2005–Dec 2015)	Scientific Beta Diversified Multi-Strategy: Multi-Beta Multi-Strategy (four factors)		Multi-Beta Multi-Strategy (six factors)	
Years	Equal weight	ERC	Equal weight	ERC
2006	4.02%	4.15%	3.61%	3.50%
2007	–2.99%	–3.15%	–2.33%	–2.26%
2008	2.96%	3.05%	3.79%	3.71%
2009	–1.95%	–0.87%	–0.59%	0.05%
2010	5.86%	5.11%	6.26%	5.47%
2011	2.91%	1.80%	2.86%	1.89%
2012	0.11%	0.22%	0.10%	0.21%
2013	0.05%	0.12%	0.89%	0.88%
2014	3.07%	2.90%	3.17%	3.07%
2015	2.67%	2.24%	2.80%	2.47%

The table compares the year-wise excess returns of the SciBeta Developed Multi-Beta Multi-Strategy indices. The six-factor Multi-Beta Multi-Strategy EW (ERC) index is the equal-weighted (equal relative risk contribution) combination of the six Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, value, low investment and high profitability respectively. The four-factor Multi-Beta Multi-Strategy EW (ERC) index is the equal-weighted (equal relative risk contribution) combination of the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility and value respectively. The daily total returns from 31 December 2005 to 31 December 2015 are used for the analysis. SciBeta Developed Cap-Weighted index is used as benchmark. Source: www.scientificbeta.com.

8. Live performance of SciBeta Developed four-factor Multi-Beta Multi-Strategy index

Performance and risks – Live period Developed (20 Dec 2013–30 June 2016)	SciBeta Cap-Weighted index	Multi-Beta Multi-Strategy four-factor index	
		EW	ERC
Annualised returns	3.13%	6.44%	6.10%
Volatility	12.16%	11.13%	11.27%
Sharpe ratio	0.25	0.57	0.53
Relative returns	–	3.31%	2.97%
Tracking error	–	2.13%	2.01%
Information ratio	–	1.55	1.48

The table presents the live performance and risks of SciBeta Developed Multi-Beta Multi-Strategy (four-factor) indices. The live date is 12 December 2013. Analysis is based on daily total returns in US dollars from 20 December 2013 to 30 June 2016. SciBeta Developed Cap-Weighted index is used as benchmark and yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Source: www.scientificbeta.com.

◄ multi-smart factor indices can be extended to incorporate a tailored approach that takes investors' risk objectives and constraints into account.

Multi-smart factor solutions – the future

The purpose of Scientific Beta's flagship multi-smart factor indices is to be a fairly neutral and robust starting point for investing in smart beta. The Scientific Beta Multi-Beta Multi-Strategy six-factor EW index actually corresponds to an equal-weight allocation between 30 smart factor indices corresponding to six factor tilts and five weighting schemes per factor tilt. It is obvious that more sophisticated investors will want to take account of the relationships between these indices to improve their allocation or to make it correspond to a risk objective. In that case, it no longer involves considering that all the indices have identical volatilities and correlations, like in an equal-weight allocation between smart factor indices, but taking their differences into account. These differences depend not only on the choice of factor, but also on the weighting used in the smart factor indices. Figure 9 suggests that the returns of the different weighting schemes for the same factor tilt are not perfectly positively correlated, which means that further reduction of non-systematic risk can be achieved by combining different weighting schemes at the diversification-stage of smart factor index construction (or equivalently by allocating to smart factor indices that tilt towards the same factor, but that are diversified with different weighting schemes).

The solutions offered by ERI Scientific Beta will therefore be able to rely not only on the decorrelation between factors, but also between the weighting schemes. It is this double layer of diversification that enables the allocation between smart factor indices to be optimised.

The research conducted by EDHEC-Risk Institute in multi smart-factor allocation led to the creation of Scientific Beta Multi-Beta Multi-Strategy Solutions. Scientific Beta Multi-Beta Multi-Strategy Solutions provide the possibility to deliver tailored state-of-the art allocation solutions building on Scientific Beta Smart Factor indices. Thus, Scientific Beta Multi-Beta Multi-Strategy Solutions are the advanced and flexible extensions of the flagship multi-smart factor indices.

Figure 10 shows a framework of different possible customisation themes.

The first, and arguably most important, dimension relates to whether risk is defined by the investor from an absolute perspective in the absence of a benchmark, or whether it is instead defined in relative terms with respect to an existing benchmark, which is more often than not a cap-weighted index. In the former situation, one would use volatility as a relevant risk measure, while tracking error with respect to the cap-weighted index would instead be used in the latter case.

The second dimension concerns whether one would like to incorporate views regarding factor returns in the optimisation process. While additional benefits can be obtained from the introduction of views on factor returns at various points of the business cycle, we focus in what follows only on approaches that are solely based on risk parameters, which are notoriously easier to estimate with a sufficient degree of robustness and accuracy (Merton [1980]).

The third dimension is related to the objective of the allocation procedure. Indeed, there are several possible targets for the design of a well-diversified portfolio of factor exposures, eg, risk minimisation, or equalisation of relative or absolute contributions. In order to implement this risk allocation, ERI Scientific Beta has chosen to design allocation techniques that use equal weighting between indices as a starting point. Indeed, this starting point presents a number of advantages:

- It corresponds to the flagship Scientific Beta Multi-Beta Multi-Strategy indices, which have an excellent live track record, therefore enabling an investor to capitalise on a robust solution that has been implemented with success for several years by a large number of asset owners and asset managers.
- It is a neutral starting point that enables robust allocation, which is very dependent on parameter estimation error, to be built. Indeed, the proposed solution is simply a deviation of equal weighting, of which the sole objective is to satisfy a risk constraint, or constraints, that are representative of the solution desired by the investor.
- The allocation does not correspond to a complex optimisation since the technique used is an allocation with a maximum deconcentration objective (maximisation of an effective number of indices) with a constraint that is representative of the desired risk objective. In the absence of constraints, maximum deconcentration is equivalent to equal weighting. Depending on the risk allocation objective, it will of course be necessary to estimate different risk parameters, but in any case the equal-weight starting point contained in the allocation method selected by Scientific Beta provides a robust allocation minimising the consequences of estimation errors on the diversification of the smart beta solution offered.

The fourth and last dimension is related to the presence of various forms of constraints such as minimum/maximum weight constraints, turnover constraints, or factor exposure constraints, which are obviously highly relevant in the context of risk factor allocation. Let us look at two examples of smart beta solutions, each focusing on a different risk objective.

Absolute risk solution

The Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) Solution is an example of an absolute risk solution. The objective of the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) Solution is to dynamically allocate to Scientific Beta Smart Factor indices on the basis of their evolving risk characteristics in changing markets in an effort to deliver at least the targeted constant reduction in volatility relative to the market. Traditional defensive solutions seek to reduce or minimise absolute volatility unconditionally. The Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution targets a constant reduction in volatility (10%) relative to the Scientific Beta capitalisation-weighted, market benchmark. This objective is implemented by using maximum deconcentration allocation on the 30 smart factor indices based on six factors (value, mid cap, low volatility, high momentum, low investment and high profitability) and five difference weighting schemes (maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio and diversified risk weighted diversification strategies). The maximum deconcentration allocation maximises diversification measured by the effective number of constituents – defined as the inverse of the sum of squared constituent weights: $ENC(w) = 1/wT*w$ – of smart factor indices subject to a constraint of 10% ex-ante reduction in volatility relative to the reference index.

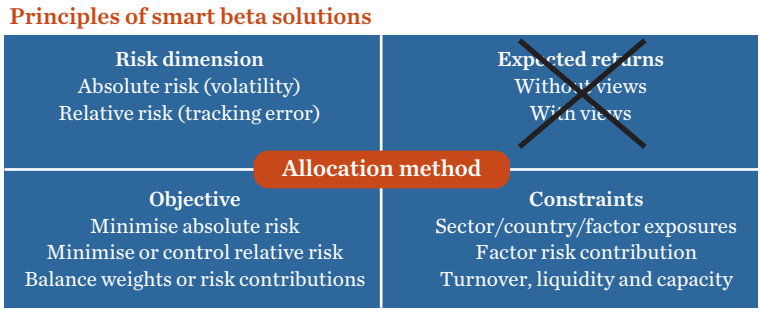
Figure 11 shows that over the very long term (43-year record), the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) Solution respects its objective constraint of a minimum volatility reduction of 10%. It achieves a robust 15% reduction and produces higher returns than the low volatility smart factor indices. Its Sharpe ratio is twice that of the benchmark, on par with that of the two low volatility indices. The solution's high outperformance of 3.51% pa and lower tracking error is a result of its dissymmetric defensive profile and the exploitation of decorrelation opportunities across both the factor and weighting scheme dimensions. Volatility reduction for the two traditional defensive strategies – efficient minimum volatility and low volatility diversified multi-strategy is 16% and 18% respectively. The 95th percentile of three-year rolling tracking error of the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) Solution is also 15% less than that of

9. Average pairwise correlations of excess returns across five weighting schemes

US Long Term Dec 1970–Dec 2015)	Mid cap	High momentum	Low volatility	Value	Low momentum	High profitability
			Diversified Multi-Strategy			
Average correlation across five weighting schemes	0.89	0.87	0.96	0.88	0.9	0.85
Minimum correlation across five weighting schemes	0.76	0.68	0.91	0.75	0.78	0.64

The analysis is based on daily total returns of US Long-Term Track Records from 31 December 1970–31 December 2015. The average, minimum and maximum pairwise correlations across the five weighting schemes – maximum deconcentration, maximum decorrelation, maximum Sharpe ratio, minimum volatility and diversified risk weighted for the six factors – momentum, low volatility, value, size, low investment and low profitability are provided. Source: www.scientificbeta.com.

10. Customised smart beta solutions framework



its benchmark, showing that the reduction in volatility is indeed robust. The most striking observation, which is also the most important way in which this solution adds value, is its ability to combine significant downside protection and excellent upside capture in both bull and bear market conditions.

Relative risk solution

In addition to their exposure to rewarded factors, the vast majority of solutions have a slightly defensive bias, either because the integration of low volatility factors weighs on the overall beta of the portfolio, or because the diversification schemes themselves are of a defensive nature (risk parity, minimum volatility, etc). Ultimately, they are sold as enabling investment in rewarded factors but are not totally invested in equities and do not benefit totally from the market risk premium.

The objective of the Scientific Beta Multi-Beta Multi-Strategy Beta One Solution proposed here is to conserve exposure to factors that are rewarded over the long term while preserving a market beta of 1. This 'Beta One' exposure provides full exposure to the risk premium of the equity markets. It involves outperforming the cap-weighted index in long-only while guaranteeing a reduced level of tracking error that can then be adjusted as part of a core-satellite approach.

Such an approach enables good relative outperformance to be preserved compared to the cap-weighted index while controlling the tracking error risk, notably the extreme tracking error risk, and therefore dramatically reducing the maximum relative drawdown, the source of which is underinvestment in the market, which penalises smart beta indices, because the market is very bullish. Naturally, this index, which is a high-performance substitute for a cap-weighted index, is not representative of a defensive strategy. The objective is to have the same level of risk as the equity markets but with a higher premium derived from its diversification.

Figure 12 shows that the Scientific Beta US LTTR Multi-Beta Multi-Strategy Beta One Solution achieves low tracking error with an improved information ratio (0.79). Compared to the flagship Scientific Beta US LTTR Multi-Beta Multi-Strategy (six-factor) EW index the improvement in information ratio is 6.76%, with a reduction in TE from 4.83% to 3.30%. The most striking feature of this allocation is its maximum relative drawdown, which is 7.29% compared to 32.89% for the equal-weighted allocation. Due to market beta constraints, the strategy does not fall short on performance when the markets are bullish like most smart beta strategies do. The standard deviation and 95th percentile of three-year rolling tracking error also show improvement, meaning that tracking error is more stable over time. Its 'well behaved' tracking error makes it a good candidate for a satellite in core-satellite allocations.

Conclusion

Choosing good factor tilts combined with well-diversified weighting schemes generates attractive risk-adjusted performance, and combining the different factor tilts allows for further improvement in the relative risk-adjusted performance. As such, standard Scientific Beta Multi-Beta Multi-Strategy indices allow the rewards of factor tilts and proper diversification to be harvested and – by allocating to a range of factors – such indices smooth outperformance, especially in the short term. The Scientific Beta Multi-Beta Multi-Strategy Solutions allow the investor's particular absolute or relative risk objectives to be taken into account. These solutions are based on a double layer of diversification between factors and between diversification strategies. As such, ERI Scientific Beta recently released two Multi-Beta Multi-Strategy Solution index offerings representative of the possibilities of taking absolute or relative risk reduction objectives into account. Within the framework of fully-customised solutions it is of course possible to take account of particular constraints or objectives that are specific to the investor. ►

11. Example of an absolute risk solution: Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%)

US Long Term (Dec 1972–Dec 2015)	SciBeta US LTTR Broad CW index	SciBeta US LTTR Efficient Minimum Volatility	SciBeta US LLTR Low Volatility Multi-Strategy	SciBeta US LLTR Multi-Beta Multi-Strategy Relative Volatility (90%)
Annualised returns	10.16%	12.53%	12.94%	13.67%
Annualised volatility	17.15%	14.36%	14.08%	14.64%
Sharpe ratio	0.29	0.52	0.56	0.58
Maximum drawdown	54.63%	47.33%	48.31%	48.70%
Annualised relative returns	–	2.37%	2.77%	3.51%
Tracking error	–	5.21%	6.08%	5.13%
Information ratio	–	0.45	0.46	0.68
Maximum relative drawdown	–	40.10%	43.46%	33.18%
3-y rolling vol mean	16.40%	13.70%	13.43%	13.99%
3-y rolling vol std dev	5.33%	4.53%	4.47%	4.61%
3-y rolling vol 95%ile	29.29%	25.14%	24.63%	25.06%
Relative returns bull markets	–	–0.07%	–0.77%	1.84%
Relative returns bear markets	–	5.53%	7.52%	5.53%
Relative returns 25% bull markets	–	–4.77%	–6.39%	–0.69%
Relative returns 25% bear markets	-	5.08%	7.35%	5.14%
CAPM beta*	1	0.84	0.8	0.86
1-way annualised turnover	3.10%	31.90%	26.50%	39.40%

Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) allocation is performed using 30 [six factors x five weightings] Standard Smart Factor indices in the US LTTR universe. The factors are: mid cap, momentum, low volatility, value, high profitability, low investment. The weightings are: maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio, diversified risk weighted. The analysis is based on daily total returns in US dollars in the period 31 December 1972–31 December 2015 (43 years). *Regressions are performed using weekly total returns in US dollars. All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 172 rebalancings in the 43-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Quarters with positive benchmark index returns are bull quarters and the remaining are bear quarters. Top 25% quarters with best benchmark index returns are extreme bull quarters and bottom 25% quarters with worst benchmark index returns are extreme bear quarters. The Scientific Beta US LTTR universe contains 500 stocks. Source: scientificbeta.com.Scientific Beta US Long-Term Track Records. Scientific Beta US Long Term Smart Factor indices have a 45-year track record, of which two years are used for calibration of parameters. Consequently US Long Term Smart Beta Solutions have a 43-year track record.

12. Example of a relative risk solution: Scientific Beta Multi-Beta Multi-Strategy Beta One

US Long Term (Dec 1972–Dec 2015)	SciBeta US LTTR Broad CW index	SciBeta US LTTR Multi-Beta Multi-Strategy six-factor (EW)	SciBeta US LLTR High Liquidity Multi-Beta Multi-Strategy six-factor (EW)	SciBeta US LLTR Multi-Beta Multi-Strategy Beta One
Annualised returns	10.16%	13.72%	13.12%	12.76%
Annualised volatility	17.15%	15.34%	15.99%	16.87%
Sharpe ratio	0.29	0.56	0.50	0.45
Maximum drawdown	54.63%	50.93%	52.27%	54.10%
Annualised relative returns	–	3.55%	2.96%	2.60%
Tracking error	–	4.83%	3.88%	3.30%
Information ratio	–	0.74	0.76	0.79
Maximum relative drawdown	–	32.89%	23.83%	7.29%
3-y rolling TE mean	–	4.41%	3.56%	3.15%
3-y rolling TE std dev	–	1.91%	1.55%	0.93%
3-y rolling TE 95%ile	–	9.41%	7.92%	5.17%
Relative returns bull markets	–	2.62%	2.77%	4.12%
Relative returns bear markets	–	4.54%	2.98%	0.41%
Relative returns 25% bull markets	–	1.35%	1.98%	6.35%
Relative returns 25% bear markets	-	3.86%	2.47%	–0.11%
CAPM beta*	1	0.90	0.94	0.99
Carhart market beta*	1	0.90	0.94	1.00
1-way annualised turnover	3.10%	27.20%	29.60%	41.20%

Scientific Beta Multi-Beta Multi-Strategy Beta One is performed using 30 [six factors x five weightings] Hihg Liquidity Smart Factor indices in the US LTTR universe. The factors are: mid cap, momentum, low volatility, value, high profitability, low investment. The weightings are: maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio, diversified risk weighted. The Scientific Beta US LTTR High Liquidity Multi-Beta Multi-Strategy six-factor (EW) index is an equal combination of the six factor tilted High Liquidity Multi-Strategy indices. The analysis is based on daily total returns in US dollars in the period 31 December 1972–31 December 2015 (43 years). *Regressions are performed using weekly total returns in US dollars. All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Reported turnover is one-way annual and it is averaged across 172 rebalancings in the 43-year period. All allocations are systematically rebalanced quarterly. Maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. Quarters with positive benchmark index returns are bull quarters and the remaining are bear quarters. Top 25% quarters with best benchmark index returns are extreme bull quarters and bottom 25% quarters with worst benchmark index returns are extreme bear quarters. The Scientific Beta US LTTR universe contains 500 stocks. Source: scientificbeta.com.Scientific Beta US Long-Term Track Records. Scientific Beta US Long Term Smart Factor indices have a 45-year track record, of which two years are used for calibration of parameters. Consequently US Long Term Smart Beta Solutions have a 43-year track record.

Pay for What you Get

Because we believe in the quality of our research and the robustness of the performance of the associated smart beta indices, we are proposing that investors who wish to do so can replicate our flagship Scientific Beta Multi-Beta Multi-Strategy indices on the basis of a pure performance fees mandate.

Institutional investors who choose this option from June 1, 2016, will pay zero fixed fees and will only pay variable fees if the flagship Scientific Beta Multi-Beta Multi-Strategy index outperforms the reference cap-weighted index.

For more information on this new pricing offer, please contact
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Smart beta and low carbon investing

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ERI Scientific Beta recently introduced a series of low carbon indices that reduce the carbon footprint of an index using a mild exclusion of high carbon stocks, thereby putting pressure on highly polluting companies to reform, while at the same time retaining exposure to rewarded risk factors and maintaining an appropriate level of diversification. The Scientific Beta Low Carbon Multi-Beta Multi-Strategy indices thus enable both environmental and financial objectives to be reconciled.

Providing a financial incentive for investing in the low carbon economy

For some years now, there has been an increasing interest among investors in low carbon investments. There seem to be two possible reasons for this shift. The first reason, involving a financial consideration, is investors' belief that exposing portfolios to securities with a low carbon footprint would be profitable in the long term, as it would avoid regulatory risk associated with high carbon stocks and would mean investing in green stocks which would profit from the transition to a low carbon economy. The second reason, involving a non-financial consideration, is investors' belief in contributing towards the transition to a low carbon economy. The argument is that if investors shift investments to low carbon or green stocks, it would put pressure on polluting firms to reform.

Unfortunately, a thorough analysis of the way financial markets operate and of the pricing of financial assets would suggest that higher profitability of green stocks vis-à-vis the rest is not certain.

In the academic literature there is broadly a consensus about non-financial considerations involved in socially responsible investing (SRI), which also includes low carbon investments, and the possibility of change in a firm's behaviour through a shift in investments away from securities of high carbon emitting companies. For example, Heinkel et al (2001) show that in an equilibrium setting, if 'green investors' shun polluting companies, there would be a downward pressure on the share price, and 'non-green investors' would demand higher expected returns to hold shares of such companies, thus raising the cost of capital for the polluting firms, and thus incentivising them to reform.

On the other hand, when it comes to purely financial considerations, there is evidence in the academic literature that questions the profitability of low carbon investments.

First of all, the argument about profitability of low carbon investments does not fit well into financial asset pricing theory. Both equilibrium models such as Merton's (1973) inter-temporal capital asset-pricing model and no-arbitrage models such as Ross's (1976) arbitrage pricing theory allow for the existence of risk factors with multiple prices. The economic intuition for the existence of a reward for a given risk factor is that exposure to such a factor is undesirable for the average investor, because it leads to losses in 'bad times' when marginal utility is high (Cochrane [2001]). A potential future climate crisis is a good illustration of such a 'bad time'. Because they could serve as a hedge by providing income during bad times, investors could actually be willing to pay a premium for holding low carbon stocks: investors would want, if and when climate change-related distress occurs, to be invested in those companies likely to benefit, or at least to lose less, in such an economic context. If one makes the assumption that companies that currently have lower carbon emissions than their peers would be among those sought-after companies in future distressed times (governments might for instance want in the future to 'punish', through taxation, fines, regulations or otherwise, those companies deemed responsible for the dire times, much as banks have been burdened by governments in the aftermath of the recent financial crisis), an investor should be willing to pay a premium for companies which currently have lower carbon emissions.

> *"From an empirical perspective, a large part of the academic literature establishes that there are no or insignificant benefits from low carbon investing as compared to conventional investing, from a financial point of view*

From an empirical perspective, a large part of the academic literature establishes that there are no or insignificant benefits from low carbon investing as compared to conventional investing, from a financial point of view. For example, Ibikunle and Steffen (2015) focus particularly on green funds and compare their performance to 'black' (ie, fossil energy and natural resources) and conventional European mutual funds for the period 1991–2014. During this sample period, the green mutual funds exhibited lower returns than black and conventional mutual funds. The green mutual funds also had higher volatility than the conventional mutual funds, which can be attributed to lower diversification potential due to restricted investment universes.

In the same vein as this empirical literature, we compare the historical profitability of low and high carbon stocks. To do so, we create a portfolio of 25% worst-in-class carbon intensity stocks and another portfolio of 25% best-in-class carbon intensity stocks in the Scientific Beta Developed universe. We analyse their performance from 31 December 2005 to 31 December 2015. In figure 1 on page 10, we note that during this 10-year period, the annualised return of the low carbon factor is 1.15%, but a high p-value of 0.28 suggests that we cannot reject the hypothesis that there is no premium, at a reasonable significance level.

One can also argue that the financial markets do not yet incorporate the consequences of global warming. In that case, selecting securities of companies with low greenhouse gas emissions would be a 'free option' or serve as cheap protection from the negative economic consequences of climate change. That said, however, the success of this approach assumes that the players in the financial sector will be able to identify the companies that do not have their ►

environmental properties correctly priced into the current market value of their securities. However, this assumption runs contrary to findings of academic studies, which demonstrate the inability of asset managers to persistently outperform the market through stock picking.

These findings lead us to the conclusion that if we truly want the finance industry to contribute to saving the planet, we have to encourage it to do so, and not solely rely on the potential profitability of green stocks. Our research shows that it is possible to reconcile environmental and financial objectives using low carbon indices introduced by ERI Scientific Beta, which aim to outperform the market not because they are green, but because they are exposed to traditional risk premia and are better diversified than traditional cap-weighted indices.

How to reduce carbon emissions, while providing exposure to consensual risk premia and remaining well diversified?

A first question that comes to mind when designing a low carbon index is how much of the universe needs to be impacted in order to achieve a meaningful reduction in carbon emissions. To answer this question, we look at the cross-sectional distribution of both total emissions and carbon intensity of stocks. We observe that both total emissions and carbon intensity are highly concentrated in a few stocks. Thus, to achieve a significant reduction in the carbon emissions of the index, it is not necessary to remove a large number of companies, which could also possibly alter the performance and risk characteristics of the index.

"We observe that both total emissions and carbon intensity are highly concentrated in a few stocks. Thus, to achieve a significant reduction in the carbon emissions of the index, it is not necessary to remove a large number of companies"

Based on these findings, ERI Scientific Beta designs a low carbon index which mildly excludes high carbon stocks and uses techniques to construct an index which is well exposed to the rewarded factors and is well diversified.

ERI Scientific Beta's approach to excluding high carbon stocks relies on excluding stocks that are identified by any of the following criteria, applied independently: coal mining companies[1], top 2% companies by total emission in the broad developed universe, top 2% companies by total emission in each of Scientific Beta's geographic basic blocks and top 25% companies by Ccon intensity in each of the level 2 sub-sector (or level 1 sector in case any one of the level 2 sub-sectors does not have at least four stocks).

Once we exclude stocks based on carbon emissions, we apply ERI Scientific Beta's approach termed Smart Beta 2.0. The approach distinguishes two steps in the construction of smart beta strategies, with the first step tilting towards the targeted risks by way of transparent security selection, and the second step diversifying away the undesired and unrewarded risks through the application of a diversification strategy (smart weighting scheme). Smart factor indices are thus tilted towards the desired risk factors but also well diversified.

ERI Scientific Beta's multi-smart factor indices are constructed using smart factor indices, representing a set of six academically-documented and popular risk factors – size, momentum, volatility, valuation, investment and profitability.

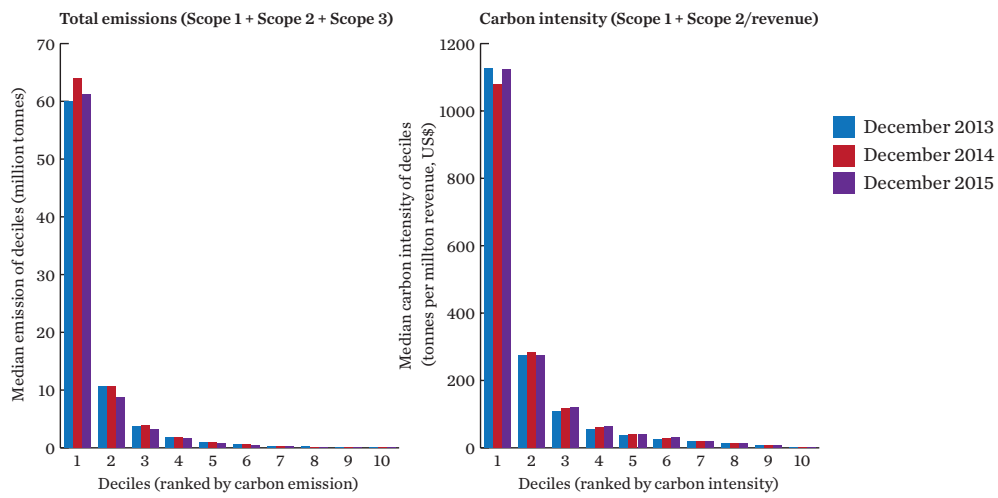
1 We identify coal mining companies as companies with Thomson Reuters Business Classification (TRBC) code of 501010.

1. Performance of low carbon factor

	SciBeta Developed Cap-Weighted	25% best-in-class intensity stocks (cap-weight)	25% worst-in-class intensity stocks (cap-weight)	Low carbon factor
Annualised return	5.58%	6.53%	5.43%	1.15%
(31 Dec 2005–31 Dec 2015)				[0.28]*
Yearly performance				
2006	20.53%	17.23%	20.94%	-3.06%
2007	11.41%	15.78%	10.11%	5.19%
2008	–40.40%	–44.21%	–38.82%	–8.31%
2009	30.52%	32.04%	30.48%	1.34%
2010	12.64%	17.87%	11.54%	5.73%
2011	–5.59%	–3.79%	–6.34%	2.89%
2012	16.43%	18.56%	15.29%	2.92%
2013	26.81%	27.12%	27.43%	–0.24%
2014	5.09%	6.84%	7.10%	–0.23%
2015	0.02%	3.37%	–2.69%	6.27%

The time period of analysis is 31 December 2005–31 December 2015 (10 years). To report performance, we use daily total return series in US dollars. The 25% best- (or worst-) in-class intensity stocks consist of the bottom (or top) 25% carbon intensity stocks from each Level 2 sector of TRBC industry classification, wherein intensity is measured as the ratio of the sum of Scope 1 and Scope 2 emissions to the company's revenue. In case any one of the Level 2 sectors does not have at least four stocks, we exclude stocks at the Level 1 sector of TRBC industry classification. The Scope 1 emission refers to greenhouse gas (GHG) emission from sources that are owned and/or controlled by the company. The Scope 2 emission refers to indirect GHG emission from generation of electricity, steam or heat purchased/consumed by the company. The source of a company's emission data is South Pole. We use a company's carbon emission data for the year t only starting December rebalancing of year t+1. For years prior to 2011, we make an assumption that a company's carbon emissions were the same as in 2011. The low carbon factor is created as difference in daily return of 25% best-in-class and 25% worst-in-class intensity stocks. The portfolios are cap-weighted. *Reports p-value for the null hypothesis that the mean return is zero.

2. Cross-sectional distribution of total emission and carbon intensity



The figure plots cross-sectional distribution of total emission and carbon intensity of stocks in the Scientific Beta Developed universe of 2,000 stocks at the last three December rebalancings. We rank stocks by their total emission (and carbon intensity) and report the median total emission (and carbon intensity) of each decile. Total emission is defined as the sum of Scope 1, Scope 2 and Scope 3 emissions. Carbon intensity is defined as the ratio of the sum of Scope 1 and Scope 2 emission to the company's revenue. The Scope 1 emission refers to greenhouse gas (GHG) emission from sources that are owned and/or controlled by the company. The Scope 2 emission refers to indirect GHG emission from generation of electricity, steam or heat purchased/consumed by the company. The Scope 3 emission refers to GHG emissions that are a consequence of the activities of the company, but occur from sources not owned or controlled by the company. The source of a company's emission data is South Pole.

3. Smart Beta 2.0 approach

Scientific Beta Low Carbon indices: better financial performance, lower carbon emissions

We first analyse the financial performance of Scientific Beta Low Carbon Indices and compare it with the standard (without any low carbon filter) Scientific Beta indices. Figure 4 reports performance of standard (panel 1) and low carbon (panel 2) indices for the developed market for the 10-year period from 31 December 2005–31 December 2015. We note that both standard and low carbon Scientific Beta indices

4. Performance analysis

Panel 1	Standard Indices (Developed)								
				Diversified Multi-Strategy				Multi-Beta Multi-Strategy EW	
	CW-	Mid cap	Value	High momentum	Low volatility	High profitability	Low investment	Four-factor	Six-factor
Absolute performance									
Annualised return	5.58%	7.53%	6.37%	7.19%	8.58%	8.30%	9.03%	7.46%	7.86%
Annualised volatility	17.34%	16.26%	17.43%	16.25%	13.98%	15.57%	15.64%	15.85%	15.74%
Sharpe ratio	0.26	0.39	0.30	0.37	0.53	0.46	0.51	0.40	0.43
Relative performance (wrt CW)									
Annualised relative return	–	1.96%	0.80%	1.61%	3.01%	2.72%	3.46%	1.88%	2.29%
Tracking error	–	3.29%	2.24%	3.69%	4.43%	2.99%	3.19%	2.60%	2.61%
Information ratio	–	0.59	0.36	0.44	0.68	0.91	1.08	0.72	0.88

Panel 2	Low Carbon Indices (Developed)								
				Diversified Multi-Strategy				Multi-Beta Multi-Strategy EW	
	CW-	Mid cap	Value	High momentum	Low volatility	High profitability	Low investment	Four-factor	Six-factor
Absolute performance									
Annualised return	5.58%	7.68%	6.33%	7.57%	8.85%	9.33%	8.22%	7.64%	8.02%
Annualised volatility	17.34%	16.36%	17.19%	16.04%	13.76%	15.48%	15.45%	15.70%	15.59%
Sharpe ratio	0.26	0.40	0.30	0.40	0.56	0.53	0.46	0.42	0.44
Relative performance (wrt CW)									
Annualised relative return	–	2.11%	0.75%	1.99%	3.27%	3.75%	2.65%	2.06%	2.45%
Tracking error	–	3.42%	2.33%	3.81%	4.69%	3.44%	3.26%	2.78%	2.82%
Information ratio	–	0.62	0.32	0.52	0.70	1.09	0.81	0.74	0.87
Low carbon vs standard indices									
Difference in annual return	–	0.15%	–0.05%	0.38%	0.26%	0.29%	–0.07%	0.18%	0.16%
Tracking error	–	0.96%	1.22%	1.12%	1.08%	0.96%	1.11%	0.80%	0.77%

The time period for analysis is 31 December 2005–31 December 2015 (10 years). We use daily total return series in US dollars. We use SciBeta Developed Cap-Weighted index as the benchmark for relative analysis.

covered here outperform the cap-weighted benchmark. Moreover, the performance of low carbon indices is very similar to that of the standard indices.

Next we report the absolute difference in risk factor exposures between the standard and low carbon indices. The average difference in exposure across all risk factors is as low as 0.02, which means the Scientific Beta methodology to construct low carbon indices does not significantly change their exposure to financially rewarded factors. This also suggests that the likelihood of significant differences in their performance over the long term is low.

We then analyse the impact on carbon metrics, with two measures: the carbon footprint and the carbon intensity. In figure 6, we report these metrics for the low carbon indices. To give perspective, we also show the change in these measures with respect to the broad cap-weighted index. We note that for the low carbon SciBeta Developed Multi-Beta Multi-Strategy indices, the reduction in carbon footprint is close to 45%. ►

5. Difference in risk factor exposures

	Difference in Beta: Low Carbon Indices vs Corresponding Standard Indices (Developed)								
			Diversified Multi-Strategy				Multi-Beta Multi-Strategy EW		
-	Mid cap	Value	High momentum	Low volatility	High profitability	Low investment	Four-factor	Six-factor	Average across indices
Market beta	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00
SMB beta	0.01	0.03	0.00	0.01	0.00	0.01	0.02	0.01	0.01
HML beta	0.00	0.00	0.03	0.01	0.05	0.00	0.01	0.02	0.02
MOM beta	0.01	0.00	0.02	0.02	0.00	0.01	0.00	0.00	0.01
VOL beta	0.02	0.00	0.01	0.02	0.01	0.00	0.01	0.01	0.01
PRO beta	0.04	0.02	0.01	0.01	0.03	0.01	0.02	0.01	0.02
INV beta	0.03	0.02	0.08	0.05	0.04	0.00	0.05	0.04	0.04
Average across exposures	0.02	0.01	0.02	0.02	0.02	0.01	0.02	0.01	0.02

The time period for analysis is from 31 December 2005–31 December 2015 (10 years). We report absolute differences in betas from risk factor exposure (seven-factor) analysis of Scientific Beta standard indices and the corresponding low carbon indices. The market factor is cap-weighted and is created as the difference in return of SciBeta Developed Cap-Weighted and risk-free rate (return of the Secondary Market US Treasury Bills (3M)). The other six factors are the long/short equal weight factors from Scientific Beta (www.scientificbeta.com).

6. Carbon metrics

	Low Carbon Indices (Developed)								
				Diversified Multi-Strategy				Multi-Beta Multi-Strategy EW	
	CW-	Mid cap	Value	High momentum	Low volatility	High profitability	Low investment	Four-factor	Six-factor
Carbon metrics									
Carbon footprint	642,126	362,584	535,571	238,776	277,185	195,381	427,990	353,529	339,581
Carbon intensity	183	112	166	69	130	48	131	119	109
Change in carbon metrics (wrt cap-weighted index)									
Carbon footprint	–	–45.53%	–16.59%	–62.81%	–56.83%	–69.57%	–33.35%	–44.94%	–47.12%
Carbon intensity	–	–38.54%	–9.17%	–62.09%	–29.05%	–73.67%	–28.16%	–34.71%	–40.11%

The figure reports carbon metrics and change in carbon metrics of Scientific Beta low carbon indices compared to the cap-weighted benchmark as of 18 December 2015. The carbon metrics report two measures: the carbon footprint and the carbon intensity. The carbon footprint of the index represents the total emissions in relation to an investment of $1bn in the index. The carbon intensity of the index is the weighted average carbon intensity of the individual companies in the index. The units for carbon footprint and carbon intensity are tonnes per billion investment in US dollars and tonnes per million revenue in US dollar, respectively. A negative (positive) change in carbon metrics implies a reduction (increase) in carbon metrics with respect to the broad cap-weighted benchmark.

Conclusion

Our results show that it is possible to reconcile environmental and financial objectives using low carbon indices introduced by ERI Scientific Beta. While on the one hand these indices achieve an environmental objective by excluding high carbon stocks, and thus putting pressure on high polluting companies to reform, on the other, these indices achieve a financial objective by retaining exposure to rewarded risk factors and by maintaining a high level of diversification. Compared to the corresponding cap-weighted index, the Scientific Beta Developed Multi-Beta Multi-Strategy EW Six Factor index reduces both the carbon footprint and the carbon intensity by more than 40%, while generating an excess return of 2.45% over the cap-weighted index.

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Applying multi (smart) factor indexing methods to emerging market stocks

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There is ample evidence in the asset pricing literature to suggest that the cross-section of equity returns in emerging markets can be explained with the help of a limited set of factors. The evidence tells us that the factors for emerging markets are same as the set of consensual factors that are documented for developed markets.

Rouwenhorst (1999), for example, shows the existence of size, value and momentum factors for a sample of 20 emerging countries from 1982 to 1997. More recent studies such as Cakici et al (2013) also provide evidence on the presence of value and momentum factors in explaining the cross-section of stock returns for emerging markets. Blitz et al (2013) document the volatility factor (negative relationship between risk and return) for emerging markets stocks. Watanabe (2013) finds a negative relationship between asset growth and stock return for a global sample of both developed and emerging markets. Broadly, the empirical evidence suggests a premium in emerging markets (see figure 1 for a summary) for the academically consensual factors that are documented for developed markets.

Given that the same factors that are rewarded in developed markets are also rewarded in emerging markets, a natural question is whether one can apply the well-established factor indexing methodologies that have been tried and tested for developed markets to an emerging market universe. We look at this question here by analysing a set of Multi-Beta Multi-Strategy indices from Scientific Beta.

ERI Scientific Beta uses portfolio construction techniques to design indices that are well exposed to rewarded risk factors and are well diversified. In this article we discuss the benefits of applying ERI Scientific Beta's approach to construct a multi-smart factor index for emerging markets stocks and then highlight the benefits of applying country neutrality to multi-smart factor indices for emerging markets.

1. Factor premium in emerging markets

Factor	Factor definition	Sample	Time period	Premium	Source
Size	Stocks with low versus high market cap.	Emerging markets	1990–2011	0.28% (Monthly mean)	Cakici, Fabozzi and Tan (2013)
Value	Stocks with high versus low book-to-market	Emerging markets	1990–2011	1.15% (Monthly mean)	Cakici, Fabozzi and Tan (2013)
Momentum	Stocks with high versus low past 12 months return (excluding last month)	Emerging markets	1990–2011	0.86% (Monthly mean)	Cakici, Fabozzi and Tan (2013)
Volatility	Stocks with low versus high volatility (standard deviation of stock returns)	Emerging markets	1999–2012	2.10% (Monthly mean)	Blitz, Pang and van Vliet (2013)
Investment	Stocks with low versus high asset growth	Global (emerging/ developed)	1982–2010	6.18% (Annual mean)	Watanabe, Xu, Yao and Yu (2013)
Profitability	Stocks with high versus low gross-profitability ratio	Emerging markets in Europe	2002–14	0.96 % (Monthly mean)	Zaremba (2014)

The figure reports the factor premium in emerging markets for the academically consensual factors. The figures are reported from the source (paper) mentioned in the last column of the table.

Constructing multi (smart) factor indices in the emerging market universe

Before we discuss Scientific Beta's multi-smart factor indices for emerging markets, we first briefly mention the structure of Scientific Beta's emerging universe. Scientific Beta's emerging universe is made up of 700 securities, grouped into three emerging geographic basic blocks. Each geographic basic block has a fixed number of securities, which are representative of the largest and most liquid securities available to non-domestic investors. These geographic basic blocks are non-overlapping and an index for emerging markets is constructed by combining the indices for geographic basic blocks in proportion to their market capitalisation. Figure 2 lists the three emerging geographic basic blocks and the countries and fixed number of securities that constitute these geographic basic blocks.

2. Scientific Beta emerging universe

Emerging geographic basic block	Number of securities	Countries
Emerging America	140	Brazil, Chile, Colombia, Mexico, Peru
Emerging Europe, Middle East & Africa	160	Czech Republic, Egypt, Greece, Hungary, Poland, Qatar, Russia, South Africa, Turkey, United Arab Emirates
Emerging Asia Pacific	400	China, Indonesia, India, Malaysia, Philippines, South Korea, Taiwan, Thailand

The figure lists Scientific Beta's emerging geographic basic blocks, the fixed number of securities and countries that constitute these blocks.

ERI Scientific Beta applies its original approach, termed Smart Beta 2.0, to construct multi-smart factor indices for emerging markets. The approach distinguishes two steps in the construction of smart beta strategies, with the first step tilting towards the targeted risks by way of transparent security selection, and the second step diversifying away the undesired and unrewarded risks through the application of a diversified weighting scheme. These indices, called smart factor indices, are thus not just tilted towards the desired risk factors but also well diversified.

ERI Scientific Beta's multi-smart factor indices are constructed by combining smart factor indices, representing a set of academically documented risk factors. The allocation across factors allows investors to diversify the sources of their outperformance and smooth their performance across market conditions.

In particular, single smart factor indices are designed for six rewarded factor exposures, namely mid cap, value, momentum, low volatility, low investment and high profitability. The single-factor indices draw on Scientific Beta's Diversified Multi-Strategy weighting scheme, which combines five diversification methods to obtain good diversification of unrewarded risks. It should be noted that the index construction methodology follows the methodology that has been used for Scientific Beta's developed market indices since late 2012. The single smart factor indices are then combined to obtain the Multi-Beta Multi-Strategy indices. The combination we discuss here is a simple equal-weighted allocation to each single factor index.

Avoiding bets on macro risk exposure

Country risk has long been recognised as a prominent risk factor impacting equity returns in emerging markets (see, for instance, Erb et al [1995], and Rouwenhorst [1999]). Implementing a smart beta strategy without imposing geographic constraints can result in significantly different country exposure relative to a reference index. For an investor who only wishes to be exposed to the well-documented microeconomic risk factors (size, value, momentum, volatility, profitability and investment), taking on unintended or implicit exposure to macroeconomic country risk does not appear to be justified. Concerning the macroeconomic risk factor exposures of emerging market stocks, it has indeed been documented that there is no link between such exposures and expected returns. Campbell Harvey (1995) in a paper on the risk exposure of emerging markets finds for example a "lack of cross sectional relation of (macroeconomic) risk exposures and expected returns" for emerging market stocks. This perhaps creates a need to construct a country-neutral strategy to avoid unintended country exposure.

Taking into account the unintended country exposure is also relevant from the perspective of controlling relative risk with respect to a benchmark (ie, tracking error), especially when one is not sure whether this risk is rewarded in comparison with the reward from a risk-controlled strategy.

ERI Scientific Beta offers country-neutral versions of indices for investors who want to avoid unintended country exposures (see Amenc et al [2016a]). It has been offering the option for country neutrality for developed market indices since December 2012, and as country neutrality is particularly relevant for emerging markets, the option of country neutrality has also been made available for emerging market indices. We note that the country neutrality methodology that is applied to the emerging markets is the same as the methodology applied to the developed market indices. ERI Scientific Beta defines country neutrality in relative terms against a broad cap-weighted index. The ERI Scientific Beta methodology to construct a country-neutral index involves two key steps. As a first step, in any regional universe, the stock selection based on the corresponding factor score is performed within each country. As a result, each country has sets of high and low scoring stocks, which are finally reassembled together to form the total high and low universe for the given regional universe. As a second step, post-optimisation, to achieve neutrality, the weight of a strategy is re-scaled to match the country weight of the cap-weighted benchmark.

In this section we first analyse the impact of applying ERI Scientific Beta's country neutrality methodology on country biases with respect to the cap-weighted benchmark. Next, we analyse whether the reward is higher from a strategy that takes higher tracking error risk or from a strategy which is country neutral.

Figure 3 reports relative country allocation of the country neutral and the standard (non-country neutral) Multi-Beta Multi-Strategy Six-Factor EW index for emerging markets. We note that without any country neutrality, the relative allocation to countries with respect to the cap-weighted benchmark can be high (eg, –8.25% for China). Applying country neutrality brings down the difference in country allocation of the strategy and its cap-weighted benchmark, and in this case the difference comes down to the range of –1.28% to 0.74%.

In figure 4 we analyse the performance of the Multi-Beta Multi-Strategy Six-Factor EW index with and without country neutrality. We note that the relative returns of both strategies are similar (a difference of 19 basis points), but the tracking error of the Multi-Beta Multi-Strategy Six-Factor EW index without any country neutrality (6.05%) is significantly higher than the tracking error of the index with country neutrality (4.23%). The result is that the information ratio of the Multi-Beta Multi-Strategy Six-Factor EW index without country neutrality (0.65) is noticeably lower than that of the index with country neutrality (0.88). This highlights the fact that applying country neutrality is better than taking high tracking error risk, by not being country neutral, when it is not rewarded.

It is also of interest to analyse the conditional performance of the indices in more detail, notably in terms of dependency of performance on market conditions as characterised by returns to the broad cap-weighted index for ►

3. Benefits of country neutrality (country allocation)

31 Dec 2005–31 Dec 2015		Scientific Beta Emerging Market Indices	
	Broad CW	Multi-Beta Multi Strategy Six-Factor EW	Multi-Beta Multi Strategy (Country Neutral) Six-Factor EW
Country	Country allocation	Relative country allocation (wrt broad CW)	
China	17.01%	–8.25%	–1.28%
South Korea	13.17%	–3.16%	–0.07%
Taiwan	14.74%	1.73%	0.62%
India	11.57%	–0.26%	0.05%
South Africa	8.97%	0.84%	0.74%
Brazil	13.58%	–5.55%	0.19%
Mexico	6.51%	–1.76%	–0.13%
Russia	7.54%	–4.85%	–1.07%
Malaysia	3.80%	5.17%	0.46%
Others	15.29%	13.18%	0.43%

The figure reports the country allocation of the SciBeta Emerging Cap-Weighted index and the relative country allocation of the SciBeta Emerging Multi-Beta Multi-Strategy Six-Factor EW indices (both standard and country neutral). The period of analysis is 31 December 2005–31 December 2015. The table reports the average value of country allocations (for the cap-weighted benchmark) and the relative country allocation (for the Scientific Beta Multi-Beta Multi-Strategy Six-Factor EW indices) calculated at each of the 40 quarterly rebalancings between 31 December 2005 and 31 December 2015. In the event that a country was not classified as emerging at any point in time, we consider only those quarters where the country was classified as emerging in calculating its average relative allocation. For example, South Korea was included in emerging markets in June 2015, and so to report its average relative allocation, we consider its relative allocation for quarters starting June 2015 to December 2015, and ignore the quarters before June 2015.

4. Benefits of country neutrality (performance analysis)

31 Dec 2005–31 Dec 2015		Scientific Beta Emerging Market Indices	
	Broad CW	Multi-Beta Multi Strategy Six-Factor EW	Multi-Beta Multi Strategy (Country Neutral) Six-Factor EW
Absolute performance			
Annualised return	4.60%	8.53%	8.34%
Volatility	21.30%	17.05%	18.70%
Sharpe ratio	0.16	0.43	0.39
Maximum drawdown	64.28%	56.98%	61.00%
Relative performance			
Relative return	–	3.93%	3.74%
Tracking error	–	6.05%	4.23%
Extreme TE*	–	13.09%	8.26%
Information ratio	–	0.65	0.88
Maximum relative drawdown	–	12.40%	7.35%
Maximum relative loss**		2.95%	0.58%

The figure reports the performance of the SciBeta Emerging Cap-Weighted index and the SciBeta Emerging Multi-Beta Multi-Strategy Six-Factor EW indices (both standard and country neutral). The period of analysis is 31 December 2005–31 December 2015. Analytics are based on daily total return index series (dividends reinvested) in US dollars. Performance figures are annualised. The Secondary Market US Treasury Bill (3M) is used as the risk-free rate in US dollars. The Broad CW is the SciBeta Emerging Cap-Weighted index, which is used for relative analysis. *Extreme TE (tracking error) is calculated as the 95th percentile of rolling tracking error calculated using a rolling window of one year (260 days) and step size of one week (five days). **To calculate maximum relative loss of a strategy during a period, we construct a long/short strategy, where the long leg is the strategy and the short leg is the cap-weighted benchmark. We define maximum relative loss as the maximum loss suffered by the long-short strategy compared to its value at the beginning of the period.

◄ emerging market stocks. Figure 5 shows that the Multi-Beta Multi-Strategy (Country Neutral) Six-Factor EW has consistent relative performance with respect to its cap-weighted benchmark in both extreme bull and bear market conditions. The results further suggest that the consistency of outperformance of the country-neutral index is more pronounced than for the index without country neutrality.

Conclusion

We analysed the benefits of multi-smart factor indices for emerging markets. We make a case for avoiding country bets for emerging market indices, by highlighting the fact that applying country neutrality is better than taking high tracking error risk, by not being country neutral, when it is not rewarded.

It should be noted also that any back test of smart beta strategies may be subject to potentially severe data mining risks if index design is conducted in an environment that allows researchers flexibility to adjust strategies so that they lead to better performance in the back test (with the benefit of hindsight on back-tested results; see Amenc et al [2015], for a discussion). However, if an index methodology is applied consistently to a new universe on which it has never been tested, this precludes any possibility of conducting a data-mining exercise. In this respect, the application of the existing Multi-Beta Multi Strategy methodology, which had been introduced for Developed Market indices, to an Emerging Market universe, actually constitutes a useful robustness test. In fact, the benefits of Scientific Beta's Multi-Beta Multi-Strategy methodology have been documented using Developed Market data (see eg, Amenc et al [2014a, 2014b]) and developed market indices applying Scientific Beta's smart factor indexing methodologies have been live since December 2012. That the results in emerging markets are highly consistent with the results previously obtained in developed markets is testament to the robustness of the methodologies employed for these indices.

5. Benefits of country neutrality (conditional performance analysis)

31 Dec 2005–31 Dec 2015	Scientific Beta Emerging Market Indices		
	Broad CW	Multi-Beta Multi Strategy Six-Factor EW	Multi-Beta Multi Strategy (Country Neutral) Six-Factor EW
Top 25 quarters by market returns (extreme bull)			
Absolute performance			
Annualised return	84.06%	84.71%	89.56%
Volatility	18.28%	14.30%	15.64%
Sharpe ratio	4.49	5.79	5.60
Relative performance			
Relative return	–	0.65%	5.50%
Tracking error	–	5.54%	4.06%
Information ratio	–	0.12	1.36
Bottom 25 quarters by market returns (extreme bear)			
Absolute performance			
Annualised return	–44.84%	–37.20%	–40.09%
Volatility	30.51%	24.38%	26.95%
Sharpe ratio	–1.50	–1.56	–1.52
Relative performance			
Relative return	–	7.63%	4.75%
Tracking error	–	8.33%	5.53%
Information ratio	–	0.92	0.86

The figure reports the conditional performance of the SciBeta Emerging Cap-Weighted index and the SciBeta Emerging Multi-Beta Multi-Strategy Six-Factor EW indices (both standard and country neutral). The period of analysis is 31 December 2005–31 December 2015. Analytics are based on daily total return index series (dividends reinvested) in US dollars. Performance figures are annualised. The Secondary Market US Treasury Bill (3M) is used as the risk-free rate in US dollars. The Broad CW is the SciBeta Emerging Cap-Weighted index, which is used for relative analysis. The top 25% of quarters with the highest market returns are considered extremely bullish and the bottom 25% quarters with the lowest returns are considered extremely bearish.

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Defensive strategies (I): concepts underlying low risk equity strategies

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In the context of defensive investing, investors often confuse being exposed to a defensive strategy, such as efficient minimum volatility, and benefitting from the reward to the low risk factor. The reason being the two types of defensive strategies, ie, low volatility and minimum volatility, which derive from different academic traditions, ie, factor investing and modern portfolio theory, respectively, can produce comparable levels of risk reduction in practice. While factor-investing strategies aiming to extract the low risk factor premium were introduced in a long/short setting and as such could be market neutral, their long-only unleveraged implementations, ie, low volatility/beta strategies, naturally acquire a defensive character as they overweight low risk stocks.

Modern Portfolio Theory minimum volatility strategies, on the other hand, are intended to be defensive, as they explicitly aim to identify the portfolio with the lowest risk on Markowitz's (1952) efficient frontier. While they give no consideration to factor exposures and explicitly rely on exploiting both low individual stock volatilities and low pair-wise correlations between stocks, traditional implementations of minimum volatility investing can be expected to produce portfolios that are dominated by low risk stocks and, as such, could indirectly produce significant exposure to the low risk factor (assuming sufficient diversification of idiosyncratic risk). ERI Scientific Beta makes a clear distinction between these two approaches, which, even though they have points in common (low beta, exposure to low volatility risk), do not have the same objectives or the same long-term performance and risk.

The low risk factor

The low risk factor occupies a particular place in the asset pricing literature as the performance of low risk strategies appears to directly contradict the central prediction of the CAPM – that returns should be linearly related to systematic market risk (as measured by market beta, ie, the covariance between the returns of the portfolio and those of the market standardised by the variance of market returns). Below we underline that the said performance is one of the strongest results in empirical finance, that it has spurred a rich literature on the rewarded factors in asset prices and that it is supported by theoretical justifications.

The lack of empirical success for the CAPM, the founding model of asset pricing theory, prompted a search for better asset pricing models which led to theoretical advances starting with Black's (1972) restricted borrowing CAPM and, following the proposal of multi-factor models with Merton's (1973) inter-temporal CAPM and Ross's (1976) arbitrage theory of capital asset pricing, to the search for and identification of priced factors in equity returns beyond market risk, the single factor posited by the CAPM, and from Fama and French (1992, 1993) onwards to their inclusion in multi-factor pricing models.

While the low risk 'anomaly' first documented by Friend and Blume (1970), Black, Jensen and Scholes (1972), Miller and Scholes (1972) and Haugen and Heins (1972, 1975), was responsible for triggering the work that led to these theoretical and empirical advances in multi-factor asset pricing, the 'anomaly' has survived to this day[1] and has been rejuvenated by recent theoretical work. Influential work by Ang, Hodrick, Xing and Zhang (2006, 2009) finds that stocks with high total volatility underperform and that stocks with high recent idiosyncratic volatility have low average returns that are not explained by the size, book-to-market and momentum effects of Fama and French (1992, 1993) and Jegadeesh and Titman (1993). While a number of papers try to explain away the idiosyncratic volatility results of Ang et al (see Martellini [2013] for a review), others, such as Chen, Jiang, Xu and Yao (2012) defend it as a common phenomenon. Stambaugh, Yu and Yuan (2015) justify the negative relation between idiosyncratic volatility and average return by restrictions on short sales that limit the shorting of overpriced stocks (which they contend exhibit the negative relationship, particularly in periods of high investor sentiment).[2] Complementing behavioural explanations of the performance of defensive strategies, Baker, Bradley and Wurgler (2011) note that tracking error constraints in benchmarked institutional management discourage arbitrage activity in both high-alpha low-beta stocks and low-alpha high-beta stocks.

Following on from Black (1972, 1993), Frazzini and Pedersen (2014) present a model of leverage-constrained investment that explains why investors seeking a high degree of market risk[3] cause low-beta assets to overperform high-beta assets on a risk-adjusted basis. They document that the 'betting against beta' strategy, which is long leveraged low-beta assets and short high-beta assets, produces significant positive risk-adjusted returns. Importantly, they show that the poor returns of the strategy when funding constraints become tight are consistent with liquidity-constrained investors having to sell leveraged positions in low-risk assets in bad times, providing a risk-based justification for the observed premium.

Nature and limits of defensive strategies

The implementation of low risk factor harvesting in a long-only and zero-leverage environment creates a defensive risk/return profile as a by-product, while the practical implementation of defensive minimum volatility strategies will typically lead to portfolios dominated by low volatility stocks, which could potentially reap some of the benefits of the low risk factor as a side effect. Note that the confusion between minimum volatility strategies deriving from Modern Portfolio Theory and low volatility factor strategies owes a lot to MSCI, which has been marketing its minimum volatility indices as low volatility factor indices. Below we discuss separately the properties and limitations of the two kinds of defensive strategies. ►

1 Black (1993) contends that this evidence is stronger than the corresponding evidence for the factors introduced by Fama and French (1992, 1993). Empirical studies documenting the performance of low risk portfolios also include Haugen and Baker (1991), Jangannathan and Ma (2003); Fama and French (2004), Clarke, de Silva and Thorley (2006) and Baker, Bradley, and Wurgler (2011). While the aforementioned studies concern US markets, the same effect has been documented for global equity markets by Blitz and van Vliet (2007), Baker, Nardin and Haugen (2012) and Baker, Bradley and Taliaferro (2014), among others.

2 Hong and Sraer (2015) show how, in the presence of short-sale restrictions, disagreement amongst investors on the future cash flows of firms leads to overpricing of stocks. As disagreement increases with a stock's beta, high beta stocks, which are more sensitive to aggregate disagreement than low beta ones, are only held in equilibrium by optimists as pessimists are sidelined. This greater divergence of opinion creates relative overpricing of high beta stocks. Analyst over-optimism regarding high-growth high-volatile stocks and insufficient discernment on the part of investors reacting to these forecasts has been put forward as a behavioural explanation of the low volatility effect by Hsu, Kudoh and Yamada (2013).

3 Note that this is different from an irrational preference for highly volatile 'lottery stocks' and 'glamour stocks' that has been offered as a behavioural explanation for the low risk phenomenon.

Minimum volatility strategies

In the Markowitz framework, the global minimum variance portfolio is a remarkable portfolio that lies on the efficient frontier and provides the lowest possible portfolio volatility. Identifying this portfolio requires the variance-covariance matrix as an optimisation input. From a theoretical standpoint and as explained by Tobin (1958), minimum variance portfolios are not optimal in the presence of a riskless asset since they are dominated by a combination of the optimal risky portfolio maximising the risk/return trade-off (tangency portfolio) and the riskless asset. In this context, the minimum variance portfolio coincides with the optimal risky portfolio only when the expected returns of all assets are identical, a rather unrealistic optimality condition.

In practice however, identifying the tangency portfolio in the traditional manner is extremely difficult as it requires estimation of the expected returns for use in optimisation. Indeed, as shown by Merton (1980), a long history is required to estimate an expected return that is known to be constant, and there is no reason why such an expected return should be a constant. The higher degree of estimation error associated with estimating the tangency portfolio in the traditional way could more than offset the benefits of an absence of optimality risk. For illustration, Jorion (1985) or Jagannathan and Ma (2003) find that tangency portfolios do not perform as well as the global minimum variance portfolios when assessed on out-of-sample Sharpe ratio.

However, even though the global minimum variance portfolio is easier to estimate, there are nonetheless challenges in constructing this type of portfolio. Unconstrained minimum variance optimisation typically produces portfolios that are extremely concentrated (in a small number of low volatility stocks) and suffer from severe sector biases (Chan, Karceski and Lakonishok [1999]). Furthermore, as optimised concentrated portfolios, they should be expected to exhibit very high turnover if parameters are time-varying and they do (eg, Clarke, de Silva and Thorley [2011]).

As for documentation of the concentration of minimum volatility portfolios, Clarke, de Silva and Thorley (2011) observe that their long-only minimum variance portfolio is constituted on average of 12% of their 1,000-security universe while DeMiguel et al (2009) note that "short-sale-constrained minimum-variance portfolios (...) tend to assign a weight different from zero to only a few of the assets". These difficulties often result therefore in unconstrained minimum volatility-type portfolios being portfolios that are concentrated and poorly diversified over a small number of low-volatility stocks. In order to remedy this problem, asset managers or index providers impose absolute and/or relative deconcentration constraints. But the cure is often worse than the illness, because this set of rigid ad-hoc constraints is in fact the veritable driver of the performance of minimum volatility strategies without there being any academic justification whatsoever for the nature or value of the constraints chosen, which depend more in this case on in-sample calibration than on a concern for out-of-sample robustness. Sold with an objective of efficient diversification, in many cases minimum volatility strategies hardly use the portfolio decorrelation budget and have fairly low levels of diversification and thus high degrees of idiosyncratic risk, the diversifiable risk that is not rewarded according to standard asset pricing theory.

Low volatility strategies

The typical low volatility strategy does not rely on an optimisation procedure but instead selects stocks with low historical volatility and then applies an ad-hoc weighting scheme that may or may not take into account differences in the individual volatilities of selected stocks. For illustration, capitalisation-weighting disregards individual volatilities whereas inverse volatility (as used by the S&P 500 Low Volatility index) or variance as well as volatility-tilted capitalisation weighting let individual volatilities impact constituent weights.

Such low volatility approaches rely solely on low volatility stocks, which should be beneficial if such stocks carry better risk-adjusted rewards than stocks that are more volatile. That is the premise of factor investing strategies tilting towards low risk stocks. Note that these approaches disregard the potential of volatility reduction that lies in correlations between securities. Naturally, ignoring correlations has practical advantages since the number of correlation coefficients in a universe of stocks increases with the squared number of stocks and correlation estimates are hard to estimate reliability (Longin and Solnik [1995]).

As is usually the case with industry implementations of factor investing, narrow factor-based stock selections and the use of weighting schemes favouring concentration lead to highly-concentrated portfolios, which have been documented to exhibit high turnover and a strong proportion of specific volatility (eg, Amenc et al [2016]). Ultimately, these strategies that are explicitly exposed to low volatility stocks suffer from the same defect as minimum volatility-type strategies, their high degree of concentration, which deprives them of one of the clear benefits of Modern Portfolio Theory since the seminal work of Harry Markowitz: diversification.

Addressing the concentration issue

Minimum volatility strategies

Irrespective of whether an investor regards low volatility stocks as attractive or unattractive, it is clear that the traditional minimum volatility strategy leads to poorly diversified portfolios and does not fully exploit correlations. As mentioned, popular implementations of low volatility strategies can be just as concentrated and disregard correlations completely.

Various approaches have been proposed to remedy the concentration issue of optimisation-based strategies, the most straightforward being to impose weight constraints. Imposing rigid security-level bounds reduces the ability of the optimiser to exploit the information in the variance-covariance matrix, but can help to obtain more 'reasonable' portfolios: absolute upper bounds promote diversification while lower bounds reduce implementation costs by doing away with small holdings. In addition, security-level upper bounds couched in relative terms, ie, as multiples of security-level weights within the capitalisation-weighted index of the underlying universe, are used to reduce concentration in small (and typically less liquid) securities. Portfolio-level weight constraints are also routinely used to reduce country and sector biases, although this can exacerbate the concentration issue at the security level. It should be underlined that, as more constraints are added, the solution is taken further away from the theoretically optimal portfolio. More worryingly, this makes the performance of the resulting portfolios highly sensitive to the choice of constraints, which comes with significant robustness risk. Indeed, choosing constraints to produce excellent in-sample performance will typically lead to disappointing out-of-sample results (on the effects of back-test over-fitting on out-of-sample performance, refer to Bailey, Borwein, López de Prado and Zhu [2014]).

A more flexible approach has been introduced by EDHEC-Risk Institute Professor Raman Uppal and his co-authors. The 'norm constraints' in DeMiguel, Garlappi, Nogales and Uppal (2009) limit the overall amount of concentration at the portfolio level, eg, by constraining the sum of squared weights, rather than imposing caps on all stocks individually. The authors show that using such flexible concentration constraints instead of rigid upper and lower bounds on individual stock weights allows for a better use of the correlation structure. The approach is found to produce portfolios that typically have higher out-of-sample Sharpe ratios than competing approaches. This is the approach that ERI Scientific Beta has selected for its efficient minimum volatility weighting scheme.

Low volatility strategies

The concentration of traditional low volatility strategies (and of other factor-based strategies) is caused by explicit choices of narrow factor-based selections and/or concentrated weighting schemes that aim to maximise the factor scores of portfolios. This concentration issue does not arise in the context of diversified factor-tilted solutions, where one relies on broad security selections and diversified weighting schemes. Diversified factor-tilted indices enjoy reduced exposure to idiosyncratic and other non-rewarded risks of all kinds, including relative industry and country biases; mitigate the risk of concentration into small and illiquid securities; and reduce turnover from changes in security-level factor scores (since broad factor-based selections are more stable and score-blind diversification strategies are unaffected by changes in scores). While the investment industry has favoured concentrated factor tilts, the seminal empirical and theoretical literature on factor investing underlines the importance of diversification and no case has been made in support of inefficient factor-tilted portfolios.

On the contrary and from a theoretical standpoint, Cochrane (1999) emphasises that any portfolio should be constructed so as to provide the efficient risk/return trade-off, in a mean-variance sense, at a given level of factor exposure. Fama (1996) shows that rewarded factors can be understood as multi-factor mean-variance efficient portfolio themselves. From an empirical standpoint, Amenc et al (2016) find that, for a given breadth of selection, diversified portfolios deliver higher returns and risk-adjusted returns and have higher probabilities of outperforming the broad market than capitalisation-weighted portfolios. Analysed in the Carhart framework, they produce much higher alphas and alphas per unit of residual standard deviation and higher reduction in idiosyncratic volatility. They also observe that moving from a broad (half-universe) to a narrow (quintile) selection produces higher gross returns. It also increases volatility and tracking error, resulting in at best marginal gains in risk-adjusted performance before taking into account the costs of severely heightened turnover and reduced liquidity associated with narrower selections. In the end, they document that the benefits of (naively) diversifying factor-tilted portfolios based on broad selections far outweigh those of shifting to narrow selections while remaining cap-weighted. Such diversified factor-tilted portfolios produce much better performance and risk-adjusted performance in the medium ►

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◄ and the long term while only marginally impacting turnover.

Conclusion

For EDHEC Risk Institute and ERI Scientific Beta, constructing a robust defensive strategy involves integration of the risk of excessive concentration and of the poor diversification of specific risk, which is very present in traditional approaches and offerings for both minimum volatility indices and low volatility factor indices. In recent months, investors have focused on the overpricing of low volatility stocks, without this overpricing being the subject of genuine academic consensus[4], and have continued to neglect the problem of the excessive concentration and poor diversification of low volatility portfolios, which the low volatility strategies based on fundamental weighting do not solve for example. Our research (Amenc, Ducoulombier and Lodh (2016)) summarised in the following article shows that better and more robust performance is built on this point of improving the diversification of defensive strategies.

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4 While the rationale for a reward to holding low risk (low beta or low volatility) stocks is based on an ample amount of peer reviewed academic evidence, claims that the low volatility effect exists solely because of increasing overpricing are solely based on providers' brochures and absent from the body of peer reviewed academic evidence. It should also be noted that, even in principle, it is unclear how the low risk effect which has been documented consistently on long term US equity data (close to a century of data), international equity data and other asset classes, could be driven by 'overpricing', which by definition should be a short-term phenomenon. For further reasons to reject claims about overpricing of factors, we refer to Asness (2016).

Defensive strategies (II): revisiting traditional defensive strategies with smart factor indices

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Having discussed the different theoretical foundations and construction steps underpinning minimum volatility and low volatility strategies in the preceding article in this issue, we introduce alternative approaches to limiting concentration in minimum and low volatility strategies in the current article. On the basis of this deconcentration objective, the article introduces ERI Scientific Beta's Smart Beta 2.0 framework, the principle of which is to reconcile a choice of factor exposure (here, low volatility) and to obtain a portfolio representing this factor choice that is not only deconcentrated but also very well diversified. We describe how the framework provides smart factor indices that are consistent with both modern portfolio theory and the factor investing literature since they represent well-diversified portfolios that are tilted along the desired factor dimensions. The article also shows how to implement low volatility and minimum volatility strategies with smart factor indices tilted towards the low risk factor, documents their risk and performance using 45 years of US large and mid-cap data and provide comparisons with popular defensive indices over 10 years for both US and developed markets as a whole.

Understanding smart factor indices

Consistent with the academic literature on factor-investing and modern portfolio theory, the ERI Scientific Beta Smart Factor indices are designed to be efficient factor proxies. As such, they do not seek to maximise factor exposures but instead to achieve higher than median exposures to the desired rewarded factors while ensuring a high degree of diversification of idiosyncratic risks. This allows smart factor indices to deliver high and robust risk-adjusted returns and to guarantee high investability and low implementation costs. It also offers the opportunity to control other non-rewarded risks such as sector or country risk without jeopardising performance.

All ERI Scientific Beta Smart Factor indices are constructed with a two-step process which is at the heart of the Smart Beta 2.0 approach promoted by EDHEC-Risk Institute: (i) factor-tilting towards the targeted priced factor by way of broad stock selection (half-universe) and; (ii) application of a smart beta diversification strategy to the selection. As documented by Amenc et al (2014), this allows the long-term premia associated with the desired factor tilts to be harvested efficiently while also reducing the contribution of unrewarded or specific risks to short-term volatility and tracking error. Stock-specific risk is reduced through the use of a suitable diversification strategy and the combination of strategies reduces the model risks inherent in single diversification strategies. As illustrated in figure 1, five diversification strategies are available for ERI Scientific Beta single-strategy Smart Factor indices and their equal-weighted combination is available as the weighting scheme underlying ERI Scientific Beta's off-the-shelf Multi-Strategy Smart Factor indices.

1. Overview of popular equity diversification strategies

Strategy	Objective	Unconstrained closed-form solution	Required parameter(s)	Optimality conditions
Maximum deconcentration	Maximise effective number of stocks	$w^* = \frac{1}{N}\mathbb{1}$	None	$\mu_i = \mu \forall i$ $\sigma_i = \sigma \forall i$ $\rho_{ij} = \rho \forall i$
Diversified risk parity	Equalise risk contributions under 'constant correlation' assumption	$w^* = \frac{diag(\sigma^{-1})}{\mathbb{1}' diag(\sigma^{-1})}$	σ_i	$\lambda_i = \lambda \forall i$ $\rho_{ij} = \rho \forall i$
Maximum decorrelation	Minimise portfolio volatility under the assumption of identical volatility across all stocks	$w^* = \frac{\Omega^{-1}\mathbb{1}}{\mathbb{1}'\Omega^{-1}\mathbb{1}}$	ρ_{ij}	$\mu_i = \mu \forall i$ $\sigma_i = \sigma \forall i$
Efficient minimum volatility	Minimise portfolio volatility	$w^* = \frac{\Sigma^{-1}\mathbb{1}}{\mathbb{1}'\Sigma^{-1}\mathbb{1}}$	σ_i, ρ_{ij}	$\mu_i = \mu \forall i$
Efficient maximum Sharpe ratio	Maximise portfolio Sharpe ratio	$w^* = \frac{\Sigma^{-1}\mu}{\mathbb{1}'\Sigma^{-1}\mu}$	μ_i, σ_i, ρ_{ij}	Optimal by construction

The figure indicates, for the diversification strategies, the optimisation objective (without taking into account any constraints, turnover control or liquidity rules), its unconstrained solution and the required parameters. The 'Optimality conditions' column indicates under which conditions each diversification strategy would result in the maximum Sharpe ratio portfolio from Modern Portfolio Theory. N is the number of stocks, μ_i is the expected return on stock i, σ_i is the volatility for stock i, ρ_{ij} is the correlation between stocks i and j, μ is the (N×1) vector of expected returns, 1 is the (N×1) vector of ones, σ is the (N×1) vector of volatilities, Ω is the (N×N) correlation matrix and Σ is the (N×N) covariance matrix.

Owing to the joint use of broad selections and of weighting schemes that ensure that opportunities for diversification of specific risk present in each selection be used, the ERI Scientific Beta Smart Factor methodology avoids or reduces issues associated with concentration (arising from optimisation and factor exposure maximisation approaches), notably high average and extreme idiosyncratic risks, undesired biases, high turnover, capacity and liquidity issues and high susceptibility to errors. Additional capacity/liquidity adjustments and optimal turnover control are applied to further increase investability of the indices and reduce replication costs.[1]

Smart factor indices deliver significant added value above cap-weighted factor-tilted stock selections. For illustration, figure 2 compares the US 45-year risk and performance profiles of the off-the-shelf Multi-Strategy Smart Factor indices (DMS for diversified multi-strategy in the table) tilted towards strategic factors, to that of the identical stock selections weighted by ►

1 Sector and country neutrality can also be imposed on off-the-shelf multi-strategy indices as well as customised indices.

2. Performance of capitalisation-weighted vs multi-strategy diversified factor selections

31 Dec 1970–31 Dec 2015		Mid cap		Positive momentum		Low volatility		Value		Low investment		High profitability	
	Broad CW	CW	DMS	CW	DMS	CW	DMS	CW	DMS	CW	DMS	CW	DMS
Annualised returns	10.45%	12.97%	14.22%	11.39%	13.39%	10.76%	13.00%	11.90%	14.28%	12.32%	14.01%	10.79%	12.99%
Annualised volatility	16.88%	17.09%	15.80%	17.26%	16.01%	15.39%	13.85%	17.17%	15.67%	15.83%	14.93%	17.02%	15.73%
Sharpe ratio	0.32	0.46	0.58	0.37	0.52	0.37	0.57	0.40	0.59	0.46	0.60	0.34	0.50
Maximum drawdown	54.63%	57.09%	53.42%	50.81%	53.25%	51.10%	48.31%	60.01%	53.75%	51.12%	50.82%	52.29%	48.86%
Effective number	114	188	188	64	198	65	200	69	189	59	187	55	199
Annualised relative returns	–	2.52%	3.77%	0.94%	2.95%	0.32%	2.55%	1.45%	3.84%	1.87%	3.57%	0.34%	2.54%
Annualised tracking error	–	5.72%	6.42%	3.49%	4.84%	4.27%	5.99%	4.42%	5.47%	3.79%	5.42%	3.22%	4.35%
95% tracking error	–	9.27%	11.54%	6.24%	8.59%	8.18%	11.38%	8.12%	10.03%	6.58%	9.88%	6.48%	7.20%
Information ratio	–	0.44	0.59	0.27	0.61	0.07	0.43	0.33	0.70	0.49	0.66	0.11	0.58
Outperformance probability (1Y)	–	61.6%	66.6%	62.8%	66.3%	50.8%	63.5%	58.9%	69.2%	62.1%	70.3%	50.6%	68.0%
Outperformance probability (3Y)	–	69.4%	75.7%	71.3%	75.7%	52.7%	75.7%	68.7%	79.5%	77.7%	82.0%	51.7%	78.9%
Outperformance probability (5Y)	–	75.9%	81.5%	81.1%	87.0%	58.4%	86.9%	67.5%	89.0%	89.1%	89.9%	58.5%	85.6%
Maximum relative drawdown	–	35.94%	42.06%	14.44%	17.28%	33.82%	43.46%	20.31%	32.68%	26.47%	38.49%	24.52%	25.21%
1-way turnover	3.1%	19.3%	27.4%	58.1%	67.1%	10.0%	26.2%	14.6%	25.3%	26.1%	33.7%	5.4%	23.0%

The analysis is based on daily total return data in US dollars from 31 December 1970 to 31 December 2015 (45 years). The benchmark is the cap-weighted portfolio of all stocks in the USA universe. The Scientific Beta LTTR USA universe consists of the largest 500 US stocks. Mid cap, positive momentum, low volatility, value, low investment and high profitability selections all represent 50% of stocks with such characteristics in a US universe of 500 stocks. The risk-free rate is the return of the 3-month U.S. Treasury Bill. Maximum relative drawdown is the maximum drawdown of the long-short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. 95% tracking error is the 95th percentile of one-year rolling tracking error. It is computed using a one-year rolling window and a one-week step size. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1, 3 or 5 years at any point during the history of the strategy. Rolling windows of 1, 3 and 5 years and a one-week step size are used. Source: www.scientificbeta.com.

◄ capitalisation (CW in the table). Note that the gain in returns and risk-adjusted returns is particularly strong for low volatility strategies. Indeed, the capitalisation-weighted low volatility selection outperforms the broad-market index by only 32 basis points (bps) per annum (pa) while the Low Volatility Diversified Multi-Strategy index delivers 255bps pa above the same benchmark. In addition, the volatility of the Low Volatility Diversified Multi-Strategy index is only 90% of that of the capitalisation-weighted selection. Thus, higher excess returns and lower volatility both contribute to producing a superior Sharpe ratio for the Low Volatility Diversified Multi-Strategy index relative to that of the capitalisation-weighted low volatility selection; these are 0.37 and 0.57, respectively.

ERI Scientific Beta Smart Factor indices also typically outperform factor indices put forward by other index providers that do not benefit from the clear separation between choice of factor and choice of diversification method or fail to optimise diversification of idiosyncratic risks. It is on the basis of this smart factor approach, which combines an explicit choice of factor exposure with the benefits of diversifying the stocks that correspond to this factor exposure, which ERI Scientific Beta proposes to revisit the traditional defensive strategies.

Implementing low volatility and minimum volatility strategies with smart factor indices

ERI Scientific Beta uses total volatility to rank individual stocks for factor-tilting, which is not only consistent with the work of Ang et al (2006, 2009) on total volatility, but also with their results on idiosyncratic volatility (since it represents the lion's share of total volatility at the individual stock level) and with the systematic risk approach of Frazzini and Pedersen (2014) (because low beta stocks tend to be low volatility stocks in the cross section and vice versa). Figures 3–5 compare the performance of two ERI Scientific Beta Low Volatility Smart Factor indices to that of popular defensive indices for the US and the developed world. These two indices correspond to two different approaches to low volatility strategies.

The Scientific Beta Low Volatility Multi-Strategy indices are intended to capture the low risk factor, while being highly diversified. This diversification is guaranteed by equally weighting five diversification methods whose combination, as was shown in Amenc et al (2014), produces an excellent level of risk-adjusted performance. This quality of diversification is explained by the fact that each of the weighting schemes used presents different model risks (optimality and estimation error risks)[2] as shown in figure 1 and is associated with different biases and conditionality. Combining multiple weighting schemes not only diversifies the aforementioned model risks, providing more robust diversification, but also allows investors to avail of the benefits of the decorrelation between the various diversification strategies that result from these weighting models.

The Scientific Beta Low Volatility Efficient Minimum Volatility indices do not seek to maximise diversification of the specific risk, but instead aim to maximise the defensive nature of the strategy while maintaining a high degree of diversification, and notably by using a state-of-the-art norm constraint deconcentration method[3]. Applying a deconcentration method enables a defensive but well-diversified index to be constructed, since, unlike traditional rigid constraints, this approach favours better use of the diversification potential offered by correlations between stocks. This is all the more so when the efficient minimum volatility approach is applied to a selection of low volatility stocks as this reduces the ability of the optimiser to simply select low volatility stocks to meet its objective and forces it to make better use of the information in the correlation matrix.

The historical out-of-sample simulations over both long and short periods confirm that the objectives of these indices are met and show notably that the Scientific Beta Low Volatility Efficient Minimum Volatility indices present a more defensive character than the Scientific Beta Low Volatility Diversified Multi-Strategy indices whose diversified multi-strategy approach produces relative performance that is less affected by market direction. In both cases, through their superior diversification qualities, these indices exhibit remarkable risk-adjusted performance that is of course superior to that of traditional defensive strategies.

As shown in figure 3, over the 45 years of the long-term track record and relative to the capitalisation-weighted benchmark, the ERI Scientific Beta Low Volatility Multi-Strategy index has a beta of 0.80 and produces a volatility reduction of around 18% while the more defensive ERI Scientific Beta Low Volatility Efficient Minimum Volatility index has a beta of 0.74 and yields a volatility reduction of 23%. The annualised outperformance of the ERI Scientific Beta Low Volatility Multi-Strategy and Efficient Minimum Volatility indices is 2.55% and 2.65%, respectively. The combination of higher performance and lower volatility translates into significant Sharpe ratio gains for these two solutions (78% and 91%, respectively).

It is important to note that over short and medium-term horizons, the ERI Scientific Beta Low Volatility Multi-Strategy index has a higher chance of outperforming the broad market benchmark than the ERI Scientific Beta Low Volatility Efficient Minimum Volatility index. This is consistent with the stronger downside protection associated with the ERI Scientific Beta Low Volatility Efficient Minimum Volatility index and the better upside capture of the ERI Scientific Beta Low Volatility Multi-Strategy index. The latter's lower tracking error, which derives from diversifying across weighting schemes and achieving lower bull/bear conditionality, contributes to its higher information ratio relative to the ERI Scientific Beta Low Volatility Efficient Minimum Volatility index (0.43 vs 0.38). Extreme risk figures are consistent with these differences in conditionality: the ERI Scientific Beta Low Volatility Multi-Strategy index has higher drawdown but lower relative drawdown than the ERI Scientific Beta Low Volatility Efficient Minimum Volatility index.

Figure 4 summarises the performance of the two ERI Scientific Beta US Low Volatility indices and two third-party indices – the MSCI USA Minimum Volatility index and the S&P 500 Low Volatility index – over a period of 10 years using the S&P 500 index as benchmark. Within a universe of US large and mid cap stocks, the MSCI USA Minimum Volatility index uses a propri-

2 For more on the benefits of diversifying weighting schemes, please refer to Amenc et al (2015), Robustness of Smart Beta Strategies; and Martellini, Milhau and Tarelli (2014), Estimation Risk versus Optimality Risk: an Ex-Ante Efficiency Analysis of Heuristic and Scientific Equity Portfolio Diversification Strategies. These White Papers are available at http://docs.scientificbeta.com/Library/External/White_Papers/ERI_Scientific_Beta_Publication_Robustness_Smart_Beta_Strategies; and http://docs.scientificbeta.com/Library/External/Research_Publications/ERI_Scientific_Beta_Publication_Estimation_Risk_vs_Optimality_Risk.

3 The efficient minimum volatility weighting scheme uses the norm-constraint approach and targets an effective number of stocks equal to one third of the eligible stocks. Concentration induced by the other optimisation-based strategies is limited through weight adjustments to ensure that all securities are included, but that no security is given a dominant weighting.

etary Barra Optimiser to determine the portfolio with the lowest total risk subject to a series of ad-hoc constraints and using an estimated co-variance matrix. Constraints include (but are not limited to) minimum and maximum weights for each included security, minimum and maximum constituent sector weights relative to the parent index and turnover limits. The S&P 500 Low Volatility index selects the 100 least volatile stocks in the S&P 500 universe based on their past one-year price volatility and weights these by the inverse of their volatilities to produce a very defensive (and highly concentrated) index. No constraints are applied.

While being a factor-harvesting solution that acquires a defensive profile in a long-only implementation, the ERI Scientific Beta US Low Volatility Multi-Strategy index produces volatility reduction of about 17% (with respect to the S&P 500 index benchmark), on par with the MSCI USA Minimum Volatility index. Providing somewhat lower downside protection but significantly better upside capture, it produces higher returns and Sharpe ratio and an information ratio that is about 1.5 times that of the MSCI USA Minimum Volatility. Due (primarily) to its more pronounced defensive character relative to the Scientific Beta US Low Volatility Multi-Strategy index, the MSCI USA Minimum Volatility index experiences longer and more severe underperformance (in periods dominated by bullish markets). Using rolling window analysis to span the available return history, the probability of outperformance measures how often the strategy has managed to outperform its benchmark for a given holding period. Although the 2006–15 decade has been marked by two significant bear episodes, the Scientific Beta US Low Volatility Multi-Strategy index shows a significantly higher probability of outperformance for a one-year holding period than the comparable MSCI index (62.55% vs 57.66%) and vastly superior three- and five-year performances (91.8% vs 74.86% and 99.24% vs 79.01%, respectively).

3. Long-term track record of ERI Scientific Beta Low Volatility indices (US)

31 Dec 1970–31 Dec 2015 (45 years)	SciBeta Long-Term US Capitalisation-Weighted	SciBeta Long-Term US Low Volatility Diversified Multi-Strategy	SciBeta Long-Term US Low Volatility Efficient Minimum Volatility
Annualised returns	10.45%	13.00%	13.10%
Annualised volatility	16.88%	13.85%	13.04%
Sharpe ratio	0.32	0.57	0.61
Maximum drawdown	54.63%	48.31%	42.42%
Annualised relative returns	–	2.55%	2.65%
Annualised tracking error	–	5.99%	6.98%
95% tracking error	–	11.38%	13.77%
Information ratio	–	0.43	0.38
Outperformance probability (1Y)	–	63.47%	59.12%
Outperformance probability(3Y)	–	75.70%	74.15%
Outperformance probability (5Y)	–	86.88%	81.86%
Maximum relative drawdown	–	43.46%	46.94%
3-year rolling volatility mean	16.36%	13.38%	12.64%
3-year rolling volatility std dev	5.23%	4.38%	3.97%
3-year rolling volatility 95%	29.22%	24.57%	22.05%
Annualised relative returns bull	–	–0.91%	–2.12%
Annualised relative returns bear	–	7.43%	9.56%
Annualised relative returns extreme bull	–	–6.18%	–9.37%
Annualised relative returns extreme bear	–	7.24%	9.47%
CAPM market beta	**1.00**	**0.80**	**0.74**
1-way turnover	3.1%	26.2%	34.2%

The analysis is based on daily total return data in US dollars from 31 December 1970 to 31 December 2015 (45 years). Regressions are performed using weekly total returns in US dollars. The benchmark is the cap-weighted portfolio of all stocks in Scientific Beta US Long-Term Track Record universe consisting of the 500 largest US stocks. The risk-free rate is the return of the 3-month US Treasury Bill. The maximum relative drawdown is the maximum drawdown of the long-short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. The 95% tracking error is the 95th percentile of one-year rolling tracking error computed using a one-year rolling window and a one-week step size. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1, 3, and 5 years at any point during the history of the strategy; rolling windows of 1, 3, and 5 years and a one-week step size are used for the computation. Rolling volatility statistics are reported as mean, standard deviation and 95th percentile of annualised volatility and computed using a three-year rolling window and one-week step size. Quarters with positive benchmark index returns are classed as bull quarters and the remaining are classed as bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The returns on the market factor are the returns of the capitalisation-weighted benchmark over the risk-free rate. Reported turnover is 1-way, annual and it is averaged across 180 rebalancings in the 45-year period. Source: www.scientificbeta.com.

The ERI Scientific Beta Low Volatility Efficient Minimum Volatility index and the S&P 500 Low Volatility index are significantly more defensive than the previous two indices and provide volatility reductions of 24% and 26%, respectively. Conditional analysis shows that the ERI Scientific Beta Low Volatility Efficient Minimum Volatility index provides less ►

4. Risk and performance of ERI Scientific Beta Low Volatility indices and comparables (US, 10 years)

31 Dec 2005–31 Dec 2015 (10 years)	S&P 500	SciBeta Long-Term US Low Volatility Diversified Multi-Strategy	SciBeta Long-Term US Low Volatility Efficient Minimum Volatility	MSCI USA Minimum Volatility	S&P 500 Low Volatility
Annualised returns	7.28%	9.62%	10.54%	8.86%	9.35%
Annualised volatility	20.71%	17.25%	15.70%	17.14%	15.28%
Sharpe ratio	0.30	0.49	0.60	0.45	0.54
Maximum drawdown	55.25%	48.31%	42.42%	46.61%	40.40%
Annualised relative returns	–	2.34%	3.26%	1.58%	2.07%
Annualised tracking error	–	5.25%	7.00%	5.32%	8.54%
95% tracking error	–	9.67%	13.87%	8.39%	17.71%
Information ratio	–	0.45	0.47	0.30	0.24
Outperformance probability (1Y)	–	62.55%	65.74%	57.66%	50.21%
Outperformance probability (3Y)	–	91.80%	86.89%	74.86%	69.67%
Outperformance probability (5Y)	–	99.24%	95.04%	79.01%	73.28%
Maximum relative drawdown	–	8.79%	12.30%	12.83%	18.75%
3-year rolling volatility mean	21.74%	17.91%	16.33%	17.62%	15.76%
3-year rolling volatility standard deviation	7.04%	6.02%	5.13%	6.47%	4.72%
3-year rolling volatility 95%	30.61%	25.48%	22.83%	25.79%	21.72%
Annualised relative returns bull	–	–2.20%	–3.28%	–4.64%	–6.59%
Annualised relative returns bear	–	8.54%	12.49%	10.37%	14.78%
Annualised relative returns extreme bull	–	–8.03%	–11.69%	–9.48%	–14.89%
Annualised relative returns extreme bear	–	9.15%	13.91%	11.97%	16.13%
CAPM market beta	**1.00**	**0.82**	**0.74**	**0.80**	**0.68**
1-way turnover	na	28.2%	36.0%	na	na

The analysis is based on daily total return data in US dollars from 31 December 2005 to 31 December 2015 (10 years). Regressions are performed using weekly total returns in US dollars. The benchmark is the S&P 500 index. The Scientific Beta US universe consists of the 500 largest US stocks. The risk-free rate is the return of the 3-month US Treasury Bill. The naximum delative drawdown is the maximum drawdown of the long-short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. The 95% tracking error is the 95th percentile of one-year rolling tracking error and is computed using a one-week step size. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1, 3 or 5 years at any point during the history of the strategy; rolling windows of 1, 3, and 5 years and a one-week step size are used for the computation. Rolling volatility statistics are reported as mean, standard deviation and 95th percentile of annualised volatility and computed using a three-year rolling window and one-week step size. Quarters with positive benchmark index returns are classed as bull quarters and the remaining are classed as bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The returns on the market factor are the returns of the capitalisation-weighted benchmark over the risk-free rate. Reported turnover is one-way, annual and it is averaged across 40 rebalancings in the 10-year period. Source: www.scientificbeta.com and Bloomberg.

5. Risk and performance of ERI Scientific Beta Low Volatility indices and comparables (developed markets, 10 years)

31 Dec 2005–31 Dec 2015 (10 years)	MSCI World	SciBeta Long-Term Developed Low Volatility Diversified Multi-Strategy	SciBeta Long-Term Developed Low Volatility Efficient Minimum Volatility	MSCI World Minimum Volatility	S&P GIVI Developed
Annualised returns	5.54%	8.58%	9.35%	7.30%	6.52%
Annualised volatility	17.80%	13.98%	12.86%	12.81%	15.62%
Sharpe ratio	0.25	0.53	0.64	0.48	0.35
Maximum drawdown	57.46%	49.55%	45.02%	47.35%	53.11%
Annualised relative returns	–	3.04%	3.81%	1.76%	0.98%
Annualised tracking error	–	4.83%	6.19%	6.74%	3.30%
95% tracking error	–	9.24%	12.29%	10.95%	6.25%
Information ratio	–	0.63	0.62	0.26	0.30
Outperformance probability (1Y)	–	65.53%	68.09%	56.60%	56.81%
Outperformance probability (3Y)	–	96.99%	93.17%	74.59%	87.16%
Outperformance probability (5Y)	–	100.00%	97.33%	79.01%	94.27%
Maximum relative drawdown	–	9.76%	13.43%	17.42%	5.75%
3-year rolling volatility mean	18.92%	14.73%	13.53%	13.27%	16.51%
3-year rolling volatility standard deviation	5.32%	4.16%	3.68%	4.44%	4.76%
3-year rolling volatility 95%	25.44%	19.82%	18.07%	18.82%	22.33%
Annualised relative returns bull	–	–1.28%	–2.41%	–6.16%	–1.49%
Annualised relative returns bear	–	8.58%	12.08%	12.73%	4.10%
Annualised relative returns extreme bull	–	–7.74%	–11.54%	–13.99%	–4.10%
Annualised relative returns extreme bear	–	10.05%	14.09%	14.72%	4.38%
CAPM market beta	1.00	0.78	0.71	0.69	0.88
One-way turnover	na	29.5%	36.4%	na	na

The analysis is based on daily total return data in US dollars from 31 December 2005 to 31 December 2015 (10 years). Regressions are performed using weekly total returns in US dollars. The benchmark is the MSCI World Index. The Scientific Beta developed universe consists of 2,000 large and mid-cap stocks. The risk-free rate is the return of the 3-month US Treasury Bill. The maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. 95% tracking error is the 95th percentile of one-year rolling tracking error and is computed using a one-week step size. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1, 3 or 5 years at any point during the history of the strategy; rolling windows of 1, 3, and 5 years and a one-week step size are used for the computation. Rolling volatility statistics are reported as mean, standard deviation and 95th percentile of annualised volatility and computed using a three-year rolling window and one-week step size. Quarters with positive benchmark index returns are classed as bull quarters and the remaining are classed as bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The returns on the market factor are the returns of the capitalisation-weighted benchmark over the risk-free rate. Reported turnover is one-way, annual and it is averaged across 40 rebalancings in the 10-year period. Source: www.scientificbeta.com and Bloomberg.

◄ downside protection but much better upside capture than the S&P 500 Low Volatility index. Note that, relative to the MSCI USA Minimum Volatility index, the ERI Scientific Beta Low Volatility Efficient Minimum Volatility index not only provides more downside protection but also achieves better upside capture. The ERI Scientific Beta Low Volatility Efficient Minimum Volatility index exhibits the highest returns and risk-adjusted returns of the four defensive US indices compared here and its Sharpe ratio of 0.60 is twice that of the broad equity market over the period. Despite its (desired) high bull/bear conditionality, the ERI Scientific Beta Low Volatility Efficient Minimum Volatility index has an information ratio of 0.47, almost twice that of the S&P 500 Low Volatility index. Furthermore, it produces vastly superior outperformance probabilities over the short and medium terms than what its S&P Dow Jones counterpart (65.74% vs 50.21% for a one-year holding period, 86.89% vs 69.67% over three years and 95.04% vs 73.28% over five years) or for that matter than the MSCI USA Minimum Volatility index.

The relative profile of the two ERI Scientific Beta Low Volatility indices is consistent with what was observed with the long-term track records. The ERI Scientific Beta Low Volatility Efficient Minimum Volatility index is more defensive and shows more bull/bear conditionality. While its more defensive profile serves it particularly well in the decade, it nevertheless shows lower probabilities of outperformance than its multi-strategy counterpart over the medium term. Again, the extreme risk indicators are consistent with this positioning: the Scientific Beta Low Volatility Efficient Minimum Volatility index has significantly lower drawdown but higher relative drawdown than the multi-strategy index.

Figure 5 extends the comparison across defensive strategies to developed markets as a whole, where the MSCI World index serves as the broad capitalisation-weighted benchmark. We compare the MSCI World Minimum Volatility index and the S&P GIVI Developed index to the Scientific Beta Developed Low Volatility Multi-Strategy and Efficient Minimum Volatility indices. The MSCI World Minimum Volatility index is constructed using the same methodological principles as the MSCI USA Minimum Volatility index but subjected to additional country relative weight constraints. The S&P GIVI Developed index is based on the 25 developed countries in the S&P Global BMI. In each country, the stocks are sorted by their betas and the lowest beta stocks that represent 70% of that country's market capitalisation are selected. The selected stocks are weighted in proportion to their 'intrinsic value', a metric which depends on book value and discounted projected earnings. Note that ERI Scientific Beta builds indices from the basic geographic block and developed indices are assembled by combining basic-block indices on the basis of each block's free-float capitalisation. This is consistent with the application of factor investing at the level of homogeneous regions and ensures the block-level geographic neutrality of multi-block indices. By construction, ERI Scientific Beta's developed smart beta indices have the same exposures to the US, Japan and UK as their capitalisation-weighted reference and their geographic biases are limited to countries that are part of multi-country blocks (ie, Developed Euro-zone, Developed Europe ex UK ex Euro-zone and Developed Asia-Pacific ex Japan).

Both Scientific Beta indices produce higher volatility reductions when applied to developed markets as a whole, but their relative profile and 10-year performance remain consistent with what was observed for the US. The ERI Scientific Beta Low Volatility Efficient Minimum Volatility index delivers volatility reduction of close to 28% while its multi-strategy counterpart reduces volatility by over 21%. Both Low Volatility Smart Factor indices deliver high excess returns in the 2006–15 decade, with the more defensive index also producing significantly better returns for developed markets as a whole (3.81% pa vs 3.04%). Both indices produce remarkable Sharpe ratio gains relative to the benchmark and have exceptional information ratios for defensive strategies. Sharpe ratios are 0.53 and 0.64 for the Developed Low Volatility Multi-Strategy and Efficient Minimum Volatility indices, respectively, to be compared to the market's 0.25, and the information ratios are 0.63 and 0.62, respectively. As observed in the US, the medium-term outperformance probabilities are higher for the ERI Scientific Beta Low Volatility Multi-Strategy index. The absolute and relative drawdown patterns are also fully consistent with the US track records.

While the MSCI World Minimum Volatility index produces volatility reduction and downside protection that are similar to those of the ERI Scientific Beta Developed Low Volatility Efficient Minimum Volatility index, it has relatively low upside capture. As a result, it underperforms its comparable ERI Scientific Beta index by over 2% pa over the period, delivers 75% of the latter's Sharpe ratio of 0.64 and 42% of its information ratio of 0.62. It also significantly underperforms the less defensive ERI Scientific Beta Developed Low Volatility Multi-Strategy index in terms of returns and risk-adjusted returns.

Owing to its mild volatility filtering, the S&P 500 GIVI Developed index is the least defensive of the four indices. However, its upside capture is lacklustre and, over the period, it posts meagre outperformance of 0.98% pa and a relatively modest Sharpe ratio gain. Its lower bull/bear conditionality is associated with tracking error that is about half that of the MSCI index, which it dominates in terms of information ratio. The said ratio is nevertheless less than half that of the ERI Scientific Beta indices. As observed with the US indices, the ERI Scientific Beta Low Volatility indices produce higher probabilities of outperformance over the short- and medium-terms.

Explicit targeting of low volatility stocks and superior diversification of specific risk

explain the superior performance of the ERI Scientific Beta Low Volatility index relative to peers. The Low Volatility Multi-Strategy indices are factor-harvesting solutions that acquire a defensive profile in a long-only implementation and trail the capitalisation-weighted benchmark reasonably in all but the strongest bull markets. The Low Volatility Efficient Minimum Volatility indices are designed to produce a strongly defensive character and as such generate sterling downside protection at a cost of reduced upside capture. This more defensive character causes the index to trail more significantly in bull markets, which leads to lower probabilities of outperformance over the medium term than what the Low Volatility Multi-Strategy indices deliver (Amenc et al [2016]).

The choice between the Scientific Beta Low Volatility Efficient Minimum Volatility index and the Scientific Beta Low Volatility Diversified Multi-Strategy index will therefore depend on investors' level of risk aversion and more globally on their risk allocation objective. It is clear that if the objective is to introduce the low risk factor into the risk factor allocation menu, then the Scientific Beta Low Volatility Diversified Multi-Strategy index is the ideal candidate. On the other hand, if the objective is to obtain the lowest absolute risk for the equity allocation, then the Scientific Beta Low Volatility Efficient Minimum Volatility index will be favoured.

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Defensive strategies (III): towards dynamic defensive strategies

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Even though it is possible to design defensive strategies that are well diversified on the specific risk level, and to produce smart factor low volatility indices that have good defensive capabilities and excellent long-term risk-adjusted performance, it must be recognised that these strategies remain concentrated in the low volatility factor by construction. Being concentrated in the low risk factor, traditional defensive strategies miss out on the rewards associated with other factor tilts. In addition, their design implies a constantly low exposure to market risk, which provides relative downside protection in bear markets but causes them to trail the broad-market capitalisation-weighted index in bull markets. In this article, we introduce the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solutions, which rely on a risk-based allocation model engineered by EDHEC-Risk Institute to dynamically allocate to the whole range of ERI Scientific Beta Smart Factor indices carrying long-term risk premia with a view to delivering a dissymmetric defensive profile.

Relying on the persistence of volatility and the negative relationship between volatility and returns, these benchmarks target a constant reduction in relative volatility that allows the defensive character of the strategy to adjust to market conditions. This is meant to achieve a high absolute reduction in volatility when market volatility is high and a low absolute reduction when market volatility is low. Because of the well documented negative relationship between volatility and returns, high volatility regimes tend to correspond to bear markets and low volatility regimes to bull markets. Hence, the solution seamlessly adjusts the risk budget to changing market conditions so as to provide significant downside protection with improved upside capture relative to traditional defensive strategies.

De facto, these solutions are derivations of the flagship Scientific Beta Multi-Beta Multi-Strategy indices, which equalise the allocation to the Scientific Beta Smart Factor Indices that correspond to the choices of factor tilts[1] (and diversification strategies).[2] In the case of smart beta solutions, the goal is no longer a fixed equally-weighted mix between the smart factor indices that make up these Multi-Beta Multi-Strategy indices, but dynamic risk allocation that is intended to respect the investor's absolute or relative risk objectives. The derivation applying to the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution takes the Scientific Beta Six-Factor Multi-Beta Multi-Strategy indices as a starting point. It therefore involves dynamic allocation drawing on the 30 indices (six factor tilts times five weighting schemes) that make up these indices.

1 Four factor tilts – mid cap, value, positive momentum and low volatility – for the Scientific Beta MBMS Four-Factor (EW) indices; two factor tilts – low investment and high profitability – for the Scientific MBMS Quality Indices; and all six factor tilts for the Scientific Beta Six-Factor MBMS (EW) indices.

2 All off-the-shelf multi-strategy indices equally weight the five diversification strategies available from ERI Scientific Beta: maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio.

The negative correlation between volatility and return

There is a long tradition in finance that models stock return volatility as negatively correlated with stock returns. This is consistent with an empirical relationship first observed for US markets (see for example the studies by Schwert [1989], covering the period from 1834–1987 and Campbell and Hentschel [1992], covering the period from 1926–88) and for which there is also international empirical evidence, in particular for developed markets (see for example Li et al [2005]; Talpsepp and Rieger [2010]; and Dimitriou and Simos [2011]). Historically, there have been two main theoretical justifications for the asymmetric nature of volatility documented by empirical studies: leverage and volatility feedback. The leverage hypothesis, which can be traced to Black (1976) and Christie (1982), notes that, as asset prices decline, companies become more leveraged as the value of their ►

◄ debt rises relative to that of their equity. With increasing leverage, stocks become riskier, so it is to be expected that they be more volatile. The feedback effect is predicated on the persistence of volatility and the existence of a positive inter-temporal relation between expected return and conditional variance. Increased volatility raises requires expected returns, which translates into reduced current stock prices (see Pindyck [1984]; French, Schwert and Stambaugh [1987] and Campbell and Hentschel [1992]); this dampens volatility in the case of good news but exacerbates it in the case of bad news.

As Aït Sahalia, Fan and Li (2013) summarise: "The leverage explanation suggests that a negative return should make the firm more levered, hence riskier and therefore lead to higher volatility; the volatility feedback effect is consistent with the same correlation but reverses the causality: increases in volatility lead to future negative returns." More recently, arbitrage restrictions and behavioural explanations have been put forward to explain the asymmetry (for a review; see Talpsepp and Rieger [2010]). In addition, Whitelaw (2000) shows how the low risk effect and the volatility-return asymmetry arise in a general equilibrium model with two-regimes as a result of hedging demand. While the jury is still out on the relative contributions of these different explanations (although there is a consensus to recognise that the leverage explanation can only contribute marginally), the phenomenon of negative correlations between volatility and returns itself is not in doubt.

Implementing the relative risk approach

The Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution dynamically allocates to ERI Scientific Beta Smart Factor indices on the basis of their evolving risk characteristics in changing markets in an effort to deliver the targeted constant reduction in volatility relative to the market. The solution dynamically allocates to 30 (single-tilt single-weighting-scheme) smart factor indices to create a multi-factor solution that exhibits dissymmetric defensive characteristics. The six rewarded factors that are subject to consensus in the academic literature are mid cap, value, positive momentum, low volatility, low investment and high profitability.[3] The weighting schemes are maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. Technically, the solution relies on a maximum deconcentration allocation which maximises diversification measured by the effective number of constituents (defined as the inverse of the sum of squared constituent weights: ENC(w)=1/(w^T*w)) smart factor indices subject to a constraint of a minimum of 10%[4] ex-ante reduction in volatility relative to the reference index. The allocation problem can be written mathematically as:

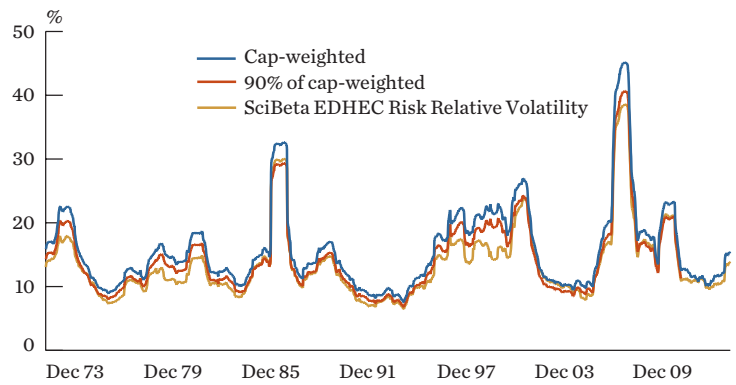
$$w^* = argmax\{1/(w^T * w)\} \quad \begin{cases} \sum_{i=1}^{N} w_i = 1 \\ w_i \geq 0 \; \forall \, i \\ \sqrt{W^T * \sum * W} \leq 90\% * Volatility_{CW} \end{cases}$$

w_i represents the weight of the i-th constituent index. N is the number of constituent indices. Σ is the covariance matrix of total returns. Weekly total returns over the previous 104 weeks are used to directly estimate this small (ie, 30×30) variance-covariance matrix and the volatility of the capitalisation-weighted benchmark. Rebalancing across smart factor indices is implemented quarterly and is not subject to turnover control. This risk-based allocation model exploits the full correlation structure of smart factor indices across the six factor tilts and five weighting schemes simultaneously. Relying on the persistent nature of volatility, the adjustment is done strictly on the basis of (ex-ante) realised volatility and as such requires no volatility forecasting.

Properties and performance of the relative risk approach

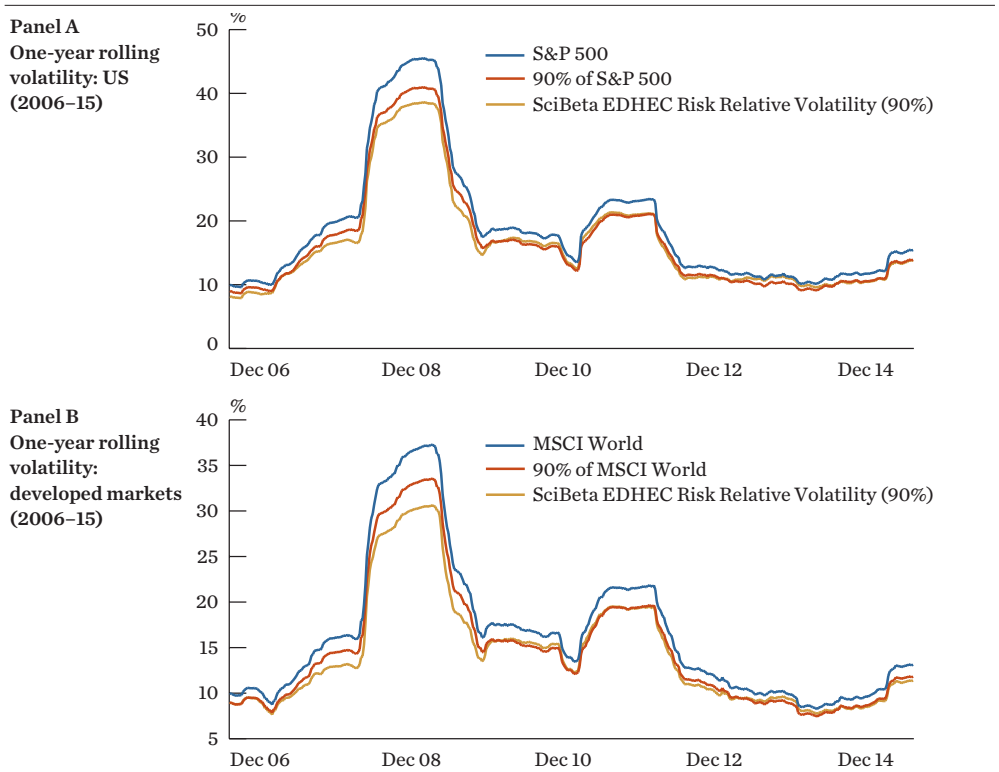
Figures 1 and 2 show plots comparing the ex-post or out-of-sample volatilities of the broad capitalisation-weighted benchmark and the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution. A one-year rolling window and one-week step size is used to plot

1. Realised one-year rolling volatility of the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution (US long-term track record)



The analysis is based on daily total return data in US dollars from 31 December 1972 to 31 December 2015 (43 years). The rolling volatility is computed using a rolling window of length one year and a one-week step size. The benchmark is the cap-weighted portfolio of all stocks in the Scientific Beta US LTTR universe, which consists of the 500 largest US stocks. Scientific Beta US Long Term Smart Factor indices have a 45-year track record. As the calibration of solutions requires two years, all US Long Term Smart Beta solutions have a 43-year track record. Source: www.scientificbeta.com.

2. Realised one-year rolling volatility of the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution (US long-term track record)



Panel A: Scientific Beta US indices – The benchmark is the S&P 500 Index. The Scientific Beta US universe consists of the 500 largest US stocks. Panel B: Scientific Beta Developed Indices – The benchmark is the MSCI World Index. The Scientific Beta Developed universe consists of 2,000 large and mid-cap stocks. The analysis is based on daily total return data in US dollars from 31 December 2005 to 31 December 2015 (10 years). The rolling volatility is computed using a rolling window of length one year and a one-week step size. Source: www.scientificbeta.com.

3 For more details, please refer to Amenc et al (2015), Scientific Beta Multi-Strategy Factor Indices: Combining Factor Tilts and Improved Diversification, available at http://docs.scientificbeta.com/Library/External/White_Papers/ERI_Scientific_Beta_Publication_Scientific_Beta_Multi-Strategy_Factor_Indices; and Goltz (2015), Long-Term Rewarded Equity Factors: What Can Investors Learn from Academic Research? P&I EDHEC-Risk Institute Research for Institutional Money Management supplement, available at www.edhec-risk.com/about_us/documents/attachments/PI_EDHEC-Risk_Supplement_August_2015.pdf.

4 This solution, calibrated with a 90% relative volatility budget, is a good compromise over the long term between limiting downside risk and capturing the upside. Naturally, the organisation of the allocation allows this parameter to be changed and it is technically possible to manage a relative constraint budget of 85% or 95%. We consider that this choice can be made in two ways:

i) either in a fixed manner: for example the investor would like a more defensive strategy and would choose 85%. This choice has a less optimal long-term trade-off, but it can be taken into account as part of the customisation of our solution's risk budget.

ii) or in a variable manner, ie, the investor has views on the future level of volatility or market returns and adapts the risk budget to his/her views. The investor will therefore choose to reduce the constraint (eg, from 90% to 95%) when s/he estimates that s/he is entering into a bull market regime and conversely, will increase the constraint, (eg, from 90% to 85%), when s/he estimates that there is a strong likelihood that the markets will fall in the medium term.

Of course, by relying on such forecasts, the investor adds alpha to the strategy; the sign and size of this alpha will depend on forecasting skills.

time-varying volatility. The plots illustrate that the ex-post delivered volatility of the solution is typically very close to the maximum targeted volatility. However, the volatility reduction has been significantly above the minimum targeted level in several periods, typically during bear markets, eg, 1973–74 (oil crisis), 2000–02 (dot-com crash), 2008–09 (global financial crisis). It has also been so during the bull run of the late nineties when diversified indices tilted towards long-term rewarded factors could not match the volatility of a cap-weighted index increasingly concentrated into high volatility technology stocks. Undershooting episodes, which should be expected when volatility suddenly rises, have been benign and extremely rare.

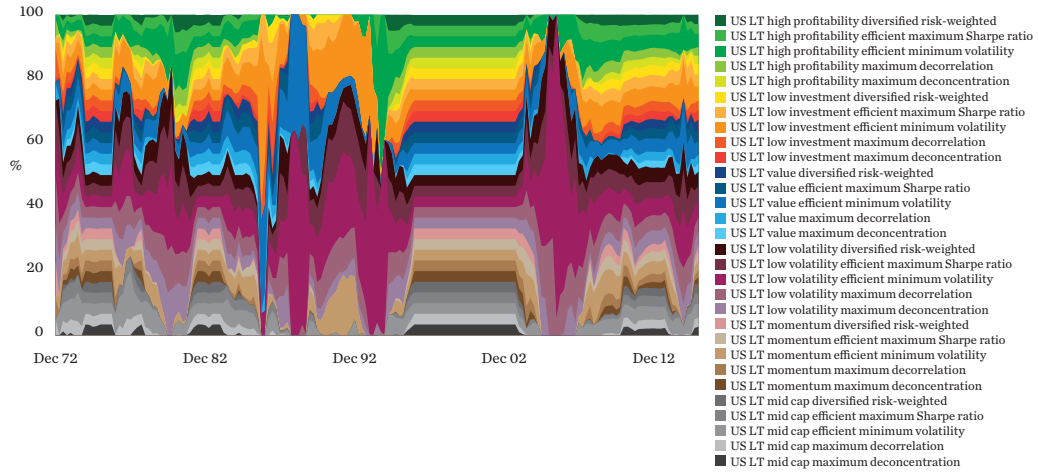
Figures 3 and 4 illustrate the dynamic multi-factor nature of the solution. When market volatility is high, delivering a 10% reduction in volatility is a strong constraint that leads the risk-based allocation to concentrate into the most defensive smart factor indices in the available pool; these defensive indices will outperform in bear markets and thus incur well-rewarded tracking error relative to the broad-market benchmark. When market volatility is low, the volatility reduction constraint is less strong and allows allocation to be broadly diversified across the smart factor indices in the available pool. Note that when the constraint is not binding, the allocation model achieves equal weighting across all available indices. In these market conditions, the solution will rely more on indices exhibiting good bull-market performance and will not be primarily invested in defensive indices that incur high detrimental tracking error because of their inferior bull-market performance.

For the above reasons, the tracking error of the solution will also be dissymmetric. The highly defensive indices in which the solution concentrates in high-volatility regimes will outperform in bear markets and thus incur well-rewarded tracking error relative to the broad-market benchmark. The mildly defensive character of the solution in low volatility environments will reduce the traditional performance drag and associated high tracking error of unconditional defensive approaches in bull markets and the broad mix of indices to which the solution allocates in these environments will harvest multiple sources of factor returns that will further improve relative performance. In other words, the solution should exhibit high bear-market tracking error corresponding to outperformance being created by the marked defensive character of the allocation and more benign bull-market tracking error reflecting a muted defensive character that allows for harvesting multiple risk premia and at least avoiding severe underperformance.

Naturally, the smart factor index diversification objective and relative risk budget approach of the strategy's allocation mechanism also allow investors to avoid the factor concentration issue of traditional defensive strategies. The strategy indeed draws on six systematic sources of long-term over-performance relative to the broad equity markets by tilting towards mid cap, value, positive momentum, low volatility, low investment and high profitability stocks. In allocating to indices representative of these tilts, the strategy does not attempt to time factor cycles but instead seeks to maximise the diversity of constituent indices, subject to the volatility budget.

As shown in figure 5, over the very long-term (43-year record), the Scientific Beta US Long-Term Multi-Beta Multi-Strategy Relative Volatility (90%) solution respects its constraint of a minimum volatility reduction of 10% – it ►

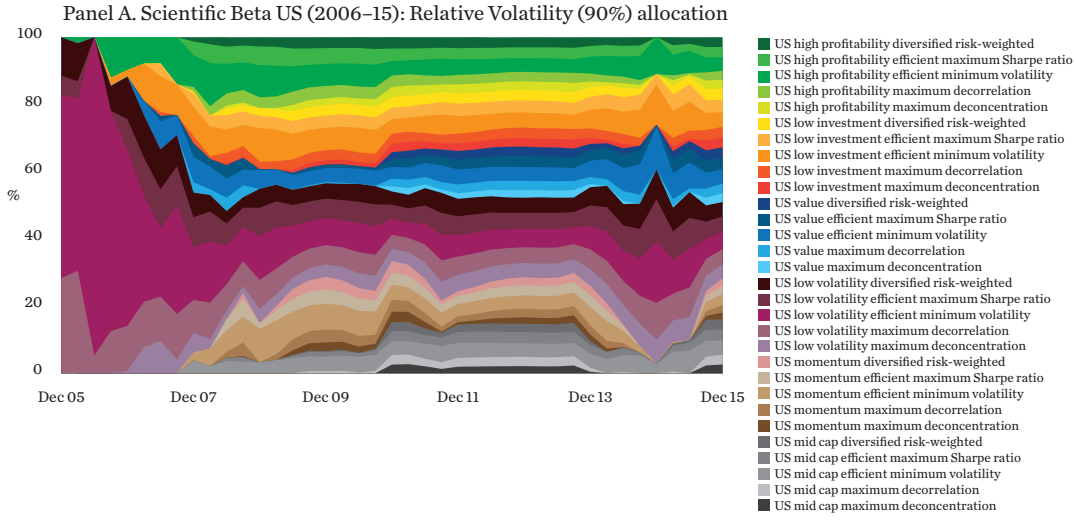
3. Smart factor index allocation of the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution (US long-term track record)



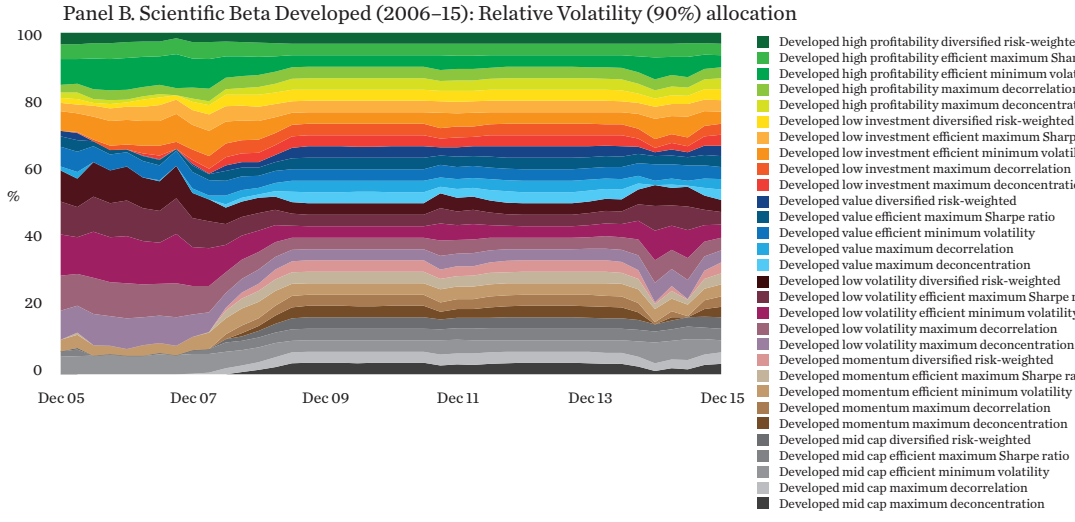
The chart shows the evolution of the allocation across the 30 (six factors x five weightings) smart factor indices for the Scientific Beta US Long-Term Track Record (LTTR) period. The factor tilts are mid cap, value, positive momentum, low volatility, low investment and high profitability. The weighting schemes are maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. The allocation is rebalanced quarterly over the period of 43 years (31 December 1972 to 31 December 2015). The benchmark is the capitalisation-weighted portfolio of all stocks in the Scientific Beta US LTTR universe, which consists of the 500 largest US stocks. Scientific Beta US Long Term Smart Factor indices have a 45-year track record. As the calibration of solutions requires two years, all US Long Term Smart Beta solutions have a 43-year track record. Source: www.scientificbeta.com.

4. Smart factor index allocation of the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution

Panel A. Scientific Beta US (2006–15): Relative Volatility (90%) allocation



Panel B. Scientific Beta Developed (2006–15): Relative Volatility (90%) allocation



Panel A: Scientific Beta US indices – The benchmark is the capitalisation-weighted portfolio of all stocks in the Scientific Beta US universe which consists of the 500 largest US stocks. Panel B: Scientific Beta Developed indices – The benchmark is the cap-weighted portfolio of all stocks in the Scientific Beta Developed universe, which consists of 2,000 large and mid-cap stocks. The chart shows the evolution of the allocation across the 30 (six factors x five weightings] smart factor indices over the 10-year period ended 31 December 2015. The factor tilts are mid cap, value, positive momentum, low volatility, low investment and high profitability. The weighting schemes are maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. The allocation is rebalanced quarterly over the period of 10 years (31 December 2005 to 31 December 2015). Source: www.scientificbeta.com.

◄ achieves a robust 15% reduction – and produces higher returns than the Low Volatility Smart Factor indices. Its Sharpe ratio is twice that of the benchmark, on par with that of the two low volatility indices. The solution's high outperformance of 3.51% pa and lower tracking error – a result of its dissymmetric defensive profile and the exploitation of decorrelation opportunities across both the factor and weighting scheme dimensions – combine to produce an information ratio of 0.68, which is exceptionally high for a defensive strategy.

The most striking observation, which is also the most important value added of this solution, is its ability to combine significant downside protection and excellent upside capture. Due to its dynamic character, the strategy gears downs its defensive exposure in low volatility environments and outperforms by 1.84% in bull markets over the period when even well-diversified defensive strategies underperform. The benefits of dynamic allocation are even more evident in extreme bull markets: the strategy trails by a mere 69bps, when the Low Volatility Multi-Strategy index is 639bps behind the benchmark and the Low Volatility Efficient Minimum Volatility index underperforms by 962bps. Relative to these indices, the strategy shows higher probabilities of outperformance in the short- and medium-terms and a lower maximum relative drawdown.

In summary, the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution comes across as an excellent compromise between performance and downside protection. It provides about the same reduction in volatility as the Scientific Beta Low Volatility Diversified Multi-Strategy index, but a much higher information ratio and a much better upside capture. This translates into greater robustness of outperformance across time and states as illustrated by the solution's superior probabilities of outperformance.

Figure 6 shows that the performance of the Scientific Beta US Multi-Beta Multi-Strategy Relative Volatility (90%) solution over the

5. Comparison of solution concept to Scientific Beta Long-Term Low Volatility indices

31 Dec 1972–31 Dec 2015 (43 years)	SciBeta US Broad CW	SciBeta Multi-Beta Multi-Strategy Relative Volatility (90%)	SciBeta Low Volatility Diversified Multi-Strategy	SciBeta Low Volatility Efficient Minimum Volatility
Annualised returns	10.16%	13.67%	12.94%	13.04%
Annualised volatility	17.15%	14.64%	14.08%	13.26%
Sharpe ratio	0.29	0.58	0.56	0.60
Maximum drawdown	54.63%	48.70%	48.31%	42.42%
Annualised relative returns	–	3.51%	2.77%	2.88%
Annualised tracking error	–	5.13%	6.08%	7.09%
95% tracking error	–	8.77%	11.43%	13.96%
Information ratio	–	0.68	0.46	0.41
Outperformance probability (1Y)	–	71.26%	66.01%	61.91%
Outperformance probability (3Y)	–	81.32%	76.53%	75.14%
Outperformance probability (5Y)	–	88.10%	86.14%	81.05%
Maximum relative drawdown	–	33.18%	43.46%	46.94%
3-year rolling volatility mean	16.40%	13.99%	13.43%	12.70%
3-year rolling volatility standard deviation	5.33%	4.61%	4.47%	4.04%
3-year rolling volatility 95%	29.29%	25.06%	24.63%	22.11%
Annualised relative returns bull	–	1.84%	–0.77%	–2.02%
Annualised relative returns bear	–	5.53%	7.52%	9.64%
Annualised relative returns extreme bull	–	–0.69%	–6.39%	–9.62%
Annualised relative returns extreme bear	–	5.14%	7.35%	9.74%
CAPM market beta	**1.00**	**0.86**	**0.80**	**0.74**
One-way turnover	3.1%	39.4%	26.5%	34.6%

The analysis is based on daily total return data in US dollars from 31 December 1972 to 31 December 2015 (43 years). Regressions are performed using weekly total returns in US dollars. The Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) allocation is performed using 30 (six factors x five weightings) smart factor indices implemented in the Scientific Beta US Long-Term Track Record (LTTR) universe. The factor tilts are mid cap, value, positive momentum, low volatility, low investment and high profitability. The weighting schemes are maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. The allocation is rebalanced quarterly over the period of 43 years (31 December 1972 to 31 December 2015). Scientific Beta US Long Term Smart Factor indices have a 45-year track record. As the calibration of solutions requires two years, all US Long Term Smart Beta solutions have a 43-year track record. The benchmark is the capitalisation-weighted portfolio of all stocks in the Scientific Beta US LTTR universe, which consists of the 500 largest US stocks. The risk-free rate is the return of the 3-month US Treasury Bill. The maximum relative drawdown is the maximum drawdown of the long/short index, the return of which is given by the fractional change in the ratio of the strategy index to the benchmark index. The 95% tracking error is the 95th percentile of one-year rolling tracking error and is computed using a one-week step size. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1, 3 or 5 years at any point during the history of the strategy using a rolling window and a one-week step size. Rolling volatility statistics are reported as mean, standard deviation and 95th percentile of annualised volatility computed using a three-year rolling window and a one-week step size. Quarters with positive benchmark index returns are classed as bull quarters and the remaining are classed as bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The returns on the market factor are the returns of the capitalisation-weighted benchmark over the risk-free rate. Reported turnover is one-way, annual and it is averaged across 172 rebalancings in the 43-year period. Source: www.scientificbeta.com.

6. Comparison of benchmark to Scientific Beta Low Volatility indices and traditional defensive strategies for US (2006–15)

31 Dec 2005–31 Dec 2015 (10 years)	S&P 500	SciBeta Multi-Beta Multi-Strategy Relative Volatility (90%)	SciBeta Low Volatility Diversified Multi-Strategy	SciBeta Low Volatility Efficient Minimum Volatility	MSCI USA Minimum volatility	S&P 500 Low Volatility
Annualised returns	7.28%	9.34%	9.62%	10.54%	8.86%	9.35%
Annualised volatility	20.71%	18.00%	17.25%	15.70%	17.14%	15.28%
Sharpe ratio	0.30	0.46	0.49	0.60	0.45	0.54
Maximum drawdown	55.25%	48.70%	48.31%	42.42%	46.61%	40.40%
Annualised relative returns	–	2.06%	2.34%	3.26%	1.58%	2.07%
Annualised tracking error	–	4.37%	5.25%	7.00%	5.32%	8.54%
95% tracking error	–	8.99%	9.67%	13.87%	8.39%	17.71%
Information ratio	–	0.47	0.45	0.47	0.30	0.24
Outperformance probability (1Y)	–	71.91%	62.55%	65.74%	57.66%	50.21%
Outperformance probability (3Y)	–	97.27%	91.80%	86.89%	74.86%	69.67%
Outperformance probability (5Y)	–	100.00%	99.24%	95.04%	79.01%	73.28%
Maximum relative drawdown	–	8.27%	8.79%	12.30%	12.83%	18.75%
3-year rolling volatility mean	21.74%	19.05%	17.91%	16.33%	17.62%	15.76%
3-year rolling volatility standard deviation	7.04%	5.62%	6.02%	5.13%	6.47%	4.72%
3-year rolling volatility 95%	30.61%	26.08%	25.48%	22.83%	25.79%	21.72%
Annualised relative returns bull	–	–0.87%	–2.20%	–3.28%	–4.64%	–6.59%
Annualised relative returns bear	–	5.92%	8.54%	12.49%	10.37%	14.78%
Annualised relative returns extreme bull	–	–5.41%	–8.03%	–11.69%	–9.48%	–14.89
Annualised relative returns extreme bear	–	6.09%	9.15%	13.91%	11.97%	16.13%
CAPM market beta	**1.00**	**0.87**	**0.82**	**0.74**	**0.80**	**0.68**
One-way turnover	na	41.5%	28.2%	36.0%	na	na

The analysis is based on daily total return data in US dollars from 31 December 2005 to 31 December 2015 (10 years). Regressions are performed using weekly total returns in US dollars. The Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) allocation is performed using 30 (six factors x five weightings) smart factor indices in the Scientific Beta US universe. The factor tilts are mid cap, value, positive momentum, low volatility, low investment and high profitability. The weighting schemes are maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. The allocation is rebalanced quarterly over the period of 10 years (31 December 2005 to 31 December 2015). The benchmark for performing the allocation is the Scientific Beta US capitalisation-weighted index. The benchmark for analytics reporting is the S&P 500 index. The Scientific Beta US universe consists of the 500 largest US stocks. The risk-free rate is the return of the 3-month U.S. Treasury Bill. The maximum relative drawdown is the maximum drawdown of the long/short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. The 95% tracking error is the 95th percentile of one-year rolling tracking error and is computed using one-week step size. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1, 3 or 5 years at any point during the history of the strategy using a rolling window and a one-week step size. Rolling Volatility statistics are reported as mean, standard deviation and 95th percentile of annualised volatility computed using a three-year rolling window and a one-week step size. Quarters with positive benchmark index returns are classed as bull quarters and the remaining are classed as bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The returns on the market factor are the returns of the capitalisation-weighted benchmark over the risk-free rate. Reported turnover is one-way annual and it is averaged across 40 rebalancings in the 10-year period. Source: www.scientificbeta.com and Bloomberg.

7. Comparison of benchmark to Scientific Beta Low Volatility indices and traditional defensive strategies for developed markets (2006–15)

31 Dec 2005–31 Dec 2015 (10 years)	MSCI World	SciBeta Multi-Beta Multi-Strategy Relative Volatility (90%)	SciBeta Low Volatility Diversified Multi-Strategy	SciBeta Low Volatility Efficient Minimum Volatility	MSCI World Minimum volatility	S&P GIVI Developed
Annualised returns	5.54%	8.23%	8.58%	9.35%	7.30%	6.52%
Annualised volatility	17.80%	15.25%	13.98%	12.86%	12.81%	15.62%
Sharpe ratio	0.25	0.47	0.53	0.64	0.48	0.35
Maximum drawdown	57.46%	50.86%	49.55%	45.02%	47.35%	53.11%
Annualised relative returns	–	2.69%	3.04%	3.81%	1.76%	0.98%
Annualised tracking error	–	3.49%	4.83%	6.19%	6.74%	3.30%
95% tracking error	–	7.52%	9.24%	12.29%	10.95%	6.25%
Information ratio	–	0.77	0.63	0.62	0.26	0.30
Outperformance probability (1Y)	–	81.06%	65.53%	68.09%	56.60%	56.81%
Outperformance probability (3Y)	–	99.18%	96.99%	93.17%	74.59%	87.16%
Outperformance probability (5Y)	–	100.00%	100.00%	97.33%	79.01%	94.27%
Maximum relative drawdown	–	7.72%	9.76%	13.43%	17.42%	5.75%
3-year rolling volatility mean	18.92%	16.29%	14.73%	13.53%	13.27%	16.51%
3-year rolling volatility standard deviation	5.32%	4.12%	4.16%	3.68%	4.44%	4.76%
3-year rolling volatility 95%	25.44%	21.23%	19.82%	18.07%	18.82%	22.33%
Annualised relative returns bull	–	0.49%	–1.28%	–2.41%	–6.16%	–1.49%
Annualised relative returns bear	–	5.32%	8.58%	12.08%	12.73%	4.10%
Annualised relative returns extreme bull	–	–2.44%	–7.74%	–11.54%	–13.99%	–4.10
Annualised relative returns extreme bear	–	6.03%	10.05%	14.09%	14.72%	4.38%
CAPM market beta	1.00	0.86	0.78	0.71	0.69	0.88
One-way turnover	na	35.0%	29.5%	36.4%	na	na

The analysis is based on daily total return data in US dollars from 31 December 2005 to 31 December 2015 (10 years). Regressions are performed using weekly total returns in US dollars. The Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) allocation is performed using 30 (six factors x five weightings) smart factor indices in the Scientific Beta US universe. The factor tilts are mid cap, value, positive momentum, low volatility, low investment and high profitability. The weighting schemes are maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility, and efficient maximum Sharpe ratio. The allocation is rebalanced quarterly over the period of 10 years (31 December 2005 to 31 December 2015). The benchmark for performing the allocation is the Scientific Beta Developed capitalisation-weighted index. The benchmark for analytics reporting is the MSCI World Index. The Scientific Beta Developed universe consists of 2,000 large and mid-cap stocks. The risk-free rate is the return of the 3-month US Treasury Bill. The Maximum Relative Drawdown is the maximum drawdown of the long-short index whose return is given by the fractional change in the ratio of the strategy index to the benchmark index. The 95% tracking error is the 95th percentile of one-year rolling tracking error and is computed using one-week step size. The probability of outperformance is the probability of obtaining positive excess returns from investing in the strategy for a period of 1, 3 or 5 years at any point during the history of the strategy using a rolling window and a one-week step size. Rolling volatility statistics are reported as mean, standard deviation and 95th percentile of annualised volatility computed using a three-year rolling window and a one-week step size. Quarters with positive benchmark index returns are classed as bull quarters and the remaining are classed as bear quarters. The top 25% quarters with the best benchmark index returns are extreme bull quarters and the bottom 25% quarters with the worst benchmark index returns are extreme bear quarters. The returns on the market factor are the returns of the capitalisation-weighted benchmark over the risk-free rate. Reported turnover is one-way, annual and it is averaged across 40 rebalancings in the 10-year period. Source: www.scientificbeta.com and Bloomberg.

last 10 years is consistent with the long-term track record. The reduction in volatility of this solution over the 10-year period is 13%, which respects the constraint, and it displays the same ability to provide both significant downside protection and excellent upside capture. Relative to the ERI Scientific Beta Low Volatility indices and third-party defensive strategies, the strategy shows lower tracking error and boasts the best information ratio; it has higher probabilities of outperformance in the short and medium terms and a lower relative drawdown. In a decade that has been particularly beneficial to highly defensive strategies, the benchmark delivers the same excess performance, about 2% pa, as the most defensive product on the market with twice its information ratio and vastly higher probabilities of outperformance over the short and medium term.

Figure 7 shows performance comparisons over the same period for developed markets as a whole. The reduction in volatility of the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution over the 10-year period is 14% and the benchmark displays the same ability to provide both significant downside protection and excellent upside capture. Relative to the ERI Scientific Beta Low Volatility indices and more defensive third-party strategies, the strategy shows lower tracking error and a lower relative drawdown. While it is dominated by the ERI Scientific Beta Low Volatility indices in terms of returns and Sharpe ratio, due to a market environment that favoured defensive strategies, it delivers a much higher information ratio and has much higher probabilities of outperformance in the short and medium term.

De facto, the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solution represents a solution that is very different from the defensive strategies proposed by the Scientific Beta Low Volatility Diversified Multi-Strategy or Scientific Beta Low Volatility Efficient Minimum Volatility indices. It no longer involves offering a defensive strategy that performs as well as possible in bear market situations (Scientific Beta Low Volatility Efficient Minimum Volatility), because it has a very low beta, but is well diversified; nor does it involve focusing on the capture of the risk premium associated with the low risk factor as efficiently as possible (Scientific Beta Low Volatility Multi-Strategy) by making sure to obtain excellent risk-adjusted performance through good diversification (ie, reduction) of weighting scheme idiosyncrasies.

The solution does not involve being concentrated in the low volatility factor, but allocating to the six rewarded factors that are subject to consensus in the academic literature. It is the dynamic nature of the multi-index allocation that will allow the strategy to be most defensive when most useful, ie, when the markets are highly volatile. The investor will ultimately avail of a dissymmetric pay-off that has the advantages of a defensive strategy as it protects against downside risk and outperforms the capitalisation-weighted benchmark strongly in bear and extreme bear markets, but does not present the disadvantage of traditional defensive strategies which underperform in bull markets. Better upside capture is produced by good factor diversification that is made possible by a lower absolute volatility reduction constraint during low volatility periods that generally correspond to periods of market rises.

Conclusion

Implemented in a long-only and unleveraged framework, the ERI Scientific Beta Smart Factor Indices that tilt towards low volatility stocks acquire a defensive character, which can be maximised by selection of the efficient minimum volatility weighting scheme. Relative to equally defensive solutions put forward by other providers, the superior diversification of the ERI Scientific Beta Low Volatility indices produces significantly higher returns, adjusted returns and probabilities of outperformance. However, these defensive solutions are concentrated on the low risk factor and thus miss out on the rewards associated with other factor tilts. Also, their unconditional defensive character – ie, a constantly low beta causes them to significantly trail the benchmark in strong bull markets.

To address these issues, the Scientific Beta Multi-Beta Multi-Strategy Relative Volatility (90%) solutions rely on risk-based allocation models engineered by EDHEC-Risk Institute to select and dynamically allocate to the whole gamut of ERI Scientific Beta Smart Factor Indices carrying long-term risk premia to deliver a dissymmetric defensive profile. Relying on the persistence of volatility and the negative relationship between volatility and returns, these benchmarks target a constant reduction in relative volatility vis-à-vis the broad market capitalisation-weighted index that allows the defensive character of the strategy to adjust to market conditions and combine downside risk protection with improved upside capture. Compared to unconditional defensive strategies, they exhibit excellent upside capture, exceptional information ratios and much higher probabilities of outperformance in the short and medium-terms. As such, these benchmarks constitute in our view core solutions for buy-and-hold investors who seek a defensive allocation but wish to reduce the risk of short and/or medium term underperformance relative to a peer group or the broad-market capitalisation-weighted index. ►

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Analysing the live performance of Scientific Beta multi-strategy indices

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This article discusses the live performance of stock market indices that aim to outperform cap-weighted indices by obtaining exposure to multiple rewarded factors and by using diversification-based weighting schemes; so-called smart factor indices. Given the potential data-mining biases that can arise when relying on backtested performance, a key issue when analysing such multi-factor indices is to look at their live performance, which does not benefit from hindsight in the way that backtests potentially can. In the end, the key question for investors is not backtested performance, but the live performances they will ultimately experience when adopting such indices.

In particular, we discuss multi-beta multi-strategy indices that allocate across several smart factor indices. One such strategy is the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index, whose live performance is the longest among the live performances of the Scientific Beta offerings and is therefore the central topic of the following pages.

This flagship index is an equal-weighted combination of four underlying single-factor indices, for the mid cap, momentum, low volatility and value factors. Equal weighting the underlying single factor indices is a straightforward way of performing a multi-factor allocation. This approach thus uses individual smart factor indices as the principal ingredients. Hence, their good design is a necessary condition for the flagship multi-factor index to work seamlessly.

The single factor indices serve as efficient and well-diversified building blocks for multi-factor allocations due to their parsimonious methodology. Academic consensus and concern for robustness underlie the design of all Scientific Beta indices. First and foremost, the single factor indices are based on the idea of diversification. Diversification is the only 'free lunch' available in investment management and investors ignore it at their peril.

ERI Scientific Beta, with its Smart Beta 2.0 approach, enables investors to obtain the right rewarded risk factor exposures in an efficient and well-diversified way. The main idea is to apply a smart weighting scheme to an explicit selection of stocks in order to construct factor indices that are not only exposed to the desired risk factors, but also avoid being exposed to unrewarded risks. This approach, referred to as 'smart factor indices' can be summarised as follows: the explicit selection of stocks provides the desired tilt (ie, the beta), while the smart weighting scheme addresses concentration issues and diversifies away specific and unrewarded risks.

The smart weighting used in the four underlying indices behind the Scientific Beta Multi-Beta Multi-Strategy EW index is called diversified multi-strategy. This weighting scheme combines five popular weighting schemes[1] and thus diversifies the specific risks of the individual weighting approaches.

There is strong intuition suggesting that multi-factor allocations will tend to result in improved risk-adjusted performance. In fact, even though the factors towards which the factor indices are tilted are all rewarded over the long term, there is extensive evidence that they may each encounter prolonged periods of underperformance. One can expect pronounced allocation benefits across factors which have imperfect correlation with one another. This is the reason why multi-factor allocation can add value for investors.

The Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index is designed to add value in this way. Launched in December 2013, its live performance is testimony to the robust methodology employed by Scientific Beta in its indices. The following sections present this multi-factor index in more detail, mostly focusing on the performance since launch.

A first look at live performance

Prior to discussing the performance of the flagship multi-factor index, we take a look at the live results of the single factor indices that constitute the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index.

The Scientific Beta multi-strategy indices for the mid cap, momentum, low volatility and value factors were launched in December 2012 and during their live period have outperformed the cap-weighted index while reducing volatility. For example, the average annualised outperformance of the four indices in the developed region, shown in figure 1, was 2.48%. The average reduction in volatility compared to the reference index amounted to 78 basis points in absolute terms, which is 6.78% in relative terms. The indices also delivered low tracking error, which led to a rather high average information ratio of 0.9. It should be noted that, despite a relatively tough time for value strategies over this period, the Scientific Beta Developed Value Multi-Strategy index managed to outperform the cap-weighted reference index, as shown in figure 1. In the end, it is the good diversification of this index which has produced this good performance, even though the cap-weighted

1 The efficient maximum Sharpe ratio, efficient minimum volatility, maximum deconcentration, diversified risk weighted and maximum decorrelation weighting schemes. More details available at www.scientificbeta.com.

value index drawing on the same stock selection has actually underperformed the broad cap-weighted index over this period.

The benefits of good design and the resulting good performance of the constituent indices carry over to the flagship multi-factor index. The Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index is an example of simple and robust allocation to smart factors. This index draws on the good live performance of the underlying indices and we can thus expect benefits from this multi-beta allocation, in line with the averages from the exhibit above.

In figure 2 we show the live performance of the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index starting with its official launch in December 2013. In each geographical universe we compare the regional version of this flagship index with its corresponding Scientific Beta Cap-Weighted index. An important take-away from the figure is the consistency of the key results across different geographical regions and the alignment of the results with the observed behaviour of the single-factor indices. The multi-factor index outperformed the cap-weighted index in all the regions with an average annualised outperformance of 3.98%. The volatility reduction effects observed among the single-factor indices are present in the multi-factor index as well. Across the different geographies, the four-factor multi-beta index lowered the volatility of the reference cap-weighted index by an average of 1.68%. This represents a 10.30% relative drop in volatility.

Consistency of live performance over time

As we have seen, the live performance of Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index has shown consistent results across the regions. It is only natural to examine the consistency of the returns over time. In the investment management industry, strategies are often assessed on the basis of short-term performance, such as calendar years. The Scientific Beta indices are built around the idea of long-term rewarded factors that might not necessarily work all the time. The multi-factor allocation tries to alleviate this problem and smooth factor cyclicality. Nevertheless, it is still an alternative strategy that will deviate from the cap-weighted reference in any given time period.

The live period of the flagship multi-factor index examined above is more than two and a half years. Examining the entire live period may hide how the strategy performed on a calendar-year basis. Therefore, in figure 3, we show the yearly live excess returns of the index along with the June 2016 year-to-date results. We can observe remarkable stability in the outperformance across regions and time-periods. Indeed, there is only one region/period combination with negative excess return.

Consistency with long-term track records

The live results presented in this article are a result of constructing factor indices based on the idea of long-term rewards associated with factor exposure. Therefore, any factor or multi-factor index should be ultimately linked to the documented historical performance.

In figure 4 we show that the short-term live results presented thus far are consistent with the results from the US long-term track records that extend 45 years back in time. Namely, the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index from the US long-term history outperformed the cap-weighted benchmark with an annualised excess return of 3.35%, which compares to the 3.98% seen in the live period across regions. Similarly, the average relative reduction in volatility of 10.49% observed over the long history compares to the regional average of 10.30% from the live period.

1. Performance and risks of single factor indices: live period, developed region

21 Dec 2012–30 June 2016	Scientific Beta Multi-Strategy indices				
	Mid cap	Momentum	Low volatility	Value	Average of four single-factor indices
Change in volatility wrt CW benchmark	–0.61%	–0.58%	–1.86%	–0.09%	–0.78%
Relative change in volatility wrt CW benchmark	–5.25%	–4.98%	–16.11%	–0.79%	–6.78%
Annualised excess returns	2.31%	2.96%	3.83%	0.83%	2.48%
Annualised tracking error	2.58%	2.71%	3.17%	1.99%	2.61%
Information ratio	0.90	1.09	1.21	0.42	0.90

Statistics are annualised and daily total returns from 21 December 2012 to 30 June 2016 are used for the analysis. The Scientific Beta Developed CW index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Source: www.scientificbeta.com.

2. Performance and risks of multi-factor indices: live period, different geographical regions

20 Dec 2013–30 June 2016	Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index						
	Annualised returns	Annualised volatility	Change in volatility wrt CW benchmark	Relative change in volatility wrt CW benchmark	Annualised excess returns	Annualised tracking error	Information ratio
US	9.95%	13.19%	–0.82%	–5.87%	2.14%	2.53%	0.85
Euro-zone	7.25%	17.63%	–2.72%	–13.38%	4.08%	4.77%	0.85
UK	5.77%	15.32%	–0.54%	–3.43%	2.62%	4.90%	0.54
Developed Europe ex-UK	0.62%	16.43%	–1.51%	–8.40%	3.79%	3.96%	0.96
Japan	8.87%	19.90%	–2.42%	–10.85%	7.71%	4.90%	1.57
Developed Asia-Pacific ex-Japan	1.48%	11.78%	–3.56%	–23.20%	4.21%	5.76%	0.73
Developed ex-UK	7.17%	10.86%	–1.06%	–8.87%	3.38%	2.15%	1.57
Developed ex-US	2.32%	12.54%	–1.42%	–10.19%	4.56%	2.86%	1.60
Developed	6.44%	11.13%	–1.03%	–8.50%	3.31%	2.13%	1.55
Average across regions	5.54%	14.31%	–1.68%	–10.30%	3.98%	3.77%	1.14

Statistics are annualised and daily total returns from 20 December 2013 to 30 June 2016 are used for the analysis. The Scientific Beta CW index in the corresponding regions is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate with the exception of the Euro-zone, UK and Japan where Euribor (3M), UK T-bill (3M) and Japan Gensaki T-bill (1M) are used respectively. Source: www.scientificbeta.com.

3. Yearly performance of multi-factor indices: live period, different geographical regions

	Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index		
	2014	2015	2016 YTD
US	2.39%	0.49%	2.83%
Euro-zone	3.15%	5.89%	1.73%
UK	6.03%	7.23%	–6.68%
Developed Europe ex-UK	2.21%	4.63%	2.92%
Japan	6.31%	6.82%	5.81%
Developed Asia-Pacific ex-Japan	2.44%	5.07%	2.45%
Developed ex-UK	2.93%	2.32%	3.35%
Developed ex-US	3.71%	5.35%	2.43%
Developed	3.08%	2.68%	2.65%
Average across regions	3.58%	4.49%	1.94%

Yearly non-annualised excess returns for the live period of the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index are shown. 2016 YTD Performance refers to the period between 31 December 2015 and 30 June 2016. The Scientific Beta CW index in the corresponding regions is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate with the exception of the Euro-zone, UK and Japan where Euribor (3M), UK T-bill (3M) and Japan Gensaki T-bill (1M) are used respectively. Source: www.scientificbeta.com.

4. Performance and risks of Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index: US long-term track records

31 Dec 1970–31 Dec 2015	Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index
Change in volatility wrt CW benchmark	–1.77%
Relative change in volatility wrt CW benchmark	–10.49%
Annualised excess returns	3.35%
Annualised tracking error	5.03%
Information ratio	0.67

Statistics are annualised and daily total returns from 31 December 1970 to 31 December 2015 are used for the analysis. The Scientific Beta USA LTTR CW index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Source: www.scientificbeta.com.

Benefits of multi-beta multi-strategy allocation

There is more to multi-factor allocation than meets the eye. We have seen that there is ►

a rationale behind combining the single-factor indices, since the average of the four performance metrics naturally avoids extreme values. This intuition materialises in the actual multi-factor EW index, whose live performance preserves the all-important positive excess returns and volatility reduction.

There is an additional benefit to combining the single factor indices into one multi-factor strategy – reduction in turnover. This reduction in turnover arises from the netting of offsetting trades required to rebalance the dollar weights of the same stocks in different constituent indices. Some of these trades are related to a stock entering or exiting the selection from which a smart factor index is built and it can naturally happen that a stock exits one index at the exact time that it enters another index.

Figure 5 documents a difference of nearly 6% between the average turnover of the four underlying single-factor indices and the turnover of the multi-factor index in the developed region during the live period of the strategy (December 2013 to June 2016). In addition to the reduced turnover, the multi-factor index improved the information ratio from 1.13 to 1.55 by reducing the average tracking error. This is accompanied by a reduction in the extreme (95th) percentile of rolling one-year tracking error.

Another important aspect of the multi-beta multi-strategy allocation is avoiding the risks of selecting individual factors or weighting schemes. The multi-factor indices represent passive investment products that do not try to predict the winning strategy or factor, bearing in mind that the rewarded factors yield positive premia in the long term in exchange for risks that can lead to considerable underperformance or relative drawdowns in shorter periods.

In essence, multi-beta multi-strategy indices represent an agnostic view on the capacity to select the winner beforehand. Diversifying across the factors also avoids factor timing and thus avoids both performance chasing and contrarian investing. Indeed, it is important to realise how crucial diversification is. Without the benefits of a crystal ball and after taking into account the large body of evidence that seems to suggest the lack of forecasting skills among the majority of investment professionals, avoiding the risks of being stuck with the wrong strategy seems like a good idea ex-ante.

To illustrate this point, let us recall that the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index is a combination of four single-factor indices, all of which employ a diversification weighting scheme based on five different approaches. We can examine the difference in performance among the 20 (4x5) alternative strategies that combine one of the four factor tilts and a unique weighting scheme. We present the annualised excess returns of these strategies in figure 6.

During the more than two and a half years-long live period of the flagship multi-factor index, the variation in outperformance among the 20 examined strategies is substantial. The worst-performing strategy turned out to be a value-tilted index using the maximum deconcentration weighting scheme, with an annualised excess return of –0.73%, while the best-performing strategy was a combination of low volatility tilt and efficient minimum volatility weighting scheme, with annualised outperformance of 7.58%.

This wide spread simply means that defensive strategies worked remarkably well in this time period, while value stocks underperformed. Naturally, investors that had invested in low volatility strategies can now be happier than value investors seeking stock-level diversification through the value maximum deconcentration strategy. This is however an ex-post analysis of a given realisation of stock returns. Other time periods would have produced different results.

With the help of the above example, it can be further shown how the performance of individual strategies can be 'rationalised'. For instance, a provider of a single strategy similar to value maximum deconcentration would likely seek to sell the idea that the strategy with the lowest recent returns is likely to make a comeback. On the other hand, a provider of a single strategy similar to low volatility efficient minimum volatility would likely emphasise his good performance over the recent patch of rough markets. The multi-beta multi-strategy index approach is immune to such strategy picking questions and just delivers the 'average' result across factors and weighting schemes.

To further illustrate the variability among the many individual strategies that an investor can choose from, consider figure 7, which shows the yearly excess returns[2] of the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index in the developed region along with the yearly excess returns of the worst (losing) and best (winning) performing single factor index among the 20 candidate strategies introduced earlier.

The exhibit shows that every year, the winner and loser would be a different single factor strategy. The Multi-Beta Multi-Strategy index takes no view of the winning strategy or factor and thus can avoid a good share of the uncertainty linked to individual strategies.

The effect of adding quality factors

Apart from the mid cap, momentum, low volatility and value factors and the associated single factor indices, Scientific Beta introduced the high profitability and low investment factors into its array of single factor indices in March 2015. The two factors represent the quality aspects of the stocks. These six factors have all been combined into a new multi-factor index,

2 Results prior to the live date come from an historical backtrack.

5. Performance and turnover of Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index: live period, developed region

20 Dec 2013–30 June 2016	Average of four single-factor indices	Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index
Annualised excess returns	3.29%	3.31%
Annualised tracking error	2.74%	2.13%
Information ratio	1.13	1.55
95th percentile of rolling one-year tracking error	3.18%	2.60%
Average annualised one-way turnover	43.32%	37.49%

Statistics are annualised and daily total returns from 20 December 2013 to 30 June 2016 are used for the analysis. The Scientific Beta Developed CW index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The average of four single-factor indices refers to the average of the metrics for the Scientific Beta Diversified Multi-Strategy indices for the mid cap, momentum, low volatility and value tilts. The extreme percentile of tracking error is calculated using weekly time steps and one-year rolling window. Source: www.scientificbeta.com.

6. Excess returns of single-factor indices – Scientific Beta Multi-Beta Multi-Strategy EW index: live period, different geographical regions

20 Dec 2013–30 June 2016	Factor tilt			
Weighting scheme	Mid cap	Momentum	Low volatility	Value
Maximum deconcentration	1.63%	1.51%	5.35%	–0.73%
Maximum decorrelation	1.91%	2.81%	5.71%	0.76%
Efficient minimum volatility	5.67%	5.39%	7.58%	3.46%
Efficient maximum Sharpe ratio	3.15%	3.69%	5.68%	1.77%
Diversified risk weighted	2.66%	2.14%	5.50%	0.29%

Statistics are annualised and daily total returns from 20 December 2013 to 30 June 2016 are used for the analysis. The Scientific Beta Developed CW index is used as the cap-weighted benchmark. Source: www.scientificbeta.com.

7. Yearly performance of single factor indices (excess returns): developed region

	Scientific Beta index name and excess returns				
	Winner		Loser		Multi-Beta Multi-Strategy Four Factor EW index
2016 YTD	Low volatility efficient min volatility	6.57%	Momentum max deconcentration	–0.08%	2.65%
2015	Momentum efficient max Sharpe	5.78%	Value max deconcentration	–2.31%	2.68%
2014	Low volatility efficient min volatility	7.23%	Momentum max deconcentration	–0.47%	3.08%
2013	Momentum max deconcentration	4.46%	Low volatility efficient min volatility	–4.12%	0.05%
2012	Value max deconcentration	1.16%	Momentum efficient min volatility	–2.00%	0.11%
2011	Low volatility efficient min volatility	10.75%	Value max deconcentration	–4.17%	2.91%
2010	Mid cap max deconcentration	10.65%	Value efficient min volatility	2.02%	5.89%
2009	Value max deconcentration	4.91%	Momentum efficient min volatility	–8.82%	–1.96%
2008	Low volatility efficient min volatility	12.19%	Momentum max deconcentration	–2.79%	2.97%
2007	Momentum max deconcentration	0.58%	Low volatility max deconcentration	–5.85%	–3.00%

The table shows the single factor indices with the highest (winner) and lowest (loser) yearly excess return among the 20 (4x5) single factor indices for the mid-cap, momentum, low volatility and value tilts combined with five weighting schemes – maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio, diversified risk weighted. The table also contains the excess return of the Scientific Beta Multi-Beta Multi-Strategy EW index. 2016 YTD Performance refers to the period between 31 December 2015 and 30 June2016. The Scientific Beta Developed CW index is used as the cap-weighted benchmark. Source: www.scientificbeta.com.

the Scientific Beta Multi-Beta Multi-Strategy Six-Factor EW index, which was launched in September 2015.

Figure 8 shows the performance comparison between the Scientific Beta Multi-Beta Multi-Strategy Four-Factor and Six-Factor EW indices, along with analysis of conditional performance depending on the market conditions. The bull (bear) markets correspond to analysis of performance of the indices in quarters when the cap-weighted benchmark returns are positive (negative). Similarly, the extreme bull (bear) markets correspond to the quarters with the top (bottom) 25% benchmark returns. This allows the performance to be assessed in different market cycles.

Comparing the results of the four and six-factor indices, we learn that the addition of the new quality factors yielded similar levels of excess returns and slightly lower tracking error. This is also true in the case of the 95th percentile of the rolling one-year tracking error.

The results in figure 8 also show that the six-factor EW index not only exhibits lower tracking error compared to the four-factor EW index, while experiencing a similar level of volatility, but the tracking error is also lower across different market regimes, suggesting smoother performance compared to the four-factor EW index. This is an important insight since investors not only care about the performance over the entire period, but are also concerned about the relative performance in bull and bear markets.

The performance of the six-factor EW index can be traced back to the performance of the underlying six single-factor indices. Figure 9 shows the yearly performances of six single-factor multi-strategy indices that serve as building blocks for the Scientific Beta Multi-Beta Multi-Strategy Six-Factor EW index, over the same period as discussed above. It contains the excess returns of these indices over the cap-weighted benchmark in the developed region.

The single-factor indices based on high profitability and low investment represent two additional sources of performance that help smooth the returns of the multi-factor index. For example, as shown in figure 9, high profitability experienced comparably high excess returns in recent years while low investment also added to the good overall results of the multi-factor index with its positive yearly excess returns.

Conclusion

In this article we have focused on the analysis of the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index and presented its performance since its launch in December 2013.

We have shown that the index displayed stable outperformance over the cap-weighted benchmark during the live period, as well as stable volatility reduction. These live results are consistent with the long-term back history of the index over 45 years. The average annual excess return across regions in the live period of 3.98% compares to the 3.35% documented in the long US history. Similarly, the observed relative volatility reduction in the live period across regions of 10.30% compares to 10.49% in the long-term track records.

The multi-factor allocation and weighting scheme diversification that the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index is built upon helps to avoid the risks associated with pursuing a single strategy and leads to smoother outperformance across time relative to an approach that bets on an individual strategy.

Importantly, the Multi-Strategy-Multi-Beta approach also avoids timing bets. The financial industry is clinging to its old ways and is currently discovering smart beta strategies as grounds for tempting investors to make tactical bets. This involves either enticing investors to chase the performance of the winning smart beta strategies or advocating contrarian bets in the form of valuation-based factor timing. This is an unfortunate development if one considers that smart beta was meant to achieve progress relative to an old world where fund managers and 'strategists' claimed that they could make the right call on which part of the market would head up or down in the short term, without any actual evidence that they possess the necessary skill or deliver any true value to investors. Smart beta at the outset has the potential to provide access to evidence-based long-term investment strategies and it would be a pity if market commentators and 'strategists' distract from the value of holding such strategies over the long term by turning smart beta strategies into yet another arena for their forecasting contests.

In contrast to such recent developments, the multi-beta multi-strategy indices do not try to predict the winning strategy or factor. They represent an agnostic view on the capacity to select the winning strategy. In fact, this approach recognises that there is value in combining different strategies that work on average over the long term. Using an index that combines 20 individual strategies (five different weighting schemes applied to four different factor tilts) means that investors will get the average outperformance across the 20 underlying strategies. In the absence of perfect foresight on the future winning strategy, such a broad exposure to a wide variety of strategies is a natural starting point for smart beta investing. Broad exposure to a wide set of strategies may allow investors to sustain their exposure in the long term and help them to consider more serious questions about their investments than asking which smart beta strategy will be the next winner. ►

8. Performance and risks of the Scientific Beta Multi-Beta Multi-Strategy EW index: live period, developed region

20 Dec 2013–30 June 2016	Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index	Scientific Beta Multi-Beta Multi-Strategy Six-Factor EW index
Annualised returns	6.44%	6.51%
Annualised volatility	11.13%	11.18%
Change in volatility wrt CW benchmark	–1.03%	–0.98%
Relative change in volatility wrt CW benchmark	–8.50%	–8.07%
Sharpe ratio	0.57	0.57
Annualised excess returns	3.31%	3.38%
Annualised tracking error	2.13%	1.98%
Information ratio	1.55	1.70
95th percentile of rolling 1Y tracking error	2.60%	2.44%
Bull markets		
Annualised excess returns	2.66%	2.68%
Annualised tracking error	2.04%	1.90%
Information ratio	1.30	1.41
Bear markets		
Annualised excess returns	4.62%	4.77%
Annualised tracking error	2.34%	2.19%
Information ratio	1.98	2.18
Extreme bull markets		
Annualised excess returns	0.27%	0.49%
Annualised tracking error	2.72%	2.51%
Information ratio	0.13	0.26
Extreme bear markets		
Annualised excess returns	2.89%	2.91%
Annualised tracking error	2.01%	1.84%
Information ratio	1.44	1.59

Statistics are annualised and daily total returns from 20 December 2013 to 30 June 2016 are used for the analysis. The Scientific Beta Developed CW index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Bull (bear) market performance is calculated based on the quarterly classification of the cap-weighted reference index – calendar quarters with positive benchmark returns comprise bull markets and the rest constitute bear markets. Similarly, extreme bull (bear) markets correspond to quarters with the top (bottom) 25% benchmark returns. The extreme percentile of tracking error is calculated using weekly time steps and one-year rolling window. The performance of the Scientific Beta Multi-Beta Multi-Strategy Six-Factor EW index comes partially from the historical backtrack (prior to 18 September 2015). Source: www.scientificbeta.com.

9. Yearly performance of single-factor multi-strategy indices – Scientific Beta Multi-Beta Multi-Strategy EW index: live period, developed region

Excess returns (yearly)	Scientific Beta Multi-Strategy indices		
	2014	2015	2016 YTD
Mid cap	3.17%	2.61%	1.91%
Momentum	1.23%	4.97%	1.58%
Low volatility	5.81%	4.26%	5.11%
Value	2.11%	–1.10%	1.91%
High profitability	4.18%	4.37%	2.31%
Low investment	2.59%	1.79%	2.43%

This table shows the yearly excess returns of six Scientific Beta single-factor multi-strategy indices that constitute the Multi-Beta Multi-Strategy Six-Factor EW index. Yearly non-annualised excess returns for the live period of the Scientific Beta Multi-Beta Multi-Strategy Four-Factor EW index are shown. 2016 YTD performance refers to the period between 31 December 2015 and 30 June 2016. The Scientific Beta CW index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. Source: www.scientificbeta.com.

Methodological differences across multi-factor index offerings

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Multi-factor indices are one of the newest additions to the family of rules-based equity strategies, and have attracted substantial inflows over the last couple of years. Factor investing advocates focusing on exposure to the underlying drivers of security returns identified in academic research, such as value, momentum and other factors. With the realisation that the returns from active management are to a very large extent attributable to exposure to well-documented systematic factors, factor indices are increasingly regarded as a cost-efficient, straightforward and transparent way of implementing desired factor tilts.

In particular, while discretionary active managers may generate performance from tilting their portfolios towards such factors, they may destroy the performance benefits of these factor tilts if they charge the high fees that are typical of active management and/or make wrong idiosyncratic calls. For example, Fama and French (2010) find that active managers on average deliver –1% return per year after adjusting for their market, value, size and momentum exposures. If investors could access index-based strategies that simply delivered the returns of their factor-tilted benchmark at low fees, this would be an interesting alternative to investing with discretionary managers, who the evidence shows underperform their factor-tilted benchmark.

Multi-factor indices are a natural extension of indices based on single factors. While any single-factor index typically targets improving risk-adjusted returns over cap-weighted reference indices, there is strong intuition suggesting that multi-factor allocations will tend to result in improved risk-adjusted performance relative to single factor indices. Intuitively, since different factors work at different times, allocating across multiple rewarded factors will increase the probability of over-performance in the short- and medium-terms. Moreover, investors who allocate across factors using single-factor indices subsumed into a multi-factor solution or held within the same mandate will enjoy implementation benefits. Indeed, some of the rebalancing trades necessary to maintain exposure to different factors may actually cancel each other out. Consider the classic example of an investor who pursues an allocation across the value and momentum tilts. If a stock included in the value strategy experiences a rapid price increase then, other things being equal, its weight in the value-tilted portfolio will tend to decrease at the next rebalancing, while its weight in the momentum-tilted portfolio will tend to increase. If both strategies are rebalanced at the same time and held within the same multi-factor index or mandate, then rebalancing trades can be crossed and turnover reduced.

While sharing the same objective, indices aiming to provide multiple factor exposures may opt for very different implementation methods, which reflect differences in underlying beliefs on multi-factor investing. This article reviews the current offerings in the world of multi-factor indices and looks at the conceptual considerations involved in designing the different approaches. The key issues that we discuss involve the robustness and consistency of the multi-factor indices as well as the (lack of) diversification among the various products.

We first provide a brief overview of several multi-factor indices published by five different index providers. Afterwards, we discuss the design choices some of the indices in this group have made, as well as the conceptual underpinnings of these choices. In particular, we look at the difference between proprietary and consensual definitions of factors and the issue related to the use of composite factor scores. Subsequently we turn our attention to the importance of consistency in index design, before focusing on the important issue of diversification, which is too frequently ignored. We show the difference between top-down and bottom-up approaches to constructing multi-factor indices and discuss the issues relating to both.

Overview of multi-factor indices

In this section, we provide a brief overview of the multi-factor indices that we are analysing. We look at the current multi-factor offerings from Scientific Beta, FTSE Russell, MSCI, Goldman Sachs and S&P.

In figure 1, we provide a short summary of the different index methodologies to provide

1. Overview of multi-beta indices

Provider	Index	Targeted factors	Short description of methodology
Scientific Beta	Multi-Beta Multi-Strategy EW	Value, size, momentum volatility (four-factor version)	Equal-weighted combination of single-factor indices
	Multi-Beta Multi-Strategy ERC	Value, size, momentum volatility, profitability, investment (six-factor version)	The weight of single-factor indices is determined so that the allocation equates the contribution of individual smart-factor portfolios to relative risk as represented by tracking error relative to the capitalisation-weighted reference
FTSE Russell	FTSE Global Diversified Factor	Value, volatility, momentum, size	Universe splits into 40 regional industry universes (four regions x 10 industries). Regional industry universes weighted in proportion to their inverse volatility. Factor ranking for individual security is a weighted average composite of individual factor rankings with weights proportional to inverse volatility of the factors
	FTSE Russell Comprehensive	Quality, value, volatility, momentum, size	Z-score for individual factors translated into s-score using cumulative standard normal distribution. Composite factor score is a product ofthe five individual s-scores. Final weights in the index are proportionate to the product of the composite factor score and the market cap
MSCI	MSCI Quality Mix MSCI Diversified Multiple-Factor	Value, volatility, quality Value, momentum, size quality	Equal-weighted combination of single-factor indices Optimisation focused on maximising an equal-weighted combination of factor exposures to the four rewarded factors. Optimisation controls exposure to non-rewarded factors and country and sector exposures
Goldman Sachs	Goldman Sachs Equity Factor	Value, momentum, size, quality, low beta	Optimisation-based index that aims to maximise aggregate basket score. Basket score is weighted average of individual factor scores weighted in proportion to the inverse volatility of the factors
S&P	S&P GIVI	Value, low beta	Low beta stock selection (70%) and weighting by intrinsic value calculated from a discounted cash flow valuation model based on residual income

the reader with a high-level overview of the situation. There are pronounced differences in methodology across the indices. For example, among other differences, some of the multi-factor indices that we look at use single-factor indices as building blocks while others apply a multi-factor methodology directly at the stock level. The choice of targeted factors also changes from index to index.

It should be noted that additional differences exist among these indices that are not captured by the schematic overview above. These differences are notable when it comes to factor definitions. We will discuss the issues relating to factor definitions in the next section. Apart from this, indices naturally differ in terms of other implementation details including universe definition, implementation rules etc.

In the next sections, we will review some conceptual issues related to the construction of multi-factor indices and provide examples of how the different indices included in our analysis approach these issues.

Factor definitions: proprietary variables versus consensual variables, and problems with composite scores

Providers across the board put strong emphasis on the academic grounding of their factor indices[1]. At the same time, product providers try to differentiate their products using proprietary elements in their strategy, often leading to the creation of products using new factors or novel strategy construction approaches which may or may not be consistent with the broad consensus in the academic literature on empirical asset pricing. As for factor definitions, many factor indices show considerable divergence from academic definitions.

A key result from analysing the literature is that well-established factor premia are not simply based on 'backtests' similar to those used by product providers, but instead have been subjected to extensive empirical analyses, including assessments over very long-term data and post-publication data as well as cross-sample validation, notably when factors uncovered in the cross-section of US stock returns have been confirmed in international data or in other asset classes. Moreover, some common factor premia have been explained using formal economic models providing a rationale for the persistence of such premia. Recalling these results is useful in clarifying the fact that novel strategies or 'enhancements' with respect to such factors should be subjected to similar levels of scrutiny before conclusions on their relative merits are drawn.

For example, the Fama and French (2012, 2014) factor definitions, which are widely used in academic research, are based on straightforward stock selection criteria such as price-to-book for value for example, even though professionals often like to make the method more complex to pick the 'best' value stocks. Extensive empirical evidence is available for the academic factor definitions, but not for the ad-hoc approaches, which are often justified by fairly limited in-sample performances.

1 For example, consider the following quotes from marketing material of index providers: "MSCI currently identifies six equity risk premia factors... They are grounded in academic research.."; "In developing the Russell High Efficiency Factor Index series ... we ensured that all of our factor specifications were consistent with academic research findings," "The FTSE Global Factor Index Series is... designed to represent ... factor characteristics for which there is a broad academic consensus"; ERI Scientific Beta: "factor indices are meant to be investable proxies for rewarded factors that have been analysed in the academic literature".

2. Mismatch with academic factor definitions – examples

Source	Value	Momentum	Profitability
Academic reference	Price-to-book (Fama and French, 1993, 2012, 2015; Carhart, 1997)	Past 12 months returns omitting last month (Carhart, 1997; Fama and French, 2012)	Return on equity* (Fama and French, 2015; Hou, Xue, Zhang, 2015) or gross profitability (Novy-Marx, 2013)
Scientific Beta Multi-Beta Multi-Strategy indices	Follows Fama and French and Carhart	Follows Fama and French and Carhart	Follows Novy-Marx
Goldman Sachs Equity Factor Index World	Value score from proprietary risk model (Axioma), relative to stock's regional industry group	Residuals from cross-sectional regression of 12-month return (omitting last month) on stock volatility	Composite based on asset turnover, liquidity, return on assets, operating cash flow to assets, accruals, gross margin, leverage
MSCI Diversified Multiple-Factor	Sector-relative composite based on enterprise value/operating cash flow, forward P/E, price to book	Exposure from the Barra Equity Model based on 12-month relative strength (25% weight), six-month relative strength (37.5% weight), historical alpha (37.5% weight)	Sector-relative composite based on return on equity, earnings variability, debt to equity
FTSE Global Factor Index Series	Composite of cash flow yield, earnings yield and sales to price	Residual momentum: mean/standard deviation of 'average residual' from 11 rolling window regressions of past 36 months' returns on country and industry index	Composite of return on assets, change in asset turnover, accruals and leverage ratio

* Operating profits minus interest expense divided by book equity in Fama and French (2015) and income before extraordinary items divided by book equity in Hou, Xue and Zhang (2015).

More generally, for most factor or multi-factor offerings, product providers typically favour more complex factor definitions which may indeed reflect a stark disagreement with how academic research defines these factors. For example, some factor scores are calculated relative to the industry or regional groups a stock belongs to. Interestingly, some providers use such industry or regional adjustments for certain variables within a given factor score while not using it for other variables making up the same factor score. Moreover, providers often use variables which are quite far removed from the original factor definition, such as, for example, change in asset turnover in quality scores, as compared to the more straightforward profitability measures used in academic research. In fact, most of the quality indices on offer have more to do with the precepts of stock-picking gurus than with the academic literature that has identified profitability and investment as asset pricing factors.

Figure 2 contains examples of selected indices among the ones included in this analysis, where the objective is to show how commonly-used definitions of factor indices deviate from the standard definitions used in the literature. Three different sets of definitions used by index providers are contrasted with the standard definitions widely employed in the literature. Rather than providing an exhaustive overview across all factors and index providers, we focus on selected examples where a deviation from academic consensus is easily apparent. However, the issue is not limited to these examples.

When considering the descriptions in figure 2, the mismatch of the provider definitions with the standard academic definitions is indeed striking. While the definitions found in the reference academic research rely on straightforward variables and make a choice of transparently and simply selecting one key metric to come up with a factor score for each stock, the proprietary definitions from most providers use different sets of variables, as well as various adjustments, and often consist of complex combinations of several variables.

The implications of the mismatch with academic factor definitions might not be immediately obvious. Nevertheless, any mismatch creates two problems. The first, which we have already mentioned, is that it is difficult to refer to academic evidence to justify one's factor offering and at the same time distance oneself from the empirical framework used for that same research with factor definitions that are different from those used by the researchers cited. The second is that this complexification and/or creation of ad-hoc proprietary factors is a source of potential data-mining problems. We discuss this second issue in the following two sub-sections. First, we look at selection bias, which relates to testing different variations in factor definitions. Afterwards, we discuss the issue of overfitting bias, which may arise when using composite scoring to capture single or multiple factors.

Data mining risk: selection bias with proprietary variables

Selecting proprietary combinations or making proprietary tweaks to variable definitions offers the possibility of improving the performance of a factor index in a backtest. In general, proprietary factor definitions increase the amount of flexibility providers have in testing many variations of factors and thus pose a risk of data-mining. In fact, it appears that providers sometimes explicitly aim at selecting ad-hoc factor definitions which have performed well over short-term backtests.

The question is whether the improvement of the 'enhanced' factor definition will also hold going forward, especially if there is no solid economic foundation for it. There is clearly a risk that one ends up with what academics have termed 'lucky factors'. Harvey and Liu (2015) show that by snooping through data on a large number of candidate factors and retaining those with the highest t-stat, one takes the risk of uncovering flukes, which will not repeat out of sample. Perhaps even more importantly, it is unclear what – if anything – factors with extensive proprietary tweaks still have in common with the factors from academic research. Therefore, the empirical evidence in favour of the academic factors and their economic grounding cannot be transposed to such new proprietary factors.

As an example of a proprietary factor found among the multi-factor indices that we analyse consider the highly original factor ►

◄ definition of the value factor used in the S&P GIVI index. This index is marketed as a multi-factor index that targets the low volatility and value factors. It combines a high beta exclusion filter with an alternative weighting scheme that weights constituents by their 'intrinsic value'. This weighting scheme is premised on the idea that markets are inefficient, that capitalisation weighting leads to systematic over-weighting of overvalued stocks and under-weighting of undervalued stocks and that it is possible to derive better proxies of the 'true value' of a stock than the market via fundamental analysis and in particular intrinsic value. The intrinsic value of each stock is calculated using the residual income model and equal to the current book value plus a function of the discounted value of analysts' consensus earnings. For each stock, the consensus earnings for the current and the next fiscal year are used to extract the projected return on equity for these two years and it is assumed that the difference between the equity's predicted profitability and the estimated cost of equity converges to zero over 22 years. The dividend payout ratio is estimated for the adjustment of the book value of equity over time. Since this is applied mechanically across the entire spectrum of stocks, proponents of fundamental analysis would probably insist that more effort would be needed to produce useful intrinsic values for stock picking. However, from a factor-exposure point of view, it is obvious that this approach will tend to tilt the portfolio towards companies with high relative book value and high relative return on equity. In this regard, the provider's proprietary definition of value should be expected to indirectly produce exposure to two conventional factors, ie, value and profitability.

Sometimes, as in the previous example, detailed analysis of the methodology will uncover relationships between proprietary factor definitions and standard academic factors. However, even when transparency is sufficient to perform such analyses, there may not be a clear relationship to standard academic factors. It should also be underlined that the interactions between the multiple dimensions of a proprietary methodology, from factor definitions, to weighting scheme, to parameter choice and calibration, could make it particularly difficult to predict relationships to conventional factors. In this context, one may consider it safer to regard proprietary factor strategies as ad-hoc constructs resulting from product back-tests. In fact, to find out whether any of these new proprietary factors are indeed related to the well-documented academic factors one would first need to assess how they align empirically with standard factors.

More data-mining risk: overfitting bias with composite scores

While the selection bias potentially exists for any strategy, there is an additional bias that is specific to so-called composite scoring approaches. These are factor definitions which draw on combinations of multiple variables. A recent paper by Novy-Marx (2015) analyses the bias inherent in back-tests of composite scoring approaches[2]. Novy-Marx argues that the use of composite variables in the design and testing of smart beta strategies yields a "particular pernicious form of data-snooping bias". He shows that creating a composite variable based on the in-sample performance of single ► page 36

2 Novy-Marx cites the MSCI Quality index and Research Affiliates Fundamental indices as industry examples of such multi-signal approaches.

Illustration of selection bias: fishing for an enhanced value factor

We illustrate the problem with selection bias, otherwise known as 'factor fishing', in the following section, where we consider alternatives to the standard value definition. The academic literature works with parsimonious and time-proven definitions of the value factor, the most popular being the book-to-market ratio, due largely to Fama and French (1993). When deviating from the established definitions, one has to keep in mind that these definitions have been confirmed by out-of-sample results ever since the seminal papers were published. This out-of-sample stability gives researchers and practitioners greater confidence that the uncovered value premium is not simply a product of a data-mining exercise.

On the other hand, if we allow ourselves the flexibility of looking for the 'best' or 'improved' definition of value, such an exercise can easily lead to relying on promising in-sample results that do not hold out-of-sample.

Commercial back-tests are typically performed over a very short time-frame, in which around 10 years of data is frequently used. Since different factor definitions will ultimately lead to different past performance, using a short time period to decide which one to pick might lead to unstable solutions.

Below, we illustrate the problems with variable selection and excessive reliance on back-tests with stylised examples. First, we study the change in back-tested performance over time, to see how (un)stable the results of variable selection are. Afterwards, we focus on the out-of-sample decay of performance benefits of in-sample variable selection.

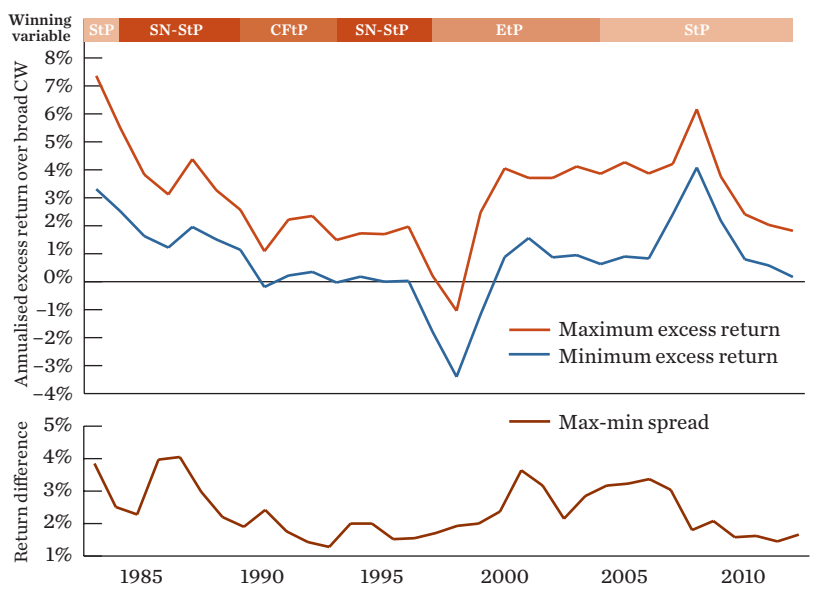
We study the rolling spreads between the annualised performances of portfolios constructed based on different value proxies. This will allow us to gain a perspective on how back-tests may have looked like over the years and, more importantly, how the change over time impacted these results.

Our question is whether we can do better than using the book-to-market measure. We select among 10 alternative value metrics – earnings-to-price, cash-flow-to-price, sales-to-price, dividend-to-price and payout-to-price, using both an unadjusted and a sector-neutral version for each. These metrics serve as the basis for forming a value-tilted portfolio where the portfolios simply select the 50% of stocks with the highest value score on an annual basis and cap-weight the selected stocks. The time series of these portfolios will serve as the basis for the empirical analysis below.

Consider figure A. Every year, we look back 10 years and plot the maximum and minimum annualised relative returns of 10 value strategies over the broad cap-weighted index in that particular period. This is done on a rolling basis between 1984 and 2013. Every year thus represents a different potential starting point for a 10-year back-test. Naturally, the excess returns change over time, but one should pay closer attention to the changing spread between the maximum and the minimum.

A. Extremes of annualised excess returns of 10 cap-weighted value strategies for 10-year lookback periods

Panel 1



This chart plots the maximum and minimum of annualised excess returns with respect to the broad US market cap-weighted benchmark of 500 stocks (from CRSP) of annually rebalanced cap weighted value tilted strategies with 50% stock selection out of the universe of 500 US stocks based on 10 value variables – earnings-to-price, cash flow-to-price, sales-to-price, dividend-to-price and payout-to-price, both plain vanilla and sector neutral versions for each. The analysis is based on daily total returns from 31 December 1973 to 31 December 2003. Ten-year trailing returns are obtained with annual step size. Every year thus represents a 10-year back-test

Panel 2. Best-performing strategies based on 10-year rolling window backtests

Period	Best performing variable
1984	Sales-to-price
1985–89	Sales-to-price, sector neutral
1990–93	Cash flow-to-price
1994–97	Sales-to-price, sector neutral
1998–2004	Earnings-to-price
2005–13	Sales-to-price

Over the years, the difference in annualised returns of the possible back-tests ranges from slightly over 4% pa to a little under 1% pa. This large spread between the different definitions suggests that considerable value can be added, at least within a back-test, when improving the variable selection. However, it is also worth noting that the best-performing variable changes over time, as the panel 2 of figure A shows.

This clearly illustrates that back-tests that search for the best past performer over a short time period might be very unstable and caution should be exercised in evaluating strategies purely constructed on the basis of in-sample performance.

Below, we focus on the performance of strategies based on alternative value definitions relative to the performance of a portfolio based on book-to-market. We know that we can enhance back-tests, but should we? Our illustrations will reveal the out-of-sample decay of the data-mined solutions that rely on picking the best in-sample winners.

In the following exercise, we use a five-year formation window at the end of which we select the best-performing strategy based on its in-sample performance. Then, we hold the strategy for five years and compare the cumulative returns of this alternative strategy with respect to the portfolio based on the book-to-market measure. We do this every year between 1984 and 2009 to obtain 26 different event studies and we study the average performance.

Figure B shows the average cumulative relative returns of the best performing alternative value definition with respect to book-to-market, both pre- and post-formation. As the chart clearly shows, the average alternative variable definition ultimately underperforms book-to-market and drives the cumulative relative returns way below zero. Picking the past winner yields cumulative outperformance over book-to-market of +1.79% in sample. However, over the following five years, picking the in-sample winner leads to cumulative underperformance of –2.72% out of sample. This is evidence that searching for a better value definition in sample does not beat book-to-market.

The previous exercise demonstrates that alternative value definitions hardly present a suitable replacement for book-to-market overall, based on the event study approach. To illustrate this point clearly and more convincingly, we now turn to simulating the experience of an actual investor in alternative value strategies.

Starting in 1984, we allow the investor to select the best-performing value variable, again using the 10 alternative variable definitions specified above. After formation, the portfolio is held for a certain period and re-evaluated again at the end of it. We thus create active strategies that the investor sticks to for the duration of the holding period. We use two lengths of the calibration period (10 and five years) as well as four lengths of the holding period (two to five years) for a total of eight active strategies, to capture the variability of the performance. We compare the performance of the active strategies based on alternative value definitions with a simple portfolio based on book-to-market in figure C. The results clearly show that none of the active strategies beats book-to-market, with the average active strategy lagging 61 basis points behind. Relative to their in-sample performance, the variable picking strategies on average create an out-of-sample degradation in performance of 128 basis points.

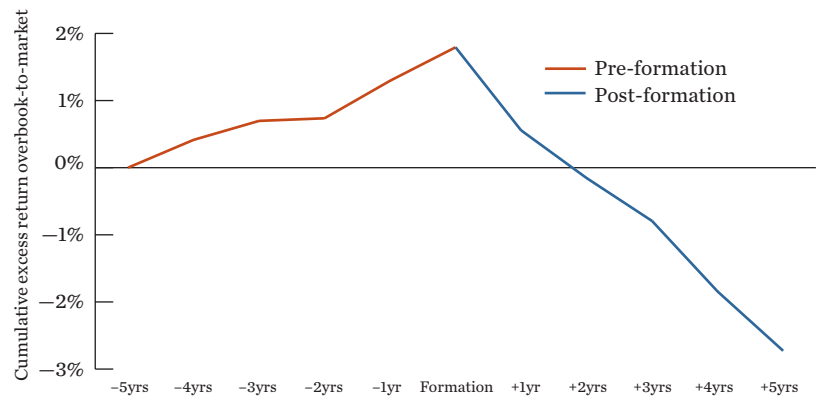
Overall, our empirical illustrations suggest that it is quite possible to enhance back-tests by selecting variables that 'work' in sample. However, the strong out-of-sample degradation of performance suggests that such an approach leads to a risk of overstated back-test performance. We emphasise also that we consider our illustration to correspond to a mostly harmless data-mining experiment, which is likely to understate the actual bias that could result in more flexible data-mining exercises. In particular, we use a relatively small number of variables that remain economically sensible proxies for value, and which are by construction highly correlated among one another. Data-mining biases would obviously be much higher if we used a much larger number of variables, economically less sensible proxies or variables that are less correlated with one another.

Ultimately and more generally, many value-tilted indices include other large sets of ad-hoc methodological choices, opening the door to data mining. It can be argued that the use of straightforward single variables in factor definitions can be an effective safeguard against data-snooping or factor-fishing biases.

Our comparison of different approaches to factor definitions thus shows two different underlying philosophies. One is to avoid creating a mismatch with academic factors. This can be achieved easily by referring to indicators for which academic research has provided thorough tests and economic explanations, and by refraining from proprietary 'tweaks'. The benefit is that this approach creates indices that are directly based on the academic groundings of factor investing. It also allows investors to understand the return drivers of the indices, and make sure that the rationale and empirical evidence of such drivers has been confirmed in vastly scrutinised and independent academic research. That is the choice that EDHEC-Risk Institute made for the Scientific Beta smart factor indices.

Another approach is to try to improve upon the academic consensus through tweaked proprietary definitions. When using novel or proprietary factors, one needs to make sure that they are thoroughly tested (ie, tested with very long-term data, across asset classes, for robustness to data-mining and to transaction costs) as well as linked to economic mechanisms. In contrast to academic factor definitions which have survived such analyses, the same amount of scrutiny has not necessarily been applied to proprietary tweaks, which carry the risk of being driven by marketing innovation rather than by genuine research advances. Therefore, investors should hold providers of proprietary factors to higher standards and conduct thorough due diligence on the soundness of their particular definitions.

B. Comparison of cumulative relative returns of the average best in-sample alternative value strategy with respect to a portfolio based on book-to-market



This chart plots the cumulative excess returns of 10 annually-rebalanced cap-weighted value-tilted strategies with 50% stock selection out of the universe of 500 US stocks based on 10 alternative value strategies, with respect to a similarly constructed portfolio based on Book-to-Market. Between 1984 and 2009, the five-year formation period is used to pick the best portfolio based on alternative value definitions and this portfolio is held for another five years. This is done every year for a total of 26 event studies. The chart plots the average outperformance pre- and post-formation with respect to the book-to-market portfolio. The alternative value definitions are earnings-to-price, cash flow-to-price, sales-to-price, dividend-to-price and payout-to-price, both plain vanilla and sector neutral versions for each. The graph is smoothed by using yearly values.

C. Performance of variable picking strategies for value-tilted portfolios

		In-sample results			Out-of-sample results of variable picking strategies								
	Book-to market	10 years	5 years	Average	Calibration period = 10 years				Calibration period = 5 years				Average
					HP=5	HP=4	HP=3	HP=2	HP=5	HP=4	HP=3	HP=2	
Annualised returns	13.1%	13.7%	13.9%	13.8%	12.7%	12.5%	12.7%	12.6%	12.4%	12.3%	12.3%	12.4%	12.5%
Annualised volatility	19.0%	18.4%	17.9%	18.1%	18.6%	18.6%	18.6%	18.5%	18.1%	18.6%	18.4%	18.3%	18.5%
Sharpe ratio	0.47	0.52	0.55	0.53	0.46	0.45	0.46	0.46	0.46	0.44	0.45	0.45	0.45
Return difference with B/M	–	0.56%	0.78%	0.67%	–0.37%	–0.60%	–0.43%	–0.46%	–0.69%	–0.80%	–0.77%	–0.75%	–0.61%
Return difference with 'in-sample'	–0.67%	–	–	–	–0.93%	–1.17%	–1.00%	–1.03%	–1.47%	–1.58%	–1.55%	–1.53%	–1.28%

This table shows the performance of eight active strategies formed on the basis of a calibration period and held for a holding period from 10 alternative value definitions. The alternative value definitions are earnings-to-price, cash flow-to-price, sales-to-price, dividend-to-price and payout-to-price, with both plain vanilla and sector neutral versions for each. At formation, the best-performing strategy based on the calibration period of 10 or five years is selected and held for a holding period of two to five years. The book-to-market portfolio is formed annually by cap-weighting the 50% selection of the stocks with the highest book-to-market ratio. The in-sample results select the best returns ex-post of the alternative value strategies. All portfolios are based on the top 500 US stocks between 1 January 1984 and 31 December 2013.

variable strategies generates an over-fitting bias. To make matters worse, this over-fitting bias interacts with the selection bias. The presence of both biases in composite variable smart beta strategies increases the data-mining problems exponentially.

Novy-Marx analyses the bias occurring in an analysis of back-tested performance by considering strategies that combine random signals. The results show that combining signals that happened to perform well in the past leads to even better past performance. Given that signals are uninformative by construction, the past performance of these composite strategies of course does not imply any capacity to generate performance out of sample. The author concludes that "combining signals that back-test positively can yield impressive back-tested results, even when none of the signals employed to construct the composite signal has real power".

The analysis also underlines the severity of the overall bias in composite scoring approaches where the selection bias and over-fitting bias interact. Novy-Marx finds that a back-test based on composite scoring using the "best k of n" variables, is almost as biased as a back-test of a strategy where one selects the single variable that had the best performance of n to the power of k candidate variables. For example, using a composite score where one selects three variables out of six candidate variables is as biased as selecting with hindsight a single variable from 216 (6 to the power of 3) candidate variables. Likewise, selecting a composite of five variables out of 10 based on back-tested performance is almost as bad as selecting a single variable among 100,000 (10 to the power of 5) candidate variables. This result underlines that the use of composite scores may lead to severe data-snooping bias. As the author concludes, by "combining spurious, marginal signals, it is easy to generate back-tested performance that looks impressive".

A simple reason why composite scores may be more prone to generating biased results is that a composite variable requires more inputs and thus increases the number of possible choices. There seems to be wide-ranging awareness that composite strategies, by having more inputs, will lead to increased data-mining risk. Pedersen (2015) makes a case against excessive back-testing, arguing that "we should discount backtests more if they have more inputs and have been tweaked or optimised more". Likewise, Ilmanen (2013) states that analysis involving "tweaks in indicator specification" is "even more vulnerable to data mining than is identification of the basic regularities".

In figure 3 we present examples of composite scores used in the construction of multi-factor indices. It is immediately apparent that it is typical for factor index providers to use composite scores. For example, and focusing on the definition of quality, MSCI combines three variables, FTSE Russell ups the ante with four metrics and Goldman Sachs mixes no fewer than seven metrics.

For investors conducting due diligence on commonly-offered smart beta strategies, it thus appears important to investigate not just the back-tested performance but also the underlying data snooping risk, given that both selection bias and overfitting bias may be present when proprietary composite scores are being used. Moreover, one can argue that backtests of strategies that do not employ complex proprietary scores are naturally more robust and the back-tested performance of such strategies needs to be discounted less than that of complex proprietary factor definitions. In the next section, we further investigate the biases stemming from methodological choices, in particular what happens when a consistent approach may be lacking in the index design.

3. Examples of the use of composite scores in the construction of a multi-factor index

Index	Composite score
FTSE Russell 1000 Comprehensive	The quality component is a composite of profitability and leverage and three individual measures make up profitability
Goldman Sachs Equity Factor Index World	Quality metric uses a composite based on asset turnover, liquidity, return on assets, operating cash flow-to-assets, accruals, gross margin, leverage
MSCI Diversified Multiple-Factor	The quality component uses a sector-relative composite based on return on equity, earnings variability, debt-to-equity

Inconsistencies in methodologies

As we discussed in the previous sections, a major source of potential data-mining bias that may result in overstated back-tested performance is the flexibility offered by the testing of many variations in search of the winning one. Such flexibility is obviously increased when a provider allows index methodologies to be inconsistent, across indices and/or time.

On the contrary, a very effective mechanism to avoid data mining is to establish a consistent framework for smart beta index creation. Such a framework can limit ad-hoc choices while providing the necessary flexibility needed for smart beta index construction. A consistent framework is the best safeguard against post-hoc index design, or model mining, ie, the testing of a large number of smart beta strategy variations, and selection of the ones that have good in-sample results. Perhaps surprisingly, while most major index providers argue that cap-weighted indices should have a consistent set of rules across regions to avoid unintended investment outcomes, consistency is often forgotten for factor indices. Below, we draw on several examples of inconsistencies for the indices discussed in this article.

Inconsistency or consistency across factors

An important aspect of a robust methodology in the case of smart beta indices is consistency in the design across the indices for different factors. Indeed, it is surprising to see the same provider rely on radically different approaches to index construction to establish exposure to different factors. It is arguably even more surprising to see indices that were built using widely different methodologies being combined into a multi-factor index.

Figure 4 outlines the design framework of the factor-based strategy indices that constitute the components of the MSCI Quality Mix index, and compares them to the factor indices used by ERI Scientific Beta as components of its multi-factor indices. Figure 4, in essence, compares two strikingly different approaches to constructing indices for different factors. While the Scientific Beta single factor indices apply the same index construction methodology and thus serve as fairly uniform building blocks for the multi-factor index, the underlying indices of the MSCI Quality Mix index all follow a different design path.

MSCI employs different stock selection schemes, weighting schemes and risk control options for the three different component indices in its Quality Mix multi factor index. For example, the value factor index includes all stocks in the universe and reweights them by their value-related scores. The quality factor

4. Comparison of consistency in index construction framework between component factor indices used in MSCI multi-factor indices and Scientific Beta multi-factor indices

Factor	Index	Stock selection	Weighting scheme	Risk controls
MSCI index methodologies (for components of Quality Mix index)				
Value	MSCI USA Value Weighted index	All stocks in CW parent index universe	Value score derived from four fundamental metrics, adjusted by investability factor	None
Low volatility	MSCI USA Minimum Volatility index	All stocks in CW parent index universe	Optimisation to minimise portfolio risk	Sector weight constraints. Cap on multiple of market cap of individual security
Quality	MSCI USA Quality index	Fixed number of stocks by three-metric factor score	Selected stocks weighted by product of market cap and three-metric quality score	Issuer weights capped at 5%
Scientific Beta index methodologies (for components of SciBeta Multi-Beta Multi-Strategy index)				
Size	SciBeta USA Mid Cap Diversified Multi-Strategy index	Half the stocks in the universe by relevant single-metric factor score	Same weighting scheme for selected stocks (Diversifed multi-strategy by default)	Cap on multiple of market cap and weight of individual securities
Value	SciBeta USA Value Diversified Multi-Strategy index			
Momentum	SciBetaUSA High Momentum Diversified Multi-Strategy index			
Low volatility	SciBeta USA Low Volatility Diversified Multi-Strategy index			

5. Inconsistency in factor definitions among MSCI multi-factor indices over time

	Scoring		Adjustments	
	2013	2015	2013	2015
Value	Sales, book value earnings and cash earnings Past three-year average values Simple average across variables	Price-to-book value price-to-forward earnings and enterprise value-to-cash flow from operations Current values Average of z-score for each variable	No sector control	Sector-relative scoring
Quality	Return on equity, debt to equity and earnings variability	Return on equity, debt to equity and earnings variability	No sector control	Sector-relative scoring
Size	None	Negative of the exposure from the Barra Equity Model. Barra uses a z-score based on the logarithm of the market cap of the relevant firm		Country control (the Barra descriptor is on a country-relative basis)
Momentum	12-month and 6-month local price performance	Exposure from the Barra Equity Model based on 12-month relative strength (25% weight), 6-month relative strength (37.5% weight), historical alpha (37.5% weight)	Momentum score is risk-adjusted	No explict risk adjustment (use of Barra exposure)

This figure shows the difference between the definitions of factors used by MSCI in its Deploying Multi Factor Allocations white paper (2013) and the definitions for the same factors used in creating the MSCI Diversified Multiple-Factor index (2015).

component takes a different approach and first selects a fixed number of stocks with the highest quality score. A relevant question is why the value index does not do the same and first selects stocks or instead, why the quality index does not use all stocks and simply reweights them by quality score, to be consistent with the value component. The third component capturing the low volatility factor follows yet another methodology. It obtains its low volatility factor tilt implicitly through a weighting scheme (minimum volatility). Again, one wonders why the same objective cannot be obtained with a methodology that would be consistent with, eg, the quality component and simply select and weight stocks by a risk measure for example. It is also worth noting that the three indices use different types of constraints on individual stock weights or sector weights. Such lack of uniformity in index design across factor indices leads to the question of what justifies the differences across factors and how back-tests of indices following such design approaches are impacted by data-mining risks.

We can also see from figure 4 that Scientific Beta follows a dramatically different approach by using a consistent methodology across each of the four component indices of its multi-factor index, containing four main rewarded factors. The implication of such consistency is that the number of potential variations that may have been tested is limited by construction. Such an approach aligns with common sense recommendations to avoid the pitfalls of data snooping. For example, Lo (1994) argued that we need "some kind of framework to limit the number of possibilities that we search over".

Inconsistency across time

Data-mining risks are further exacerbated by inconsistencies among index offerings across time. If providers change their mind frequently on what a good proxy for a given factor is, this inevitably increases the flexibility of index design and increases the potential to show inflated back-test performance. If providers launch new and enhanced versions of indices for the same factor to replace old indices capturing the same factor, one may ask whether that new version was engineered to produce a simulated track record that would distract from the poor live performance of the erstwhile flagship product, which if correct would not bode well for the robustness of the new product's performance.

To show an example of changes in methodology over time, figure 5 contrasts the factor definitions used for implementing the MSCI multi-factor approach as described in 2013 and 2015[3]. These single factor definitions are relevant in the design of two of the multi-factor indices we analyse in this article.

The MSCI Quality Mix index is an equal-weighted index of three single factor indices targeting the value, low volatility and quality factors respectively. The component indices of the MSCI Quality Mix index for the value and quality factors follow the 2013 definitions, outlined in the table. On the other hand, the optimisation-based MSCI Diversified Multi Factor index, targeting the value, quality, size and momentum factors, currently uses the four factor definitions from 2015.

It appears indeed that the 2015 factor definitions are at odds with earlier single- and multi-factor offerings. For example, in the MSCI Quality Mix index, the value component is a fundamentally-weighted index aggregating book value, sales, earnings and cash earnings, and the quality component is not sector relative and winsorised. In MSCI's Diversified Multi Factor index, the value score is a sector-neutral composite score based on earnings-to-price, book-to-market and cash flow to enterprise value.

Therefore, it appears that two multi-factor indices launched at different points in time by the same provider use different definitions of the value factor[4]. This may be surprising, especially for the value component, as value seems to be amongst the most standard factors. Just as inconsistencies across factors open the room for a large number of variations in index design, it is clear that inconsistencies over time further increase such flexibility.

Such inconsistency across time is however widely present among index offerings. Amenc, Goltz, Lodh and Sivasubramanian (2015) emphasise that "Russell launched new factor indices to create a new brand known as 'High Efficiency' (HE) indices when it already had the following factor indices in the market – Russell 1000 High Momentum, Russell 1000 Low Volatility and Russell 1000 Value. The new indices have the same objective as the old ones but different construction principles." Interestingly, these "high efficiency" factor indices have since been abandoned by the provider for most of the factors and replaced by yet another suite of factor indices for the same factors using yet another methodology. It thus appears that inconsistency over time is all but day-to-day business for index providers.

Concentrated indices and stock-level optimisation

An important issue that can be easily neglected when constructing a multi-factor index is diversification. Positive exposure to rewarded factors is obviously a strong and useful contributor to expected returns. However, products that aim to capture explicit risk-factor tilts often neglect adequate diversification. This is a serious issue because diversification has been described as the only 'free lunch' in finance. Diversification allows a given exposure to be captured with the lowest level of risk required. In contrast, gaining factor exposures exposes investors to additional types of risk, and therefore, such exposures do not constitute a 'free lunch'. They instead constitute compensation for risk in the form of systematic factor exposures. Such capturing of risk premia associated with systematic factors is attractive for investors who can accept the systematic risk exposure in return for commensurate compensation.

However, factor-tilted strategies, when they are very concentrated, may also take on other, non-rewarded, risks. Non-rewarded risks come in the form of idiosyncratic or firm-level risk, as well as potential risk for sector concentration, currency, sovereign or commodities risk exposure, etc. Financial theory does not provide any reason why such risk should be rewarded. Therefore, a sensible approach to factor investing should not only look at obtaining a factor tilt, but also at achieving proper diversification within that factor tilt. To illustrate this point, we focus on the value factor as an example below, but the discussion carries over to other factors as well.

In fact, if the objective was to obtain the most pronounced value tilt, for example, the only unleveraged long-only strategy that corresponds to this objective is to hold 100% in a single stock, the one with the largest value tilt, as measured for example by its estimated sensitivity to the value factor or its book-to-market ratio. This thought experiment clearly shows that the objective of maximising the strength of a factor tilt is not reasonable. Moreover, this extreme case of a strong factor ►

3 Please refer to Deploying Multi-Factor Index Allocations in Institutional Portfolios, Research Insight, MSCI, December 2013, and The MSCI Diversified Multi-Factor Indexes - Maximizing Factor Exposure While Controlling Volatility, Research Insight, MSCI, May 2015.

4 None of which is consistent with the definition used in MSCI's oldest value index, aptly named Value index, which is based on a composite of book value-to-price, 12-month forward earnings to price and dividend yield.

◄ tilt indicates what the potential issues with highly concentrated factor indices are. Even if the appropriateness of such an approach had been established, any value premium so captured would necessarily come with a large amount of idiosyncratic risk. This risk is not rewarded and therefore we should not expect the strategy to lead to an attractive risk-adjusted return. Additionally, it is unlikely that the same stock will persistently have the highest value exposure within a given investment universe. Therefore, a periodically-rebalanced factor index with such an extreme level of concentration is likely to generate 100% one-way turnover at each rebalancing date, as the stock held previously in the strategy is replaced with a new stock that displays the highest value exposure at the rebalancing date. While practical implementations of concentrated factor-tilted indices will be less extreme than this example, we can expect problems with high levels of idiosyncratic risk and high levels of turnover whenever index construction focuses too much on concentration and pays too little attention to diversification. This goes for both single-tilt and multi-factor indices. In the next sub-sections, we discuss concentration in the context of different approaches to designing multi factor indices.

Top-down approaches

One of the possible ways to construct a multi-factor index is to combine different single factor indices. Among the indices we analyse in this article, this is the approach chosen by Scientific Beta and is also the chosen methodology of the MSCI Quality Mix index.

For such combinations of single factor indices, there will of course be a certain level of deconcentration resulting from the fact that different indices are combined. However, a relevant question is whether such multi-factor indices are constructed using well-diversified building blocks.

Amenc et al (2016) show that well-diversified factor indices which pursue a diversification objective through an alternative weighting scheme based on a relatively broad stock selection provide considerable benefits over more concentrated single factor indices. Their results suggest that well-diversified factor portfolios or indices outperform their highly-concentrated counterparts in terms of risk-adjusted performance, because concentrated factors may be highly exposed to unrewarded factors. In addition, they show that factor-tilted portfolios on narrow stock selections present implementation drawbacks such as higher turnover.

The Scientific Beta multi-factor indices that are part of our analysis use well-diversified factor indices as building blocks. These single factor indices are also termed "smart factor indices" (see Amenc et al [2014]). In this approach, each single factor index is well diversified and multi-factor allocation across several such indices additionally smoothes returns over factor cycles.

On the other hand, looking at the MSCI Quality Mix index from a conceptual perspective, we can observe that the index does not have an explicit diversification objective which may lead to concentration, depending on the specific parameters used for weighting and stock selection. The index involves simple market-cap weighting adjusted by quality scores (in its quality component), weighting by firm fundamentals (in its value component, which uses their value-weighted approach) and an approach that is notorious for producing high concentration (to establish its low-volatility exposure) and therefore may end up being at least as concentrated as cap-weighted approaches.

6. Impact of Volkswagen scandal on Stoxx Europe 600 vs SciBeta Extended Europe Multi-Beta Multi-Strategy EW index and the MSCI Europe Diversified Multi-Factor index

31 August 2015–30 September 2015	Performance and weight analysis		
	Stoxx Europe 600	SciBeta Extended Europe Multi-Beta Multi-Strategy EW	MSCI Europe Diversified Multiple-Factor
Volkswagen AG weights as of 31 Aug 15	0.35%	0.05%	0.80%
Active weights		–0.30%	0.45%
Cumulative returns	–4.41%	–2.94%	–3.82%
Attributable to Volkswagen AG	–0.15%	–0.02%	–0.34%
Cumulative excess returns	–	1.47%	0.59%
Attributable to Volkswagen AG		0.13%	–0.19%

Taken from Amenc, Sivasubramanian and Ulahel (2015). Analysis is based on weekly total returns in US dollars from 31 August 2015 to 30 September 2015 on the Extended Europe Universe. The SciBeta Extended Europe Multi-Beta Multi-Strategy EW index is the equal-weighted combination of the four factor-tilted multi-strategy indices – mid cap, high momentum, low volatility and value. Each factor-tilted multi-strategy index selects 50% of stocks from the universe based on the factor score. The multi-strategy weighting scheme is the equal-weighted combination of five weighting schemes: maximum deconcentration, maximum decorrelation, efficient minimum volatility, efficient maximum Sharpe ratio and diversified risk weighted. Each factor-tilted score-weighted index selects 50% of stocks from the universe based on the factor score.

Bottom-up approaches

Concentration may also arise in multi-factor indexing methodologies which, rather than combining single factor indices, actually build multi-factor indices from the stock level up. If the methodology targets exposure to stocks with the highest composite multi-factor score for example, concentration may be expected quite naturally as a result of the objective of strong multi-factor exposure. If the stock-level information on factor scores is integrated in an optimisation approach, concentration issues may be exacerbated.

The optimisation approach is for example followed by the MSCI Diversified Multiple-Factor index. This index maximises the ratio of a weighted average composite multi-factor score to portfolio volatility, which corresponds to mean-variance optimisation when stock-level expected returns are proxied by the composite factor score.

As a result of such an optimisation, one may observe high levels of concentration. Indeed, it is interesting to note that MSCI reports that its MSCI World Diversified Multi-Factor index (where the top 10 stocks account for 14.5% of the index capitalisation at the end of June 2016) is more concentrated than the broad capitalisation-weighted MSCI World index (where the top 10 stocks account for 10.08% of the total index capitalisation).

More generally, methodologies that optimise on the basis of stock-level factor scores as proxies for expected returns may result in high levels of concentration if suitable deconcentration mechanisms are not included in the methodology. One may end up with a portfolio with high weighted-average factor scores, but also high idiosyncratic risk.

As an example of high idiosyncratic risk, consider the following case study analysing the exposure of the European version of the MSCI index, the MSCI Europe Diversified Multiple-Factor index, to Volkswagen AG. Figure 6, taken from Amenc, Sivasubramanian and Ulahel (2015), shows that, at the time when the emissions scandal erupted, the MSCI Europe Diversified Multiple-Factor index was very strongly exposed to the risk of Volkswagen AG stock. This poor diversification of the specific risks led to the MSCI index considerably underperforming the SciBeta Extended Europe Multi-Beta Multi-Strategy EW index.

The figure shows that the MSCI Europe Diversified Multiple-Factor index contained roughly 16 times more Volkswagen AG stock than the Scientific Beta Extended Europe Multi-Beta Multi-Strategy EW index, with respective weights of 0.05% and 0.80% as of 31 August 2015. Similarly, the MSCI multi-factor index overweighted Volkswagen stock more than twice with respect to the reference cap-weighted Stoxx Europe 600 index, with a 0.80% weight compared to 0.35% in the reference index.

Ultimately, the MSCI Europe Diversified Multiple-Factor index underperformed the SciBeta Extended Europe Multi-Beta Multi-Strategy EW index by 88 basis points, with excess returns relative to the Stoxx Europe 600 index for the month of September 2015 of 0.59% com-

7. Example of concentration of composite factor index

31 Dec 1974–31 Dec 2014	Multiplicative Scoring Strategy	SciBeta Long-Term US Multi-Beta Multi-Strategy Six-Factor EW
Annualised return	15.64%%	16.01%
Volatility	15.22%	15.52%
Sharpe ratio	0.69	0.70
Relative returns	3.48%	3.85%
Tracking error	5.22%	4.73%
Information ratio	0.67	0.81
Annualised one-way turnover	34.82%	25.03%
Weighted average market cap ($m)	11,653	11,607
Effective number of stocks	170	345

The time period of analysis is 31 December 1974 to 31 December 2014 (40 years). The composite factor index is constructed by multiplying the s-score for the six factors – size, momentum, low volatility, value, low investment, and high profitability – for each individual stock and then combining this composite factor score with the market cap to arrive at final weights. This composite factor index is rebalanced annually and is constructed using a US stock universe that contains the 500 largest stocks by total market capitalisation. The analysis is done using total returns in US dollars. The yield on secondary market US Treasury Bills (3M) is the risk-free rate. Scientific Beta US Long-Term Track records are used to obtain the Scientific Beta LTTR Multi-Beta Multi-Strategy Six-Factor EW index as well as the cap-weighted benchmark. Effective number of stocks is given by the inverse of the sum of squared constituent weights. The reported turnover and effective number of stocks is an average across the time period of analysis.

pared to 1.47% for the SciBeta Extended Europe Multi-Beta Multi-Strategy EW index. The analysis of the Volkswagen case also provides a good understanding of how the search for strong factor exposure can lead to overconcentration in a particular stock.

Another example of possible concentration issues that may arise when working from the bottom up as well as using the previously discussed composite scores is the so-called 'tilt-tilt' methodology employed by the FTSE Russell Comprehensive indices. These indices multiply several factor scores for each stock and combine them with market cap weights to arrive at a final stock weight in the index. The multiplicative scoring across factors means that stocks will be overweight not necessarily when they have high average exposure to the different factors but rather when they have positive exposure to each and every factor. This approach thus incorporates the idea of looking for champion stocks which rank well according to all factor attributes for several factors at the same time.

In figure 7 we compare the Scientific Beta Multi-Beta Multi-Strategy Six-Factor EW index (a top-down index based on well-diversified single-tilt building blocks) with a stylised bottom-up test portfolio using a multiplicative scoring methodology to adjust market cap weights by a multi-factor score. This strategy, which we call the Multiplicative Scoring Strategy, uses a composite factor score for six factors – lower size, positive momentum, low volatility, value, low investment and high profitability. The composite is based on multiplying the s-scores for each factor and adjusting the market cap-weights by this multiplicative score. The table, analysing a period of 40 years in the US, shows the risk-adjusted performance of the two indices as well as interesting weight-related measures such as turnover and effective number of stocks. The effective number of stocks, calculated as the inverse of the sum of squared stock weights, is a good measure of diversification and allows the concentration levels of the portfolios to be compared.

Comparing the effective number of stocks between the Multiplicative Scoring Strategy and the Scientific Beta multi-factor index, we learn that the levels of diversification are quite different. Indeed, Scientific Beta's multi-factor index has more than twice the effective number of stocks compared to the Multiplicative Scoring Strategy, with 345 versus 170. Apart from the concentration in fewer stocks, the Multiplicative Scoring Strategy also experienced higher turnover.

As the performance metrics reveal, the composite multiplicative factor scoring approach did not produce better risk-adjusted performance, with an information ratio of 0.67 lagging behind 0.81 for the Scientific Beta index.

In addition to concentration, stock level approaches contain further issues which we turn to now.

When using multi-factor scores in portfolio optimisation, it should not be forgotten that the score is ultimately used as a proxy for expected returns. It is well known for example that mean-variance optimisation that integrates expected returns can result in an 'error maximisation exercise' since expected return are hard to estimate at the individual stock level, and since mean-variance optimisers are very sensitive to estimation error for expected returns (Best and Grauer [1991]).

Achieving high absolute factor scores at the portfolio level by concentrating on picking champion stocks that score highly on all targeted factor dimensions is probably intuitively attractive but it is predicated on a high-precision relationship between factor scores and returns at the stock-level. There is no question that factor investing is motivated by an attempt to capture higher long-term returns through the right risk exposures. However, return estimation at the stock level is notoriously difficult. Black (1993) distinguishes between explaining returns, which is easy because it is really explaining variance, and predicting returns, which is hard. He contends that the accurate estimation of average expected return requires decades of data. For variance, he notes, "We can use daily (or more frequent) data to estimate covariances. Our estimates are accurate enough that we can see the covariances change through time." To estimate expected returns, on the other hand he writes, "Daily data hardly help at all." and "We need such a long period to estimate the average that we have little hope of seeing changes in expected return." These observations are consistent with the unavoidable statistical fact that estimators of risks are convergent/consistent (the more data points the more precise the estimation) while estimators of returns are non-convergent/consistent (the frequency of observation does not help, only the length of time does), as underlined in the first appendix to Merton (1980).

The search for champion stocks as measured by their factor scores is a stock-picking exercise that relies implicitly but heavily on the accuracy of expected return predictions. As alpha envy appears to contaminate smart beta and factor investing, it is important to pause and remember that it is precisely the lack of persistent success in stock picking that has led an increasing number of institutional investors to shift towards passive strategies and that it is the realisation that the bulk of the performance of active management programmes comes from exposure to well-documented systematic factors that has reignited the interest in factor-based investing.

Attempting to improve stock-level return forecasts, even when this is done with the support of a factor model, is a largely futile exercise. This should probably remain the preserve of professional stock pickers. If efforts are to be made to improve the adjusted returns of factor investing, it is more on the risk dimension side, where we can rely on 60 years of progress in financial econometrics to estimate convergent estimators of volatilities and covariances.

When academics have tested standard factors, they have done so by running portfolio sorts, and assessing return differences at the portfolio level, not by assessing returns at the stock level. For example, they have observed that, on average, value stocks tend to have higher returns than growth stocks over the long-term. If one now tries to design strategies based on very fine distinctions at the stock level, such relations may be drowned in noise. More generally, making very fine distinctions at the stock level is prone to capturing estimation error.

Indeed, an implication of the 'error maximisation' issue is that stock-level optimisation, which considers expected returns (or, equivalently, composite factor exposures), will not only be highly concentrated in a few stocks, but will actually tend to assign the highest weights to the stocks with the highest estimation errors. We should underline that optimisation-based smart beta strategies (such as minimum volatility, equal-risk contribution etc) had avoided using direct estimates of expected returns in optimisation precisely because it is well known that this leads to the error maximisation problem.

Thus any stock-level approach needs to be handled with care and one needs to assess whether suitable mechanisms have been built in to achieve robustness.

Furthermore, optimisation-based approaches frequently come with stringent constraints attached. These constraints are intended to avoid extreme solutions and produce more acceptable portfolios. This not only reveals a guarded faith in optimisation, but also creates model mining risks as the solution may end up being primarily driven by the constraints and not the objective. Furthermore, it is not clear how sensitive the strategy is to such constraints. Last but not least, there may be provisions in the ground rules that allow the optimiser to be changed, thus potentially introducing discretion into a supposedly rules-based design.

Conclusions

The offerings in the area of multi-factor indices are multiplying rapidly and investors have to assess how such indices match their investment needs. Given that most products have been launched recently, analysis of risk and performance is mostly limited to back-tested data. Therefore, the methodological principles behind index construction should become a key area of attention in the assessment of these indices. Analysing robustness requires an assessment of index design principles and the conceptual considerations underlying index design. Our brief review of offerings aims to shed light on several issues such as complex proprietary factor definitions, potential inconsistencies in methodologies, and concentration issues.

We have discussed the all-important issues of data-mining, which can present real problems in many cases. The proprietary factor definitions and the use of composite scores in index construction may lead to overstated back-tested performance. This is of major interest to investors as the new index products are mostly being sold on the strength of good back-tested performance. However, flexibility in design choices and the ability to test many variations of factor definitions and portfolio construction models can severely bias any historical simulation. In addition, we have argued about the importance of consistency in index design. The lack of a well-defined methodological framework, or frequent changes to it, increases the amount of flexibility that providers have and thus potentially biases the historical tests further. Similarly, uniformity and consistency across the various index offerings is a surprisingly overlooked aspect of index design.

We also compared the top-down and bottom-up approaches to multi-factor index construction and found that concentration could arise in many scenarios, thus exposing investors to undesired idiosyncratic risks. Diversification has been described as the only 'free lunch' in finance and unwanted concentration does not do much more than erode it.

In principle, multi-factor indices aim at a common goal – outperforming cap-weighted benchmarks by providing exposure to multiple rewarded factors. As discussed here, the ways to do this are nonetheless quite diverse. A key consideration for investors is how robust the performance presented in backtests is expected to be. Highly parameterised approaches naturally contain higher risks of overstated back-test performance than more parsimonious index design methods. In particular, since the bottom-up approach is more flexible, it can more easily fall prey to data-mining. It is always possible to find a combination of factor definitions, multi-factor scoring and weighting scheme that will ►

◄ select the right stocks in sample. In-sample over-fitting, however, would lead to disappointing out-of-sample performance. In terms of due diligence, the bar on innovative bottom-up methods should be set higher than for classic top-down approaches, and investors would be well advised to ask for live track records of a significant length when a provider shows a lot of creativity.

There is no doubt that more elaboration on factor definitions and the use of more granular stock-level information allow the data to be fitted better and help to produce back-tests that suggest superior performance, but the ultimate question investors should ask is that of the robustness of the advertised index performance in live conditions.

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