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RESEARCH FOR INSTITUTIONAL MONEY MANAGEMENT:
Special Issue on Liability-Driven Investing

EDHEC-RISK Institute

INTRODUCTION

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It is my pleasure to introduce the latest issue of the Research for Institutional Money Management supplement to P&I, which aims to provide institutional investors with an academic research perspective on the most relevant issues in the industry today.

This supplement is a "Liability-Driven Investing Special" in which we first examine, in a series of four articles drawn from research supported by BNP Paribas Investment Partners as part of the "Asset-Liability Management and Institutional Investment Management" research chair at EDHEC-Risk Institute, the evolution of pension fund investment management from asset management to asset-liability management and risk and asset-liability management, and from liability-driven investing to dynamic liability-driven investing, concluding with a survey of institutional investors who provide insights on current practices in these areas and how those practices compare to those prescribed in the academic literature.

We analyze the challenge of hedging long-term inflation-linked liabilities under various inflation regimes without inflation-linked instruments as part of the "Advanced Investment Solutions for Liability Hedging for Inflation Risk" research chair at EDHEC-Risk Institute supported by Ontario Teachers' Pension Plan. By focusing on the respective merits of diversifying away and hedging away expected inflation risk, we come up with a series of policy implications; notably, that a duration-matching strategy involving long-short allocation to nominal bonds of different maturities would be a reasonable approach to hedging away the risk of a large positive shock to expected inflation.

In seeking to construct improved corporate bond benchmarks, we compare the benefits of naïve and scientific diversification of bond portfolios. We select the 20 and 40 most liquid bonds based on their market capitalizations. When the number of constituents is equal to 20, then the volatility of the "naïve" portfolio is higher than that of the scientifically diversified strategies. However, when the number of constituents increases to 40, then the difference in risk-adjusted performance between naïve and scientific diversification vanishes. Our findings have important practical implications since they suggest that investors may be able to construct improved fixed-income benchmarks that would dominate standard ad-hoc benchmarks from a risk-adjusted performance perspective for a given duration constraint.

We hope that the articles in the supplement will prove useful and informative. We wish you an enjoyable read and extend our warmest thanks to our friends at P&I for their collaboration on the supplement

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PORTFOLIO MANAGEMENT

Accounting for Pension Liabilities via the Liability-Driven Investing Paradigm: From Asset Management to Asset-Liability Management

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The first and most important dimension to take into account when dealing with pension fund investment problems is the impact of the presence of pension liabilities on the allocation strategy. This question has naturally raised substantial attention in academic research, where it has given rise to the emergence of two separate strands of the literature.

A first strand of the literature, mostly from the field of operations research, has focused on developing early comprehensive models of uncertainty in an asset-liability management (ALM) context, which have served a formal basis for the development of surplus optimization methodologies. These methodologies have been developed within the stochastic programming approach to ALM, with early attempts by Kallberg et al. (1982), Kusy and Ziemba (1986), or Mulvey and Vladimirou (1992). This strand of the literature is relatively close to industry practice, with one of the first successful commercial multistage stochastic programming applications appearing in the Russell-Yasuda Kasai Model (Cariño et al. (1994, 1998), Cariño and Ziemba (1998)).

Other successful commercial applications include the Towers Perrin-Tillinghast ALM system of Mulvey et al. (2000), the fixed-income portfolio management models of Zenios (1995) and Beltratti et al. (1999), and the InnoALM system of Geyer et al. (2008). A good number of applications in asset-liability management are provided in Ziemba and Mulvey (1998) and Ziemba (1998)).

In most cases, stochastic programming models require that the uncertainties be approximated by a scenario tree with a finite number of states of the world at each time. Important practical issues such as transaction costs, multiple state variables, market incompleteness due to uncertainty in liability streams that is not spanned by existing securities, taxes and trading limits, regulatory restrictions and corporate policy requirements can be handled within the stochastic programming framework. However, this additional flexibility comes at the cost of tractability. Analytical solutions are not possible, and stochastic programming models need to be solved via numerical optimization.

On the other hand, several researchers in finance have attempted in parallel to cast the ALM problem in a stylized continuous-time framework, and extend Merton's intertemporal selection analysis (see Merton (1969, 1971)) to account for the presence of liability constraints in the asset allocation policy. A first step in the application of optimal portfolio selection theory to the problem of pension funds has been taken by Merton (1990) himself, who studies the allocation decision of a university that manages an endowment fund. In a similar spirit, Boulier et al. (1995) have formulated a continuous-time dynamic programming model of pension fund management, which contains some of the basic elements for modeling dynamic pension fund behavior, and can be solved by means of analytical methods (see also Siegmann and Lucas (2002)). Rudolf and Ziemba (2004) extend these results to the case of a time-varying opportunity set, where state variables are interpreted as currency rates that affect the value of the pension's asset portfolio.

Also related is a paper by Sundaresan and Zapatero (1997), which is specifically aimed at asset allocation and retirement decisions in the case of a pension fund. More recently, Detemple and Rindisbacher (2008), Martellini and Milhau (2009), Martellini, Milhau and Tarelli (2012) and Deguest, Martellini and Milhau (2013) have analyzed pension fund investment strategies in a continuous-time setting and in the presence of short-term risk constraints.

This continuous-time stochastic control approach to ALM is appealing because it enjoys the desirable property of tractability and simplicity, allowing one to fully and explicitly understand the various mechanisms affecting the optimal allocation strategy. However, because of the simplicity of the modeling approach, such continuous-time models do not allow for a full and realistic account of uncertainty facing institutions in the context of ALM. In an attempt to circumvent the concern of the black-box flavor of stochastic programming models, while preserving some of the core insights obtained with the continuous-time models, it is feasible to test for the optimality of various rule-based strategies within the context of a comprehensive and realistic ALM model (see, for example, Mulvey et al. (2005)). As a result, these two approaches can be perceived more as complementing each other then competing with one another.

The most useful insight to be obtained from the continuous-time stochastic control approach to ALM is undoubtedly the existence of a *fund separation theorem*. This fund separation theorem in ALM provides formal justification for the liability-driven investing (LDI) approach to ALM, which has rapidly gained interest with pension funds, insurance companies and investment consultants alike. While they can vary significantly across providers, LDI solutions typically involve a focus on matching the risk exposures of the pension liabilities within the pension fund asset portfolio via dedicated fixed-income liability-hedging portfolios (LHPs), while keeping some assets free for investing in higher-yielding asset classes. In this context, surplus optimization appears as a somewhat inefficient investment approach which can be regarded as an attempt at *diversifying* liability risk away, while the proper way to manage liability risk is to *hedge* it away. On the other hand, if a dedicated LHP is added to the menu of risky assets within a stochastic programming surplus optimization model, then the insights obtained are broadly consistent with the LDI paradigm.

Introducing the basic LDI paradigm

The LDI paradigm prescribes that investors endowed with liability-driven objectives need to invest in two distinct portfolios, in addition to the risk-free asset, which can always be held long or short (in the latter case, we obtain a leveraged version of the LDI strategy): one well-diversified performance-seeking portfolio (PSP), needed to generate access to risk premia; and one LHP, needed to hedge against unpredictable changes in risk factors that impact pension liability values. The presence of these two portfolios reflects the dual focus in investment management, namely, on the one hand, the desire to generate performance in order to alleviate the burden on contributions, and on the other hand, the need for safety. Overall, the basic LDI paradigm is defined in terms of three main ingredients — a safe LHP, a risky PSP and an allocation to these two building blocks as well as the risk-free asset.

Designing meaningful liability-hedging portfolios

Risk diversification is only one possible form of risk management, focusing merely on achieving the best risk/return tradeoff regardless of investment objectives and constraints. However, it should be recognized that diversification is simply not the appropriate tool when it comes to protecting long-term liability needs. In particular, it is clear that the risk factors impacting pension liability values should be hedged rather than diversified away. Three of these factors stand out (namely, interest rate risk, credit risk and inflation risk).

Matching interest-rate-risk exposures of the liabilities

A first approach to interest rate-risk management for pension funds, known as cash-flow matching, involves ensuring a perfect static match between the cash flows from the asset portfolio and the pension commitments on the liability side. Let us assume, for example, that a pension fund has a commitment to pay out a monthly pension to a number of retirees. Leaving aside the complexity relating to the uncertain life expectancy of the retirees, the structure of the liabilities is defined simply as a series of cash outflows to be paid, the value of which is known today if we assume away inflation indexation. It is possible, in theory, to construct a portfolio of assets which will have future cash flows identical to this structure of pension liability commitments. To do so (assuming that securities of that kind exist on the market) would involve purchasing pure discount bonds with a maturity corresponding to the dates on which the monthly pension installments are paid out, with amounts that are proportional to the amount of the pension commitments.

This technique, which provides the advantage of simplicity and allows, in theory, for perfect risk management, nevertheless presents at least two main limitations from a practical perspective. First of all, it will generally prove impossible to find fixed-income instruments whose maturity dates correspond exactly to the dates of the pension payments. Moreover, most of those securities pay out coupons, thereby leading to the problem of reinvesting the coupons. To the extent that perfect matching is not possible, pension fund managers can resort to a technique called *immunization,* which allows the residual interest rate risk created by the imperfect match between the assets and liabilities to be managed in a dynamic way. Broadly speaking, the key difference is that immunization strategies aim at ensuring a match between factor exposures on the asset and liability sides, which is a weaker requirement than ensuring a match between cash-flow payments; in other words, cash-flow matching obviously implies interest rate exposure matching, while the opposite is not true.

While the most basic form of implementation of the immunization approach can be performed in terms of duration matching, the interest rate risk management technique extends to more general contexts, including, for example, hedging larger changes in interest rates (through the introduction of a convexity adjustment) or hedging against changes in the shape of the yield curve (see, for example, Fabozzi, Martellini and Priaulet (2005) for interest rate risk management in the presence of non-parallel yield curve shifts).

It should be noted that these approaches can be implemented either via cash instruments, typically sovereign bonds, or via derivatives, such as interest rates swaps or futures contracts. The use of derivatives can prove a useful way to implement a leveraged version of the strategy, whereby 100% of the assets of the pension plan can be dedicated to liability hedging while still leaving some non-zero access to the upside potential of risk assets.

Matching credit risk exposures of the liabilities

According to international accounting standards SFAS 87.44 and IAS 19.78, which recommend that pension obligations be valued on the basis of a discount rate equal to the market yield on AA bonds, the most straightforward way for pension funds to match liability payments is actually to build a portfolio of long-dated, investment-grade corporate bonds, as opposed to sovereign bonds. The fact that liabilities are discounted at the risk-free rate plus a credit spread implies that the regulatory value of pension liabilities is impacted by the presence of unexpected changes in credit spreads, in addition to unexpected changes in interest rates. It also implies that a portfolio of AA corporate bonds hedge liability risk better than sovereign bonds do, precisely because their yield includes a credit spread component that evolves in line with the discount rate on liabilities.[1] It should be emphasized, however, that pension funds need dedicated corporate bond portfolios, as opposed to off-the-shelf corporate bond indexes. Indeed, there is an inherent conflict of interest between issuers and investors about the duration of corporate bonds, which is known as the duration problem.

Each pension fund has a need for a specific time horizon equal to the duration of its pension liabilities, and there is no reason to expect that these needs correspond to the optimal financing plan of the issuers. For pension funds that have some fixed, typically long-term, liabilities originating from their defined benefit plans, long-term bonds are a much better hedge than short-term debt. On the other hand, the duration structure of outstanding bonds reflects the preferences of the issuers in their aim to minimize the cost of capital. Since the duration of the indexes is nonetheless a result of the sell side of corporate bonds, corporate bond indexes are typically not well suited to serving as benchmarks for liability-driven investors. More worrisome, perhaps, is that the characteristics of corporate bond indexes can change unpredictably over time (see Campani and Goltz (2011)). In this context, efforts are required to design stable corporate bond indexes optimized in an attempt not only to maximize their risk-adjusted performance but also to achieve a target duration and rating class composition over time.

Matching inflation (and expected inflation) risk exposures of the liabilities

Liabilities of defined benefit pension funds often include a clause of indexation on consumer prices or wage levels. The presence of uncertainty in future prices and wages raises the problem of finding appropriate hedging instruments for these risks. The theoretical cash-flow matching portfolio consists of a set of pure discount bonds with maturities and face values matching the liability commitments, and a principal repayment indexed on the reference index. However, this strategy is hard to set up in practice, for various reasons. First, as far as wage inflation is concerned, perfectly indexed financial securities may not exist. Second, for consumer price inflation, although inflation-indexed bonds are available, mainly issued by sovereign states, the market may not have the sufficient capacity to meet the demand from institutional investors. In this context, OTC derivatives (such as inflation swaps) can be used as substitutes for indexed bonds, but they involve a counterparty risk that not all institutional investors are willing to bear.

The practical difficulties encountered in the construction of a cash-flow matching portfolio motivate the search for asset classes with good inflation-hedging properties. A vast empirical literature has examined the inflation-hedging ability of traditional and alternative asset classes, but the results are mixed. Broad stock and bond indexes are not good inflation hedgers in the short run (see Hoevenaars et al. (2008) and Martellini and Milhau (2013)), and individual stocks have highly time-varying inflation betas, making their inflation-hedging properties difficult to forecast. Conclusions are more positive for commodities, which display good correlation with inflation at all horizons (see Hoevenaars et al. (2008), Amenc et al. (2009)).

However, the quest for inflation-hedging assets should not divert investors from the construction of an LHP. In fact, liability hedging and inflation hedging are two objectives that should not be confused in general, even for inflation-indexed liabilities. When liability maturity approaches zero, the discount factor is close to one, so the present value of promised payments almost coincides with the payment itself, and hedging liability risk and hedging realized inflation risk are two equivalent objectives. On the other hand, for a mature pension fund, with long-term commitments, liability maturity is long, so the value of the liabilities largely depends on the interest rate. Martellini and Milhau (2013) show that in this context, interest rate risk accounts for the largest part of total liability risk, a fraction which is growing in the maturity. The practical implication of this finding is that the LHP should not merely be an inflation-hedging portfolio, but should contain a substantial fraction of bonds, which are assets with easily measurable interest rate exposure. Failing to include them in the LHP leaves a large part of liability risk unhedged, which leads to high funding ratio volatility, and makes it more difficult to respect the short-term funding constraints imposed by many regulations and otherwise scrutinized by stakeholders of the pension plan (trustees, CFO of the sponsor company, etc.).

A further difficulty arises if one takes into account the difference between nominal and real interest rates. Indeed, the interest rate risk present in inflation-linked liabilities is real interest rate risk, while nominal bonds are by definition hedgers for nominal interest rate risk. This difference would not be a problem if the breakeven inflation rate — that is, the nominal rate minus the real rate — were constant, but the fact that it is time-varying implies that nominal bonds do not perfectly hedge real rate risk. In mathematical terms, the investor faces a duration mismatch problem: nominal bonds have non-zero duration with respect to expected inflation, while this duration is zero for indexed bonds (Fabozzi and Xu (2012)). The risk is thus to experience a rise in expected inflation with a constant real rate, which will lead to a bear market for nominal bonds, while liability value will remain constant. The potential result is a drop in the funding ratio.

As a conclusion, the construction of an LHP for inflation-linked liabilities in the absence of perfectly indexed securities is a non-trivial problem, which requires finding assets with satisfactory inflation-hedging properties, and mixing them with bonds to hedge interest rate risk.

Designing meaningful performance-seeking portfolios

The purpose of the LHP is to provide hedging with respect to changes in the liability value. If 100% of the portfolio is invested in the LHP, and if the LHP is a perfect hedging portfolio for the liabilities, then the current funding ratio will be locked forever, with no relative upside potential with respect to the value of the liabilities in the absence of additional contributions. Given the need to alleviate the required burden on contributions, it is in general desirable for pension funds to allocate a non-vanishing fraction of their assets to a PSP, in an attempt to benefit from risk premia on risky assets across financial markets.

Diversification (as opposed to *hedging*, which is used to manage liability risk, or *insurance*, which is used to manage short-term risk constraints) is the risk management technique that allows investors to efficiently extract long-term risk premia out of performance-seeking assets. Indeed, by holding well-diversified portfolios, investors may be able to eliminate or at least reduce *(diversify away)* unrewarded risk in their portfolios, which allows them to enjoy higher rewards per unit of risk, and therefore a higher average funding ratio at horizon for a given risk budget.

Portfolio diversification and the quest for efficient portfolios

While the benefits of diversification are intuitively clear, there is no straightforward definition of what exactly a well-diversified portfolio is. The most common intuitive explanation of *naïve* diversification is that it is the practice of not "putting all eggs in one basket." Having eggs (dollars) spread across many baskets is, however, a rather loose prescription. It should be noted, fortunately, that a fully unambiguous definition of scientific diversification has been provided by modern portfolio theory: more precisely, the prescription is that the PSP should be obtained as the result of a portfolio optimization procedure aiming to generate the highest risk-reward ratio.

Portfolio optimization is a straightforward procedure, at least in principle. In a mean-variance setting, for example, the prescription consists of generating a maximum Sharpe ratio (MSR) portfolio based on expected return, volatility and pair-wise correlation parameters for all assets to be included in the portfolio. One key issue is the presence of estimation risk in parameter estimates, which is particularly true for expected return parameters (see Merton (1980)). Once a set of input parameters is given, the optimization procedure can be handled analytically in the absence of portfolio constraints. More generally, it can be handled numerically in the presence of minimum and maximum weight constraints. Introducing weight constraints can actually be regarded as a way to reduce

[1] While this analysis suggests that corporate bonds are attractive additions to pension fund LHPs, it should be noted that corporate bonds are also useful within PSPs because of their diversification benefits with respect to other asset classes, and also because of the access they offer to the credit risk premium.

estimation risk (see, for example, Jagannathan and Ma (2003)). Expressing the MSR portfolio mathematically is useful because, in principle, it provides a straightforward expression for the optimal portfolio starting from a set of N risky assets. In the presence of a realistically large number N of securities, the curse of dimensionality, however, makes it practically impossible for investors to implement such direct one-step portfolio optimization decisions involving all individual components of the asset mix.

Portfolio diversification across and within asset classes

The standard alternative approach widely adopted in investment practice consists instead of first grouping individual securities in various asset classes, as well as sub-classes, according to various dimensions, e.g. country, sector and/or style within the equity universe, or country, maturity and credit rating within the bond universe, and subsequently generating the optimal portfolio through a two-stage process.

On the one hand, investable proxies are generated for MSR portfolios within each asset class in the investment universe. We call this step, which is typically delegated to professional money managers, the portfolio construction step. While market-cap indexes are natural default choices as asset class benchmarks, academic and industry research has offered convincing empirical evidence that these indexes tend to exhibit poor risk-adjusted performance, because of the presence of an excessive amount of unrewarded risk due to their extreme concentration in the largest cap securities in a given universe, as well as the absence of a well-managed set of exposures with respect to rewarded risk factors (for example, cap-weighted indexes have a natural large cap and growth bias, while academic research (e.g., the seminal work by Fama and French (1992)) has found that small cap and value were instead the positively rewarded biases). The combination of these empirical and theoretical developments has significantly weakened the case for market cap-weighted indexes (Amenc, Goltz and Le Sourd (2006)), and a consensus is slowly but surely emerging regarding the inadequacy of market cap-weighted indexes as efficient investment benchmarks. In this context, a new paradigm known as smart beta equity investing has been proposed, the emergence of which blurs the traditional clear-cut split between active vs. passive equity portfolio management (see, for example, Amenc et al. (2012).

After efficient benchmarks have been designed for various asset classes, these building blocks can be assembled in a second step, the asset allocation step, to build a well-designed multi-class PSP. Although the same methodologies can in principle be applied in both portfolio construction and asset allocation contexts, a number of key differences should be emphasized. In particular, the number of constituents (asset classes) is small in the asset allocation context, and the curse of dimensionality is not as problematic. In this context, attempts to capture time and state-dependencies in risk and return parameters using sophisticated covariance matrix estimates based on GARCH or Markov Regime Switching models are legitimate and likely to add value (e.g., Ang and Bekaert (2002)). On the other hand, such refinement does not necessarily improve the situation in portfolio construction contexts, in which the number of constituents is large, as they lead to an additional increase in the number of parameters to estimate.

Similarly, although it is not in general feasible to perform portfolio optimization with higher-order moments in a portfolio construction context, due to the dramatic increase in the number of parameters to estimate (co-skewness and co-kurtosis parameters, in addition to covariance parameters), it is feasible to go beyond mean-variance analysis in an asset allocation context, in which the number of constituents is limited (Martellini and Ziemann (2010)).

From asset allocation to risk allocation

It should be noted that an interesting new framework, known as risk-allocation framework, is currently gaining popularity among large sophisticated pension funds as a modern approach to the design of PSPs. This trend is related to the recognition, supported by recent research (e.g., Ang et al. (2009)), that risk and allocation decisions could be best expressed in terms of rewarded risk factors, as opposed to standard asset class decompositions, which can be somewhat arbitrary. For example, convertible bond returns are subject to equity risk, volatility risk, interest rate risk and credit risk. As a consequence, analyzing the optimal allocation to such hybrid securities as part of a broad bond portfolio is not likely to lead to particularly useful insights. Conversely, a seemingly well-diversified allocation to many asset classes that essentially load on the same risk factor (e.g., equity risk) can eventually generate a portfolio with very concentrated risk exposure. More generally, given that security and asset class returns can be explained by their exposure to pervasive systematic risk factors, looking through the asset class decomposition level to focus on the underlying factor decomposition level appears to be a perfectly legitimate approach, which is supported by standard asset pricing models such as the intertemporal CAPM (Merton (1973)) or the arbitrage pricing theory (Ross (1976)). Asset pricing theory also suggests that factors are (positively) rewarded if and only if they perform poorly during bad times, and more than compensate during good times in order to generate a positive excess return on average across all possible market conditions.

In academic jargon, the expected excess return on a factor is proportional to the negative of the factor covariance with the pricing kernel, given by marginal utility of consumption for a representative agent (see, for example, Cochrane (2000) for more details). Hence, if a factor generates an uncertain payoff that is uncorrelated to the pricing kernel, then the factor will earn no reward even though there is uncertainty involved in holding the payoff. On the other hand, if a factor payoff co-varies positively with the pricing kernel, it means that it tends to be high when marginal utility is high — that is, when economic agents are relatively poor. Because it serves as a hedge by providing income during bad times, when marginal utility of consumption is high, investors are actually willing to pay a premium for holding this payoff. Standard examples of such rewarded factors in the equity space are the "HML" (high minus low) or "value" factor (represented by a portfolio going long value stocks and short growth stocks) and the "SMB" (small minus big) or "size" factor (represented by a portfolio going long small cap stocks and short large cap stocks), which can be regarded as possible proxies for a "distress" factor (Fama and French (1992)).

In this context, one can argue that the ultimate goal of portfolio construction techniques is to invest in risky assets in order to ensure an efficient diversification of specific and systematic risks within the portfolio. Note that the word "diversification" is used with two different meanings. When the focus is on the diversification of specific risks, "diversification" means *reduction* of specific risk exposures, which are not desirable because they are not rewarded. On the other hand, when the focus is on the diversification of systematic risks, "diversification" means *efficient allocation* to factors that bear a positive long-term reward, with modern portfolio theory suggesting that efficient allocation is in fact maximum risk-reward allocation (maximum Sharpe ratio in a mean-variance context).

If the whole focus of portfolio construction is ultimately to harvest risk premia that can be expected from holding an exposure to rewarded factors, it seems natural indeed to express the allocation decision in terms of such risk factors. In this context, the term "factor allocation" is a new paradigm advocating that investment decisions should usefully be cast in terms of risk factor allocation decisions, as opposed to asset class allocation decisions, which are based on somewhat arbitrary classifications. A second interpretation for what the *risk allocation* paradigm might mean is to precisely define it as a portfolio construction technique that can be used to estimate what an efficient allocation to underlying components (which could be asset classes or underlying risk factors) should be. The starting point for this novel approach to portfolio construction is the recognition that a heavily concentrated set of risk exposures can be hidden behind a seemingly well-diversified allocation. In this context, the risk allocation approach to portfolio construction, also known as the risk budgeting approach, consists in advocating a focus on risk, as opposed to dollar, allocation. In a nutshell, the goal of the risk allocation methodology is to ensure that the contribution of each constituent to the overall risk of the portfolio is equal to a target risk budget. In the specific case when the allocated risk budget is identical for all constituents of the portfolio, the

It should be noted in closing that the existence of uncorrelated long-short factor-replicating portfolios is not a necessary condition to perform risk budgeting, which is fortunate since such uncorrelated pure factors are hardly investable in practice.

strategy is known as *risk parity*, which stands in contrast to an equally-weighted strategy that would recommend an equal contribution in terms of dollar budgets (see Roncalli (2013) for further details). To better understand the connection between this portfolio construction technique and standard recommendations from modern portfolio selection techniques, it is useful to recognise that, when applied to uncorrelated factors, risk budgeting is consistent with mean-variance portfolio optimization under the assumption that Sharpe ratios are proportional to risk budgets. Thus, risk parity is a specific case of risk budgeting, a natural neutral starting point that is consistent for uncorrelated factors with Sharpe ratio optimization assuming constant Sharpe ratios at the factor level.

Overall, it appears that risk allocation can be thought of as *both* a new investment paradigm advocating a focus on allocating to uncorrelated rewarded risk factors, as opposed to correlated asset classes, and a portfolio construction technique stipulating how to optimally allocate to these risk factors (see Amenc and Martellini (2014) for a discussion). It should be noted in closing that the existence of uncorrelated long-short factor-replicating portfolios is not a necessary condition to perform risk budgeting, which is fortunate since such uncorrelated pure factors are hardly investable in practice. Indeed, one can use any set of well-diversified portfolios as constituents, as opposed to factor-replicating portfolios, thus leaving the hurdle to reach target factor exposures until the asset allocation stage. For example, Amenc, Deguest and Martellini (2013) use long-only factor-tilted smart beta benchmarks as constituents and choose the allocation to these constituents in order to ensure that the contribution of standard rewarded equity factors to the tracking error of the portfolio with respect to the cap-weighted benchmark are all equal.

In the end, two main benefits can be expected from shifting to a representation expressed in terms of risk factors, as opposed to asset classes. On the one hand, allocating to risk factors may provide a cheaper, as well as more liquid and transparent, access to underlying sources of returns in markets where the value added by existing active investment vehicles has been put in question. For example, Ang et al. (2009) argue in favor of replicating mutual fund returns with suitably designed portfolios of factor exposures such as the value, small cap and momentum factors. On the other hand, allocating to risk factors should provide a better risk management mechanism, in that it allows investors to achieve an ex-ante control of the factor exposure of their portfolios, as opposed to merely relying on ex-post measures of such exposures. Therefore, while working at the level of underlying risk factors that impact/explain the returns on all asset classes appears to be an intuitively meaningful approach, the practical challenges of this paradigm shift for the organization of the asset allocation processes are substantial, and more work is needed to turn this paradigm into a fully operational approach that can be used by pension funds to construct their PSP.

Designing meaningful allocation to PSP and LHP portfolios

Once the PSP and LHP have been carefully designed, using some of the aforementioned portfolio construction techniques, the next step is to determine what percentage of the pension fund assets should be allocated to each one of these building blocks. Portfolio theory provides useful guidance with respect to the question of the optimal allocation to the safe LHP vs. risky PSP building blocks. In the fund separation theorem in asset pricing theory, the allocation to the "risky" building block appears to be an increasing function of the PSP Sharpe ratio and a decreasing function of the PSP volatility, as intuitively expected. The allocation to the "safe" building block, on the other hand, is an increasing function of the beta of the liability portfolio with respect to the LHP. If there is an asset portfolio that perfectly matches the liability portfolio, then the beta is 1.

Overall, the allocation to the performance-seeking portfolio is not only a function of objective parameters, the PSP volatility and Sharpe ratio and the beta of the LHP with respect to the liabilities, but also a function of one subjective parameter, the investor's risk aversion. As expected, the optimal allocation to the PSP should be inversely proportional to the investor's risk aversion. If risk aversion rises to infinity, the investor only holds the safe LHP. This is consistent with the intuition that for an investor facing liability commitments, the LHP, as opposed to cash, is the true risk-free asset. On the other hand, cash is still used to make the balance (if non-zero) between the total amount invested in the PSP and LHP building blocks and the pension fund assets.

In the case that a short position in cash is obtained, then the pension fund is implementing a leveraged version of the LDI strategy. It is important to emphasize that the risk aversion parameter is not observable, and not even well-defined for institutional investors; it should, in fact, be treated as a *degree of freedom*, the value of which can be inferred from a given risk budget, often expressed in terms of expected shortfall with respect to liabilities. •

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The chair examines advanced ALM topics such as dynamic allocation strategies, rational pricing of liability schemes and formulation of an ALM model integrating the financial circumstances of pension plan sponsors.

The full version of the research is available on the EDHEC-Risk Institute website at the following address: http://www.edhec-risk.com/ALM/BNPPAM_Research_Chair

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PORTFOLIO MANAGEMENT

From Isolated to Integrated Liability-Driven Investing

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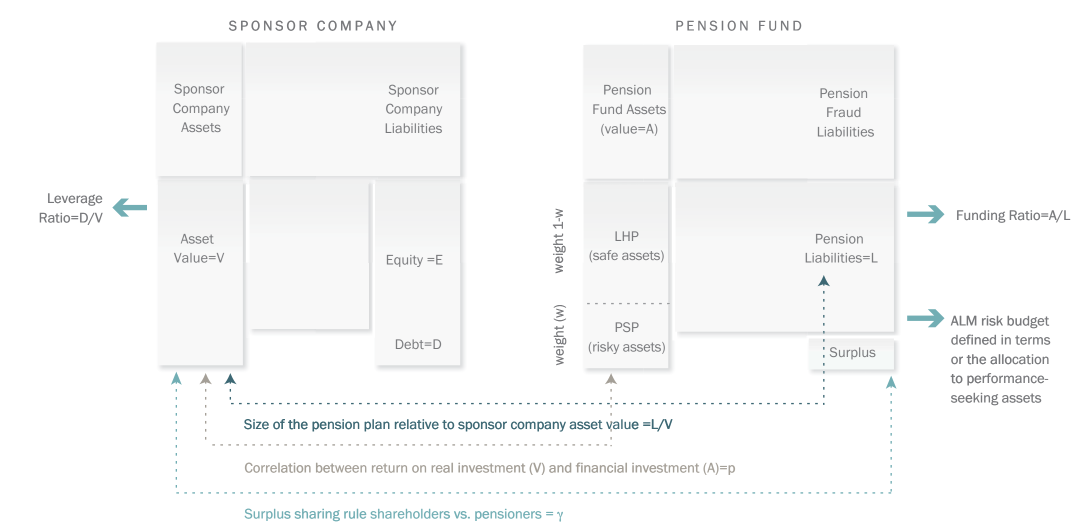
EXHIBIT 1

An integrated ALM framework



To focus solely on pension funds for optimal liability-driven investing (LDI) is a severe simplification, in particular for corporate pension plans, where it is expected that the presence of the sponsor company can help solve underfunding problems through additional contributions if and when needed. Conversely, one of the main risks for plan participants, actually the only source of uncertainty for a defined benefit plan with unconditional liability payments, is that of sponsor bankruptcy when the pension plan is underfunded. In an attempt to address this concern, a number of dramatic changes have occurred over the past few decades in the legal, regulatory, accounting and fiscal environments of corporate pension funds. These changes have collectively led to significantly heightened scrutiny over the valuation of pension liabilities, with a focus on greater transparency in terms of the impact of both the market and credit risk components on pension obligation values.

Correctly assessing the value of a pension plan in deficit with a weak sponsor company remains, however, a real challenge, given that no comprehensive model is currently available for the joint quantitative analysis of capital structure choices, pension fund allocation decisions and their impact on rational pricing of liability streams. In fact, international accounting standards SFAS 87.44 and IAS 19.78 recommend that pension obligations be valued on the basis of a discount rate equal to the market yield on AA corporate bonds, the same rate for all firms. While the use of a market rate is arguably progress with respect to using a constant rate (including a credit spread component or not) independently of market conditions, the use of the same market rate to discount all pension liabilities regardless of the sponsor credit rating, pension funding situations and asset allocation policy is not likely to lead to a correct assessment by the various stakeholders of the impact of specific default risk on the value of pension obligations.

This question can be addressed by recognizing that pension liabilities can be regarded as defaultable claims issued by the sponsor company to workers and pensioners in the context of an integrated model of capital structure (Martellini and Milhau (2010b)). The analysis focuses on the interaction between the allocation decisions of the pension plan and the valuation of these liabilities, thereby extending the capital structure literature and the defaultable bond pricing literature to account for the presence of a pension plan. Such a model is a stylized representation of the relationships between various stakeholders of a company with a pension plan, notably including shareholders of the sponsor company, bondholders and beneficiaries of the pension fund (workers and pensioners).

The model can be summarized as follows. The sponsor company issues a debt with face value D, and also issues pension claims, perceived as a collateralized form of debt held by workers and pensioners with face value L. The initial capital of the firm is allocated to funding investment projects (company asset value denoted by V) and to funding the pension plan (pension asset value denoted by A). The pension fund allocates a fraction ω of the initial endowment to some performance-seeking portfolio (PSP) and a fraction 1-ω to some liability-hedging portfolio (LHP). In the event that the assets of the pension fund A are insufficient to deliver the promised pension payment L, the sponsor makes a contribution equal to the deficit L-A. If the sponsor is unable to make this contribution, default is triggered. If the pension fund enjoys a surplus, equity holders receive a fraction γ of this surplus, which can be used to pay back bondholders. If debt cannot be fully repaid, bankruptcy is also triggered. When default has not been triggered, equity holders are left with the remaining assets of the pension fund and the sponsor, plus their access to surpluses. Otherwise, they receive nothing. One can also incorporate tax effects, bankruptcy costs, as well as contributions triggered by the presence of regulatory funding ratio constraints.

Under standard assumptions regarding the dynamics followed by all variables of interest, including the return on the performance-seeking portfolio and the return on the real assets held by the firm, one can use option pricing theory to find the rational value of the claims held by all stakeholders, and also analyze the impact on the value of these claims of funding and leverage decisions at the sponsor company level, as well as asset allocation decisions at the pension fund level. The main ingredients of the model are the size of the pension fund relative to the assets of the sponsor company (L/V), the relative size of the pension assets with respect to the pension liabilities (also known as the funding ratio, A/L), and the relative size of the outstanding debt of the sponsor company relative to the assets of the sponsor company (a.k.a. the leverage ratio, D/V). Other important parameters are those that define the allocation strategy of the pension fund, as well as the correlation between the return on pension assets and the return on the sponsor company assets (see Exhibit 1).[2]

These findings have two main kinds of implications — macro implications on the one hand, with a number of possible policy recommendations for pension fund regulators, and micro implications on the other hand, with a number of strategy recommendations for pension fund managers. Focusing on the latter dimension, we analyze in what follows how investment decisions at the pension level impact stakeholders' wealth.

When the correlation between the value of the firm process and the stock index return process is positive, we find that the fair value of promised payments to bondholders and pensioners is a decreasing function of the allocation to risky assets by the pension fund. This is a clear case of asset substitution, since a higher allocation to risky assets leads to an increase in the total riskiness of the total assets held by the firm (financial assets held off the balance sheet through the pension funds and real assets directly held on the balance sheet), which is the underlying state variable on which the value of such claims is based.

When the correlation is negative, however, a higher allocation to risky assets may induce diversification benefits. This competition between the asset substitution effect and the diversification effect, which has never been analyzed in the related literature, leads to an interior optimal solution with respect to maximizing total firm value (and also with respect to maximizing pensioners' value), at least for reasonably low funding ratios.

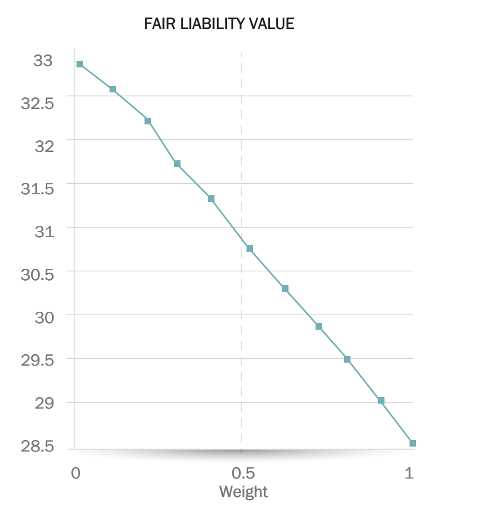
Overall, there is in general clear evidence of conflicts of interest between the various stakeholders, and in particular between shareholders and pensioners. Assuming they do not have access to any surplus of the pension fund, risk-taking is detrimental from the pensioners' perspective, because it involves increasing the likelihood of partial recovery of pension claims, while risk-taking allows shareholders to reduce the burden on contributions needed to meet expected pension payments due to exposure to the upside potential of the performance-seeking assets (see Exhibit 2).

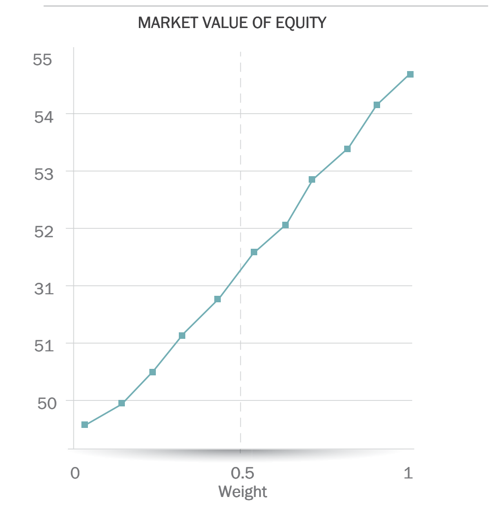
These conflicts of interest could be mitigated by granting pensioners some partial access to the surplus (see conditional indexation rules in the Netherlands), thereby allowing plan beneficiaries to benefit from the increases in expected performance related to more aggressive investment strategies. More generally, our results have implications in terms of the optimal design of pension plans, since they advocate the emergence of more subtle surplus sharing rules, which could include, for example, the use of hybrid retirement plans, and/or the use of contribution holidays for defined benefit

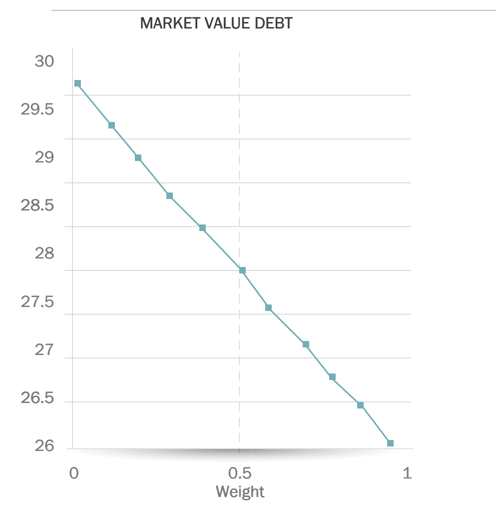
[2] *Institutional elements such as those governing the surplus sharing rule, the tax rate and bankruptcy costs will also have an impact on the numerical results.*

EXHIBIT 2

Impact of allocation decisions on stakeholders' welfare.
These figures perform comparative static analysis with respect to the allocation to the risky asset, when the firm is positively correlated with the market. The pension fund is fully funded in the regulatory sense at the initial date.







plans, that would allow equity holders to reduce the burden of contributions while protecting the interests of pensioners.

We also find that an effective way to align the incentives of shareholders and pensioners without any complex adjustment to the pension plan structure consists of enlarging the set of admissible investment strategies to include dynamic risk-controlled strategies such as constant-proportion portfolio insurance (CPPI) strategies, or their extension in a pension management context, sometimes referred to as contingent immunization strategies or dynamic LDI strategies. In fact, implementing risk-controlled strategies aiming to ensure a minimum funding ratio level above 100% allows shareholders to get some (limited) access to the upside performance of risky assets, while ensuring that pensioners will not be hurt by the induced increase in risk (see Exhibit 2).

Accounting for long horizons via the life-cycle investing paradigm: from policy portfolios to policy portfolio strategies

One important element of pension investing is the presence of long horizons. Indeed, except for very mature pension plans close to termination, the duration of pension liabilities tends to be long, sometimes exceeding 20 or 30 years. It is therefore important to analyze what the impact is, if any, of the presence of such long horizons on LDI.

Although it may be acceptable to assume a constant opportunity set when investors have a short-term horizon, the presence of a long horizon makes it necessary to go beyond Markowitz's static portfolio selection analysis. The next important step after Markowitz (1952) is Merton (1969, 1971), who introduced dynamic programming techniques in a continuous-time setting to solve dynamic portfolio optimization problems. In terms of industry implications, the development of dynamic asset pricing theory has led to the emergence of improved investment solutions that take into account the changing nature of investment opportunities, which we discuss in what follows.

Benefits of fixed-mix strategies vs. buy-and-hold strategies

Investors with consumption/liability objectives need to invest in two distinct portfolios, in addition to cash: one PSP and one LHP, with an optimal allocation strategy as follows:

$$\max_{(w_t)} E\left[u\left(\frac{A_T}{L_T}\right)\right] \Rightarrow W_t^* = \frac{\lambda}{\gamma\sigma} W^{PSP} + \left(1-\frac{1}{\gamma}\right)\beta W^{LHP} \quad (1)$$

Formally, under the assumption of a constant opportunity set, we obtain, from the above, the following expression for the fund separation theorem in the intertemporal context when trading is possible between current date and investment horizon:

$$\text{Fixed - mix LDI strategy:} \quad W = \frac{\lambda}{\gamma\sigma} W^{PSP} + \left(1-\frac{1}{\gamma}\right)\beta W^{LHP} \quad (2)$$

where w is the vector of weights allocated to the risky assets, w^{PSP} is the vector of weights of the PSP, w^{LHP} is that of the LHP, and λ and σ are respectively the Sharpe ratio and the volatility of the PSP, β is the beta of liabilities with respect to the LHP, and γ is the relative risk aversion. Note that in this expression, parameters λ, σ and β are taken constant.

It is important to emphasize that Equation (1) above is already the solution to a *dynamic* optimization problem (note the explicit dependency on time that has been introduced in the expression of the optimal strategy on the left-hand side of Equation (1)) where the corresponding strategy is a fixed-mix strategy with trading taking place periodically to rebalance the portfolio allocation back to the constant target.

This stands in contrast with buy-and-hold strategies, where an initial trade is performed in order to implement a given policy portfolio, which is left unbalanced so that the current portfolio weights progressively drift away from the target, as expressed in the following equation:

$$\text{Buy - and - hold LDI strategy:} \quad W = \frac{\lambda}{\gamma\sigma} W^{PSP} + \left(1-\frac{1}{\gamma}\right)\beta W^{LHP}$$

In fact, fixed-mix strategies are superior in general to buy-and-hold strategies, and the gains obtained from rebalancing can be very substantial for long horizons. To illustrate the superiority of fixed-mix strategies with respect to buy-and-hold strategies, we simulate the long-term performance of a hypothetical pension fund allocating an initial amount normalised at $100 to a stock index and a bond index. More precisely, we run 10,000 Monte Carlo simulations using a general model with stochastic equity Sharpe ratio, volatility and interest rates. The parameters used to generate the sample paths are taken from Deguest, Martellini and Milhau (2013) and recalled in Exhibit 3.[3]

EXHIBIT 3

Long-term parameter values.
This table displays the parameter values used in the numerical experiments. For each stochastic process (nominal short-term rate, equity Sharpe ratio and equity variance), the dynamics is recalled: the three processes z^r, z^λ *and* z^v are three correlated Brownian motions. The last panel contains the values of the instantaneous correlations between the three stochastic processes and the stock price process.

Nominal Short-Term Rate $dr_t = a(b - r_t)dt + \sigma_r dz_t^r$	
b	3.06 %
a	0.13
σ_r	0.98%
λ_r	-53.00%
Equity Sharpe Ratio $d\lambda_t = \kappa(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda dz_t^\lambda$	
κ	0.35
σ_λ	23.22%
$\bar{\lambda}$	40%
Equity Variance $dV_t = \alpha(\bar{V} - V_t)dt + \sigma_V \sqrt{V_t} dz_t^V$	
α	5.07
σ_V	48.00%
$\sqrt{\bar{V}}$	21.38%
Correlations	
$\rho_{S\lambda}$	−1
ρ_{Sr}	0
ρ_{SV}	−76.70%
ρ_{rV}	0

[3] Note that our simulated portfolios are rebalanced on a quarterly basis, and to robustify the numerical experiments, we set lower and upper bounds at 0.25 and 0.75, respectively, for the simulated equity Sharpe ratio and 0.05 and 0.65, respectively, for volatility before implementing dynamic allocation strategies.

We first assume that the pension fund manager incorrectly infers that risk and return parameters are constant, and implements either the buy-and-hold LDI strategy or the fixed-mix LDI strategy. We subsequently turn to a situation where the pension fund manager properly recognises that risk and return parameters vary through time and benefits from time-varying policy portfolio strategies.

In this numerical illustration, we take the time horizon to be equal to 20 years, while the risk aversion parameter, which is not observable, is calibrated in such a way that the average allocation to equity for the optimal dynamic portfolio strategy over the 20-year life of the strategy is equal to a target of 10%, 20% or 30%. The three corresponding strategies are referred to as defensive, moderate and aggressive, respectively.

In Exhibit 4, we show the distribution of wealth levels obtained at horizon for 3 pension funds, corresponding to the defensive, moderate and aggressive risk aversion levels, following a buy-and-hold versus a fixed-mix strategy.

We first find the usual risk-return trade-off for both the buy-and-hold and fixed-mix strategies: strategies implemented by less risk-averse investors will contain a higher allocation to equities, which will result in a higher average wealth level as well as a higher uncertainty around the terminal wealth level.

The analysis of the results in Exhibit 4 also shows a very different wealth distribution at horizon for the buy-and-hold strategy and for the fixed-mix strategy. Broadly speaking, it appears that fixed-mix strategies lead to much narrower distributions for average levels that are somewhat equivalent to those generated by buy-and-hold strategies. For example, for the pension fund following a so-called aggressive strategy, the average wealth level at a 20-year horizon is $348.18 for the fixed-mix strategy, while it reaches $357.61 for the buy-and-hold strategy, which represents a mere 2.87% higher value. On the other hand, the standard deviation of the wealth level distribution is only $19.99 for the fixed-mix strategy, while it is more than twice as high at $53.24 for the buy-and-hold strategy!

From this analysis, one can expect that the welfare gains from implementing fixed-mix strategies as opposed by buy-and-hold strategies would be substantial for risk-averse investors. It should be noted that this conclusion, which has been obtained in the case of a very simple experiment with no frictions, would also hold true if realistic levels of transaction costs were introduced.

Benefits of time-varying strategies versus fixed-mixed strategies

A large body of empirical research has shown that interest and inflation rates, as well as expected return, volatility and correlation parameters, are stochastically time-varying, as a function of key state variables that describe the state of the business cycle. Unexpected changes in these variables have an impact on portfolio risk and performance (through changes in interest rates and risk premium process parameters), which should be managed optimally. We now analyze how the allocation strategy is optimally impacted by the explicit recognition of the presence of time-variation in expected return and volatility for the risky assets entering the composition of the PSP.

One can show that the explicit recognition of the presence of time-variation in risk and return parameters has two main implications for dynamic asset allocation problems. On the one hand, the presence of time-variation of risk and return parameters implies that the target split between the PSP and LHP, as well as the composition of these two building blocks, become explicit functions of time. In other words, Equation (1) is replaced with the following so-called myopic strategy:

$$\text{Myopic dynamic LDI strategy:}\quad w_t = \frac{\lambda_t}{\gamma\sigma_t} w_t^{PSP} + \left(1 - \frac{1}{\gamma}\right) B_t w_t^{LHP}$$

It turns out that, in general, this time-varying strategy is not the optimal strategy. Indeed, in the presence of stochastic equity volatility and an equity risk premium, the optimal strategy involves the introduction of new hedging demand, in addition to the LHP, dedicated to the optimal hedging of the state variables impacting the equity risk premium process, as can be seen from Equation (3)[4]:

$$\begin{aligned} w_t^* &= \frac{\lambda_t}{\gamma\sigma_t} w_t^{PSP} + \beta_t\left(1 - \frac{1}{\gamma}\right) w_t^{LHP} + HD_{t,T}^{\lambda} \\ &= \underbrace{\frac{\bar{\lambda}}{\gamma\bar{\sigma}} \bar{w}^{PSP}}_{\text{long-term strategic portfolio}} + \underbrace{\frac{\lambda_t}{\gamma\sigma_t} w_t^{PSP} - \frac{\bar{\lambda}}{\gamma\bar{\sigma}} \bar{w}^{PSP}}_{\text{short-term (tactical) deviations}} + \underbrace{\beta_t\left(1 - \frac{1}{\gamma}\right) w_t^{LHP}}_{\text{hedging against unexpected changes in liability values}} + \underbrace{HD_{t,T}^{\lambda}}_{\text{hedging against unexpected changes in risk premia values}} \end{aligned} \tag{3}$$

See Deguest, Martellini and Milhau (2013) for the proof and the detailed expression of the hedging demand $HD_{t,T}^{\lambda}$.

The hedging demand against unexpected changes in the equity risk premium is the portfolio of risky assets that has the highest squared correlation with changes in the equity risk premium. The design of that portfolio is a matter of empirical calibration. It is widely accepted that an increase (respectively, decrease) in realized returns on equities implies a corresponding decrease (respectively, increase) in expected return of equities, given that everything else equal equity prices have become more (respectively, less) expensive following the increase (respectively, decrease) in stock prices. In this context, it is hardly surprising that empirical research has found a strong negative correlation between expected returns on stocks and realized returns on stocks (Kim and Omberg (1996)). As a result, the intertemporal hedging demand against changes in the equity risk premium mostly contains a long equity position. More generally, a dedicated hedging demand is needed for each variable impacting the interest rate or risk premium process of any of the risky asset classes (Detemple and Rindisbacher (2008)). For example, let us assume that the expected return on most risky assets is negatively impacted by increases in oil prices. To compensate for the deterioration of the investment opportunity set in the event of a sharp increase in oil prices, the investor will benefit from holding a long position in a portfolio optimized to exhibit the highest possible correlation with oil prices.

To provide a numerical analysis of the benefits of time-varying strategies over fixed-mix strategies, we test the following strategies, introduced in increasing order of complexity:

1. Strategies based on time changes in equity volatility only, assuming a constant equity risk premium — such strategies are often referred to as *target volatility strategies* and defined by the following expression[5]:

$$\text{Target volatility dynamic LDI strategy:}\quad w_t = \frac{\lambda}{\gamma\sigma_t} w_t^{PSP} + \left(1 - \frac{1}{\gamma}\right) \beta_t w_t^{LHP}$$

2. Strategies based on time-changes in equity volatility and equity risk premium, without the introduction of a dedicated hedging demand - such strategies are referred to as *myopic dynamic strategies* and defined by the following expression:

$$\text{Myopic dynamic LDI strategy:}\quad w_t = \frac{\lambda}{\gamma\sigma_t} w_t^{PSP} + \left(1 - \frac{1}{\gamma}\right) \beta_t w_t^{LHP}$$

3. Strategies based on time-changes in equity volatility and equity risk premium, with the introduction of a dedicated hedging demand; - such strategies are referred to as *optimal dynamic strategies* and defined by the following expression:

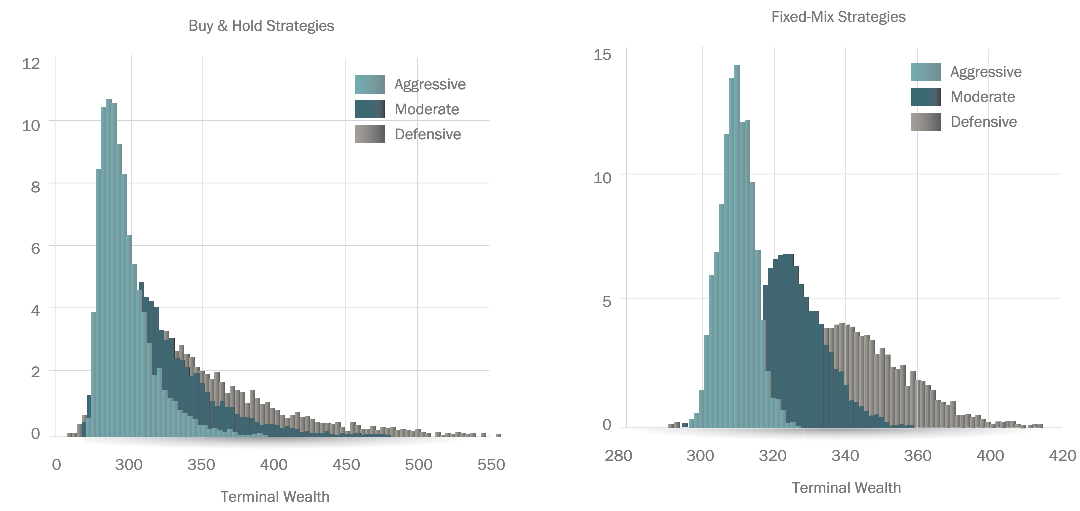
$$\text{Optimal dynamic LDI strategies:}\quad w_t^* = \frac{\lambda_t}{\gamma\sigma_t} w_t^{PSP} + \beta_t\left(1 - \frac{1}{\gamma}\right) w_t^{LHP} + HD_{t,T}^{\lambda}$$

[4] In principle one could also allow for the presence of time variation in the investor's risk aversion.

[5] In all these strategies, the beta of the liability portfolio with respect to the LHP is assumed to be time-varying. In practice, interest rate risk is the main driving force for the liability returns, and this beta often corresponds to the portfolio duration, which is indeed dynamically managed by internal or external fixed-income money managers.

EXHIBIT 4

Distributions of terminal wealth for buy and hold and fixed-mix strategies.



We show in Exhibit 5 the resulting distribution of terminal wealth for various risk aversion levels. Comparing Exhibit 5 to Exhibit 4, it appears that the benefits obtained from implementing dynamic portfolio strategies are spectacular either in terms of increases in average wealth levels or in terms of decreases in uncertainty around these average wealth levels, or both.

This can be seen in Exhibit 6, where we show the average and standard deviation for the distribution of wealth levels at horizon for the three considered levels of risk aversion and for the various LDI strategies, including the buy-and-hold (B&H) LDI strategy, the fixed-mix (FM) LDI strategy, the target volatility (TV) dynamic LDI strategy, the myopic dynamic (MD) LDI strategy and the optimal dynamic LDI strategy.

In Exhibit 6, we also report the opportunity cost for each tested strategy with respect to the optimal dynamic LDI strategy. This *opportunity cost* is measured as the percentage of initial wealth necessary for sub-optimal strategies to reach the same average terminal wealth as the optimal strategy for the same standard deviation as the optimal strategy[6]. Conversely, we also report the opportunity gain for each tested strategy with respect to the buy-and-hold LDI strategy. This opportunity gain is measured as the percentage of initial wealth that can be saved by an investor willing to implement a better strategy so as to reach the same terminal wealth level as the buy-and-hold strategy for the same standard deviation as the buy-and-hold strategy.

The opportunity gains obtained from shifting to increasingly sophisticated strategies are substantial. For example, we learn that a pension fund wishing to implement an aggressive strategy can save up to 16.44% with a fixed-mix policy portfolio strategy compared to the corresponding buy-and-hold policy portfolio. The opportunity gain is then multiplied by a factor of almost two, to reach 32.73%, if the pension fund manager is willing to utilize information about time-varying volatility levels. This result suggests that adjusting the policy mix as a function of changes in equity volatility is a valuable improvement over implementing a simple fixed-mix strategy.

Such an improvement in policy portfolio implementation does not really require any additional skills, given that volatility has been made quasi-observable with the existence of volatility indexes such as the VIX index, and can be safely estimated using various proven econometric techniques, such as GARCH models, for example. On the other hand, adjusting the strategy in a myopic way as a response to changes in the equity risk premium appears to only generate modest additional benefits, with an opportunity gain that increases in the aggressive case from 32.73% to 34.34%; this is an interesting result, given that changes in the equity risk premium, which is not observable, are much harder to track compared to changes in equity volatility. Intuitively, this result can be related to the fact that the volatility of changes in equity volatility is more than twice as high as that of changes in the equity risk premium, given the assumed parameter values that are reported in Exhibit 3. On the other hand, it appears that a substantial welfare gain is generated when the intertemporal hedging demand is accounted for in the optimal strategy, with an opportunity gain increasing from 34.34% to 42.62%.

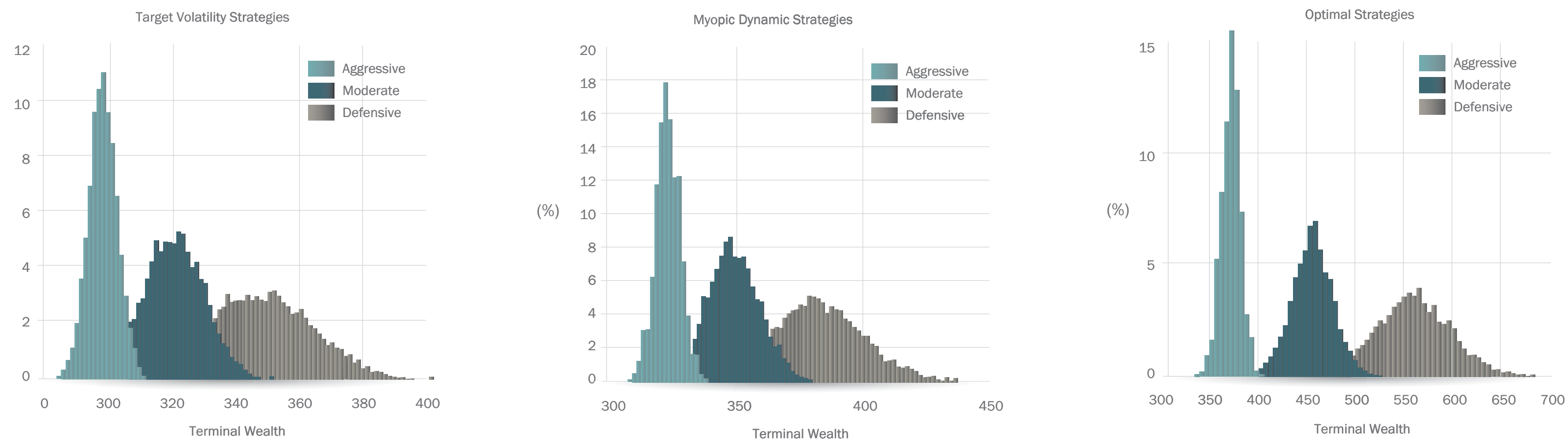
In the end, the loss for an investor sticking to a standard buy-and-hold strategy reaches more than 50% (more precisely 55.59%), given the assumed parameter values compared to a more sophisticated investor who would implement the optimal dynamic LDI strategies. The opportunity gains/losses are naturally smaller for less aggressive investors, and would converge to zero in the limit of an infinite risk aversion, since all strategies would then converge toward a 100% allocation to the LHP. On the other hand, they remain substantial for realistically high levels of risk aversion, with opportunity gains as high as 35.83% for the moderate investor and 24.57% for the defensive investor. While these results again have been obtained in the absence of transaction costs and other forms of frictions, the magnitude of the welfare gains involved in dynamic vs. static strategies are so substantial that they are expected to be of economic significance even after accounting for implementation constraints.

From deterministic to stochastic glidepaths: improved forms of target-date funds

One of the key insights from the previous analysis of long-term allocation decisions with mean-reverting equity returns is the fact that equities serve as a hedge against unfavorable equity returns in the presence of mean-reverting equity returns. As a result, the optimal allocation to stocks is

EXHIBIT 5

Distributions of dynamic optimal and sub-optimal LDI strategies.



[6] In detail, we mix the sub-optimal strategy with a long or short allocation to the 20-year pure discount bond so as to match the standard deviation for the distribution of the terminal wealth level for a given sub-optimal strategy and for the optimal strategy. We then record differences in average wealth levels, and report the percentage of initial contribution needed as an additional contribution to generate with the sub-optimal strategy mixed with the pure discount bond the same average wealth level as with the optimal strategy.

EXHIBIT 6

Statistics and risk measures for long-term LDI strategies.
This table displays over a 20-year horizon the average terminal wealth together with its standard deviation for five long-term allocation strategies, and three different levels of riskiness: aggressive, moderate and defensive. It also shows the maximum drawdown at a 99.5% level (0.5% of the worst max DD have been discarded).

Panel A: Aggressive Strategy	Optimal	MD	TV	FM	B&H
Av. Wealth	530.87	376.62	355.09	348.18	357.61
Std of Wealth	38.02	16.64	12.63	19.99	53.24
Max DD at 99.5%	23.45%	15.87%	15.94%	16.42%	19.41%
Opportunity Cost	0	11.83%	14.07%	35.62%	55.59%
Opportunity Gain	42.62%	34.34%	32.73%	16.44%	0

Panel B: Moderate Strategy	Optimal	MD	TV	FM	B&H
Av. Wealth	438.49	346.29	333.67	329.61	336.09
Std of Wealth	20.59	9.66	7.44	11.85	33.28
Max DD at 99.5%	19.06%	16.13%	16.11%	16.33%	16.93%
Opportunity Cost	0	10.07%	11.63%	24.84%	36.01%
Opportunity Gain	35.83%	26.77%	25.30%	12.18%	0

Panel C: Defensive Strategy	Optimal	MD	TV	FM	B&H
Av. Wealth	365.66	322.22	316.34	314.44	317.88
Std of Wealth	8.84	4.46	3.48	5.58	16.38
Max DD at 99.5%	16.51%	16.45%	16.59%	16.81%	16.72%
Opportunity Cost	0	6.35%	7.17%	13.29%	18.05%
Opportunity Gain	24.57%	16.56%	15.49%	7.05%	0

higher compared to the myopic case, and investors with longer time horizons hold more stocks compared to investors with shorter horizons. This prescription has very often been taken at face value by target-date funds or life-cycle funds, an investment solution advocating a deterministic decrease of equity allocations (glidepath) when approaching retirement date. For example, a popular asset allocation strategy for managing equity risk on behalf of a private investor in the context of a defined contribution pension plan is known as deterministic life investing. In the early stages, when the retirement date is far away, the contributions are invested entirely in equities. Then, beginning on a predetermined date (say, 10 years) before retirement, the assets are switched gradually to bonds at some predefined rate (say, 10% a year). By the date of retirement, all the assets are held in bonds.[7] This is somewhat reminiscent of the rule of thumb put forward by Shiller (2005), advocating a percentage allocation to equity given by 100 minus the investor's age in years.

Embedding the life-cycle allocation decisions within a one-stop decision is a valuable attempt at providing added value to unsophisticated investors who otherwise will likely make sub-optimal decisions. However, current forms of target-date funds are not the right answer to the problem at hand. An initial limitation with (most) existing target-date funds is that they do not account for risk aversion. In other words, lifestyle investing funds (balanced funds) focusing purely on differences in risk aversion across investors, have been replaced by life-cycle funds (target-date funds) focusing purely on differences in time horizon across investors.

Obviously, one needs to encompass both dimensions in the design of a family of funds to address a wide range of investor needs. Another, arguably more important, restriction is that existing target-date funds do not allow for revisions of the asset allocation as a function of changes in market conditions. This is entirely inconsistent with the aforementioned academic prescriptions, and also common sense, which both suggest that the optimal strategy should also display an element of dependence on the state of the economy. In particular, in the presence of a mean-reverting equity premium, the allocation to equities should be increased when equities become cheap, and it should be decreased when they become expensive, as measured through a proxy like the dividend yield or price/earnings ratios (see, for example, Campbell and Viceira (1999), Chacko and Viceira (2005), or Munk, Sørensen, and Vinther (2004)). These life-cycle strategies can be extended to account for uncertainty in income streams, as outlined by Cocco, Gomes and Maenhout, (2005) or Viceira (2001), among others. Abundant academic research (see for example Cairns, Blake, and Dowd (2006), Viceira and Field (2007), Basu (2009), Bodie, Detemple, and Rindisbacher (2009) or Martellini and Milhau (2010a)) has documented that omitting to take market conditions into account in life-cycle investing, as is the case with available target-date funds, leads to genuine underperformance of the funds.

Overall, the extended forms of life-cycle strategies that adjust the allocation to equities, not only as a function of time horizon but also as a function of the current value of equity volatility as well as risk premium as proxied by the relative cheapness of equity markets, strongly dominates the standard approaches by avoiding buying too high and selling too low. This argument has been made explicit in the following quote extracted from Viceira and Field (2007): "Research suggests that long-term equity investors should invest more on average in equities than their short-horizon counterparts, but they should also consider periodic revisions of this allocation as market conditions change. It is logically inconsistent to count on reduced long-term risk while ignoring the variation in returns that produces it. This market-sensitive allocation policy is very different from the asset allocation policy of life-cycle funds, whose target mix moves mechanically away from stocks as an inverse function of investment horizon, regardless of market conditions. Thus mean-reversion arguments provide, if anything, only a partial justification for the roll-down schedule characteristic of life-cycle funds." •

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The chair examines advanced ALM topics such as dynamic allocation strategies, rational pricing of liability schemes and formulation of an ALM model integrating the financial circumstances of pension plan sponsors.

The full version of the research is available on the EDHEC-Risk Institute website at the following address:

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[7] In fact, the rationale behind the strategy is to reduce the impact on the pension of a catastrophic fall in the stock market just before the plan member retires and to hedge the interest-rate risk inherent in the pension-related liability value. As we will argue later, holding no exposure to equity is a very trivial way of managing equity risk, and one that keeps investors from enjoying equity upside potential.

PORTFOLIO MANAGEMENT

Accounting for Short-Term Constraints via the Risk-Controlled Investing Paradigm: From Asset-Liability Management to Risk and Asset-Liability Management

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One of the key insights from the academic literature on long-term allocation decisions with mean-reverting equity returns is the fact that a long-term allocation to equities serves as a hedge against unfavorable short-term equity returns in the presence of mean-reverting equity returns. As a result, the optimal allocation to stocks is higher compared to the myopic case, and investors with longer time horizons hold more stocks compared to investors with shorter horizons. This prescription has very often been taken at face value by target-date funds or life-cycle funds as an investment solution advocating a deterministic decrease of equity allocations (also known as glide path) when approaching retirement date. One key problem, however, is that this prescription can lead to extremely difficult situations when risk is assessed from a shorter-term perspective, in particular in the context of a severe bear equity market such as the one experienced in 2008. Hence it appears that the key element missing from the analysis presented so far is the incorporation of short-term constraints to the design of the optimal allocation strategy by pension funds.

Introducing short-term funding ratio constraints

The introduction of funding ratio constraints is not only an obviously desirable feature from a risk management standpoint, but has also been the focus of recent regulation in most developed countries. For example, in the United States, the Pension Benefit Guaranty Corp. (PBGC), which provides partial insurance of pensions, charges a higher premium to funds that report a funding level of under 90% of current liabilities; this consequently provides strong incentives for maintaining the funding ratio over that minimum 90% threshold. In the U.K., there was a formal general Minimum Funding Requirement (MFR) that came into effect in 1995, which was eventually replaced in the 2004 Pensions Bill with a scheme-specific statutory funding objective to be determined by the sponsoring firm and fund trustees. A regulatory requirement over a minimum funding ratio rule is also present in other European countries, e.g. in Germany, where Pensionskassen and Pensionsfonds must be fully funded at all times to the extent of the guarantees they have given, in Switzerland, where the minimum funding level is 100%, with an incentive for conservative management (investment in equities, for example, is limited to 30% of total assets for funds with less than 110% coverage ratio), or in the Netherlands, where the minimum funding level is 105% plus additional buffers for investment risks.

These regulatory pushes toward an increased focus on risk management by pension funds have given rise to a fierce debate between advocates of tighter regulation and those arguing that imposing short-term constraints on long-term investors would only result in severe welfare loss. For example, in a report prepared on pension funding rules for the OECD, Pugh (2008) makes the following argument: "Minimum funding standards in many countries are designed around … current market yields on long-term bonds. In order to avoid problems, especially in jurisdictions that require immediate correction of the (perceived) underfunding, a plan sponsor is tempted to over-invest in such long-term bonds. … However, pension plans in the long term … need substantial investments in equities. Otherwise, the investments may be inefficient, and the cost of the pension plan to the plan sponsor will therefore increase. (...) In the area of minimum funding, as with other areas of legislation, there is a fine line between (over)protecting the interests of DB (defined benefit) plan members and destroying the incentives for employers to sponsor such plans. … In many countries, the minimum funding standards focus on the pension fund assets exceeding the pension plan's accrued liabilities on every measurement date. … If an asset/liability type of minimum funding measure is to be introduced or retained, then legislation should not require the immediate and complete correction of any underfunding that the test purports to reveal. Asset values fluctuate, and funding shortfalls may disappear as quickly as they had appeared. It is counterproductive for a plan sponsor to make high additional contributions and then find (one or two years later) that the markets have recovered and the plan now has an embarrassing funding excess."

The conflict between the presence of long-term objectives and short-term constraints is in fact one of the most critical challenges faced by long-term investors. Focusing on equity investments, the dilemma can be summarized as follows. On the one hand, investing substantial fractions of their wealth in equity markets makes it difficult for investors to ensure the respect of short-term risk budgets, and leads to a dominant allocation to safe assets that show better correlation with the investors' liabilities. On the other hand, shying away from investing in equity involves a substantial opportunity cost, especially for long-term investors, because the equity risk premium is positive, hence attractive for all investors, but also mean-reverting, hence even more attractive for long-term investors.

More generally, it is widely perceived that tension exists between a focus on hedging long-term risk and a focus on insurance with respect to short-term constraints: typically, dynamic risk-controlled strategies, which imply a reduction to drop in equity prices has led to a substantial diminution of the risk budget, have often been blamed for their pro-cyclical nature, and long-term investors are often reluctant to sell equity holdings in those states of the world where equity markets have become particularly attractive in the presence of mean-reversion in the equity risk premium.

Managing short-term constraints

While tension exists between hedging against long-term risks and insuring against short-term risks, it can be shown that long-term objectives and short-term constraints need not be mutually exclusive and can be integrated in a comprehensive asset allocation framework (see Deguest, Martellini and Milhau (2013)). Depending on market conditions and parameter values, the pro-cyclical risk-control motivation may outweigh the revision of strategic asset allocation motivation, or vice-versa. On the other hand, risk management always ultimately prevails in the sense that when the margin for error has entirely disappeared, the optimal strategy in the presence of risk constraints requires the use of a secure investment strategy.

This argument is based on the recognition that short-term risk constraints are best managed not through *diversification* strategies (which are dedicated to the design of the performance-seeking portfolio (PSP)) or *hedging* strategies (which are dedicated to the design of the *hedging* demand against unexpected changes in liability values or to the design of the hedging demand against unexpected changes in the equity risk premium) but through *insurance* strategies.

From a technical standpoint, the introduction of short-term constraints can be formalized in a portfolio selection problem based on a key insight into the profound correspondence between pricing and portfolio problems. On the one hand, asset pricing problems are equivalent to dynamic asset allocation problems: Merton's (1973) interpretation of the Black and Scholes (1973) option pricing formula. On the other hand, dynamic asset allocation problems (Merton 1973) are equivalent to asset pricing problems: the *martingale* or *convex duality* approach to dynamic asset allocation problems (Karatzas, Lehoczky, and Shreve (1997) and Cox and Huang (1989)).

The practical implication of the introduction of short-term constraints is that the optimal investment in the PSP becomes a function not only of risk aversion but also of risk budgets (defined as the distance of minimum funding floors regarded as an estimate for the margin for error), as well as of the likelihood of the risk budget being spent before the horizon. In a nutshell, a pre-commitment to risk management allows pension funds to adjust their risk exposure in an optimal state-dependent manner, and therefore to generate the highest exposure to upside potential of the PSP while respecting risk constraints.

Overall, the key insight is that the payoff of the optimal strategy in the presence of minimum funding ratio constraints can be regarded as an option on the payoff of the strategy that is optimal in the absence of such constraints. Obviously, no such long-maturity options written on customized dynamic LDI/LCI (liability-driven investing/life-cycle investing) strategies can be found, even as over-the-counter contracts, and investors will have to implement some form of dynamic allocation strategy that will allow replication of the optimal payoff.

While the original approach was developed in a simple framework with a focus on respecting a minimum funding ratio floor, it can be extended in a number of important directions, allowing for the introduction of more complex floors. A large variety of floors, such as capital guarantee floors or maximum drawdown floors, for example, can in fact be introduced (simultaneously, if necessary) in order to accommodate the needs of different pension funds. The maximum drawdown floor, which allows pension fund managers to set a strict limit on maximum consecutive losses, is arguably of particular importance in practice, and it is the one that will be used in the illustration that follows below.

Reducing the cost of insurance against short-term constraints

In addition to accounting for the presence of floors, the dynamic risk-controlled strategies can also accommodate the presence of various forms of caps or ceilings. These strategies recognize that the investor has no utility over a cap target level of wealth, which represents the investor's goal (actually a cap), which can be a constant, deterministic or stochastic function of time. From a conceptual standpoint, it is not clear a priori why any investor should want to impose a strict limit on upside potential. The intuition is that by forgoing performance beyond a certain threshold, where they have relatively lower utility from higher wealth, investors benefit from a decrease in the cost of the downside protection. This is equivalent to adding a short position in a convex payoff in addition to the long position, in order to generate a collar-like payoff, with a truncation of the wealth level distribution on the left-hand side (below the minimum funding ratio constraint) as well as on the right-hand side (above the maximum funding ratio constraint). Putting it differently, without the performance cap, investors have a greater chance of failing an almost-attained goal when their wealth level is very high. It should be noted that the presence of upper (in addition to lower) bounds on performance, consistent with the kind of utility satiation often exhibited by long-term investors, is another, independent reason why a fall in equity prices should not always lead to a decrease in equity allocation, even without mean-reverting equity risk premium. As a result, general risk-controlled strategies do not necessarily share the pro-cyclical aspect of basic forms of risk-controlled strategies such as standard constant proportion portfolio insurance (CPPI) strategies.

The quintessence of investment management is finding optimal ways to spend risk budgets that investors are reluctantly willing to set, with a focus on allowing for the highest possible access to performance potential while respecting such risk budgets. Risk diversification, risk hedging and risk insurance are three useful approaches to optimal spending of investors' risk budgets. In this context, improved forms of LDI strategies rely on sophisticated exploitation of the benefits of the three competing approaches to risk management, namely risk diversification (key ingredient in the design of better benchmarks for PSPs), risk hedging (key ingredient in the design of better benchmarks for hedging portfolios) and risk insurance (key ingredient in the design of better dynamic asset allocation benchmarks for long-term investors facing short-term constraints). In the end, risk management, which focuses on maximizing the probability of achieving investors' long-term objectives while respecting the short-term constraints they face, appears to be the key source of added value in investment management.

More generally, one can show that the opportunity costs implied by the short-term constraints are significantly lower when these constraints are optimally addressed through insurance strategies, as opposed to being inefficiently addressed through excessive hedging and an unconditional decrease of the equity allocation. These insights confirm that dynamic asset allocation benchmarks can be designed to allow for a more efficient spend of investors' risk budgets. Intuitively, this is because the pre-commitment to reduce the allocation to equity in times and market conditions that require such a reduction in order to avoid over-spending risk budgets allows investors to invest, on average, more in equities compared to a simple static strategy that is calibrated to respect the same risk budget constraints. The welfare gains involved in this higher allocation to equities are found to be substantial for reasonable parameter values, especially for long-term horizons and in the presence of a mean-reverting equity risk premium.

As a numerical illustration of the benefits of risk-controlled strategies, and their benefits in terms of reduction of the opportunity cost implied by the presence of short-term risk constraints, we consider three long-term unconstrained optimal dynamic strategies that we refer to as defensive (leading to an average stock weight of 10%), moderate (leading to an average stock weight of 20%) and aggressive (leading to an average stock weight of 30%), respectively.

While these long-term strategies are engineered to achieve optimal risk/return tradeoffs over the long term, short-term losses and drawdown levels can remain extremely large, especially for the aggressive investor, with a maximum drawdown at 23.45%, as can be seen from Exhibit 1. A pension fund wishing (or obliged) to maintain the maximum drawdown at a level of around 15%, for example, would have to opt for the defensive strategy, which has a reported drawdown of 16.51%, even if the average level of wealth achieved with this strategy is much less attractive at $365.66 than what is allowed by the aggressive strategy, $530.87. The objective measure of the opportunity cost associated with a maximum drawdown limit set at around 15% can be formally defined as the additional initial contribution needed by the defensive strategy to reach the same average wealth level as with the aggressive strategy for the same dispersion level, a cost which turns out to be a prohibitive 45.18% in this particular example (see Exhibit 8). As a result, we find that the pension that would need to finance a liability payoff around $500 at the 20-year horizon is facing a challenge and a dilemma. On the one hand, implementing a defensive strategy would (almost) allow for the respect of the 15% short-term maximum drawdown risk budget, but it would imply a violation of the $100 initial contribution dollar budget since it would require an additional initial contribution of more than $45 to reach an average wealth level at horizon sufficient on average to cover the required pension payment. On the other hand, implementing an aggressive strategy would allow for achieving (in fact, exceeding) the target average wealth level of $500 while respecting the $100 initial dollar contribution budget, but it would imply a severe violation of the 15% short-term maximum drawdown risk budget, with a maximum drawdown estimated at 23.45% on

EXHIBIT 1

Statistics and risk measures of long-term strategies with maximum drawdown constraints.

This table displays the average terminal wealth over a 20-year horizon together with its standard deviation for four long-tem allocation strategies, and three different levels of riskiness: aggressive, moderate and defensive. It also shows the maximum drawdown at a 99.5% level.[8] We also compute the opportunity cost, which is the percentage of initial wealth necessary for each strategy to reach the same average terminal wealth as the optimal aggressive strategy for the same standard deviation. Additionally, we compute the opportunity gain, which is the percentage of initial wealth saved if one decides to invest in non-defensive strategies to reach the same average terminal wealth as the defensive optimal strategy for the same standard deviation.

	Optimal Aggressive	Opt Agg w/ 15% MDD Control	Opt Agg w/ 10% MDD Control	Optimal Defensive
Av. Wealth	530.87	501.10	401.92	365.66
Std of Wealth	38.02	41.09	38.24	8.84
Max DD at 99.5%	23.45%	14.57%	10.05%	16.51%
Opportunity Cost	0	5.94%	32.08%	45.18%
Opportunity Gain	31.12%	27.03%	9.02%	0

the Monte Carlo simulated paths.

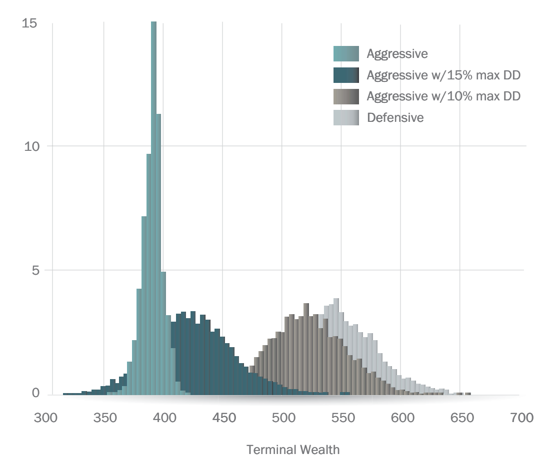
The only way out of this dilemma is to recognize that a more efficient, less costly approach to the management of the short-term maximum drawdown risk budget is to use insurance, as opposed to hedging. Hence, as an alternative to opting for the defensive strategy, the investor can choose the aggressive strategy, which allows for a much higher access to the equity risk premium, to which will be added the implementation of a dynamic risk-controlled investing overlay designed to ensure that the maximum drawdown will be kept below 15% (see Exhibit 2 for the resulting distribution of terminal wealth, where we have also tested maximum drawdown levels at 10%). We now observe from Exhibit 1 that the average wealth of the aggressive strategy with a 15% maximum drawdown constraint is substantially higher than the unconstrained defensive one, for essentially the same level of extreme losses (in fact, the maximum drawdown is kept below 15% for this risk-controlled strategy while it did exceed 15% for the defensive strategy without risk explicit risk controls). This result makes a strong case for the management of short-term constraints through dynamic risk budgeting rather than through the choice of unnecessarily conservative investment policies. To provide an objective assessment of the opportunity cost of imposing stricter drawdown constraints when these constraints are optimally managed through insurance techniques, we find that the use of the aggressive strategy with a maximum drawdown constraint of 15% requires a mere 5.94% of additional investment to reach the same average wealth level as with the aggressive strategy without maximum drawdown constraints. This value compares very favorably to the aforementioned 45.18% opportunity cost involved in managing maximum drawdown constraints inefficiently through an excessive level of hedging.

In fact, one can achieve a higher wealth level compared to the defensive strategy if one uses the aggressive strategy as an underlying for a risk-control layer with a target maximum drawdown at a mere 10%, with a corresponding utility cost at 32.08% vs. 42.18% for the defensive strategy. Overall, these results illustrate that not disentangling long-term risk aversion and short-term loss-aversion may lead to poor investment decisions. Relatively simple solutions exist that can be implemented as dynamic asset allocation strategies in order to control short-term risk levels while maintaining access to long-term sources of performance. These solutions are a substantial improvement with respect to traditional strategies without dynamic risk-control, which inevitably lead to underspending of investors' risk budgets in normal market conditions, with a strong associated opportunity cost, and overspending of investors' risk budgets in extreme market conditions.

Our conclusion is that rising to the challenges of modern investment practice for pension funds involves designing new forms of investment solutions relying on improved, more efficient PSP and LHP (liability-hedging portfolio) building blocks, as well as on improved dynamic allocation strategies. Although each of these ingredients can be found in current investment products, it is only by putting the pieces of the puzzle together and by combining all these sources of expertise that the asset management industry will address investors' needs satisfactorily. From the technical perspective, these advanced investment solutions rely on a sophisticated exploitation of the benefits of the three competing approaches

EXHIBIT 2

Distributions of terminal wealth generated by long-term investment strategies in the presence of maximum drawdown constraints.



to risk management: risk diversification (a key ingredient in the design of better benchmarks for PSPs), risk hedging (a key ingredient in the design of better benchmarks for hedging portfolios) and risk insurance (a key ingredient in the design of better dynamic asset allocation benchmarks for long-term investors facing short-term constraints), each of which represents a hitherto largely unexplored potential source of added value for the asset management industry.

Risk *management* is often mistaken for risk *measurement*. This is a problem since the ability to measure risk properly is at best a necessary but not sufficient condition to ensure proper risk management. Another misconception is that risk management is about risk reduction. In fact, risk management is at least as much about return enhancement as it is about risk reduction. Indeed, risk management is about maximizing the probability of achieving investors' long-term objectives while respecting the short-term constraints they face. In the end, the traditional static LDI strategies without dynamic risk-controlled ingredient inevitably lead to underspending investors' risk budgets in normal market conditions, with a high associated opportunity cost, and overspending their risk budgets in extreme market conditions. •

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[8] 0.5% of the worst drawdowns have been discarded.

PORTFOLIO MANAGEMENT

Key Insights from a Survey on Institutional Investors' LDI and DLDI Practices

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Between November 2013 and January 2014, EDHEC-Risk Institute surveyed 104 investors to determine how liability-driven investing (LDI) and dynamic liability-driven investing (DLDI) strategies are used in practice and the reasons that motivate the adoption or non-adoption of such techniques. The respondents to the survey were predominantly European, with a breakdown as follows: 87% Europe, 8% North America and 5% Australia.

The survey finds that LDI is popular, but in concrete terms the fund separation approach, which is consistent with the LDI paradigm, is not yet sufficiently widely applied to manage the LDI approach optimally, especially in southern European countries. In the same way, one of the key points in LDI for pension funds is hedging with respect to the duration of the liabilities, but it is not always implemented by pension funds, even though they affirm that they use LDI-type solutions.

The risk allocation approach, which has come up with innovative offerings in the area of factor investing in recent years, is gaining ground, because it contributes to a better understanding of institutional investors' risks and diversification. From that perspective, we observe acceleration in the adoption by professionals of concepts that are well documented in the academic world.

Too many pension funds are still more concerned with standalone performance than risk management, which explains why they favor tactical allocation or reviews of strategic allocation over risk management in asset-liability management (ALM). On this subject, we observe a clear difference in the rates of adoption of dynamic LDI between the north and the south of Europe. Too many pension funds remain asset-only rather than ALM and do not take sufficient account of the impact of their liabilities in their asset allocation policy or risk management.

Among the key insights and concerns highlighted in the survey:

• 80% of the respondents are fully aware of the LDI paradigm.

The LDI principle is specifically designed to provide the possibility of achieving good performance while maintaining a low level of relative risk. The purpose of the survey question was to see whether participants are familiar with this investment paradigm. A vast majority of them (78.43% in Exhibit 1) answered positively. This figure suggests that the LDI paradigm has become an accepted norm in ALM for pension funds; it also suggests that the gap between theory and practice is not particularly large on this particular issue since the LDI principle is deeply rooted in portfolio theory (it is one manifestation of the many "fund-separation theorems").

• Roughly 50% of respondents are not only aware of the importance of measuring and managing liability risk, but have effectively adopted the LDI approach, which encompasses an explicit focus on liability hedging that is achieved through a dedicated liability-hedging portfolio, as opposed to seeking to diversify away liability risk within some well-balanced policy portfolio.

Familiarity with the LDI concept does not imply that participants actually adopt fund separation as an investment guideline. As a matter of fact, Exhibit 2 shows that there are virtually as many respondents who explicitly split their allocation into a performance-seeking portfolio (PSP) and a liability-hedging portfolio (LHP) as respondents who do not.

This score is subject to variations across the geographical zones considered. The only two countries where a clear majority of respondents adopts a formal fund separation are the United Kingdom (16 vs. three who do not) and Denmark (seven vs. one). The scores in the Netherlands and Germany are the same as those at the global level: in these two countries, "Yes" and "No" attracted the exact same number of answers. Finally, in some other regions, fund separation does not seem to be a widespread practice: in Switzerland, two-thirds of respondents answered "No," and a similar rate was recorded in North America.

• More than 50% of the respondents explicitly measure liability risk through a probability of a shortfall or the magnitude of this shortfall.

A variety of risk measures are available to measure "liability risk." The first question we asked participants is which measure(s) they use to assess liability risk. Exhibit 3 first shows that while a large majority of respondents (81.37%) do measure liability risk, a non-negligible proportion of them declare they do not. This result is perhaps surprising, since the measurement of the relative risk of the portfolio with respect to liabilities appears to be a critically important element in the monitoring of a portfolio in the presence of liabilities.

Risk measures such as the shortfall probability and the expected shortfall are popular, being used respectively by 39.22% and 29.41% of respondents; the former measure is defined as the probability that assets underperform liabilities, and the latter is the average amount by which assets fall short of liabilities in such a situation. These risk measures focus on downside risk with respect to the liabilities, that is, risk of losses relative to the liability portfolio value. This stands in sharp contrast to measures such as volatility, which captures both upside and downside risk, although only the former is obviously a concern, and which are based on an asset-only assessment of the portfolio risk. More than 40% of respondents say they also use other methods, in place of or in addition to shortfall measures. One can assume that volatility, in spite of the aforementioned shortcoming, and Value-at-Risk or Conditional Value-at-Risk, which are also asset-only risk indicators, but with a focus on downside risk, are among the techniques often adopted.

• Risk allocation, a novel approach to diversification within the performance-seeking portfolio, has already been adopted by more than 35% of the respondents to the survey.

An approach with growing popularity consists in viewing portfolio construction as a factor-allocation exercise. This new paradigm leads investors to frame the allocation decisions in terms of underlying risk factors that drive asset returns, as opposed to allocation to the asset classes themselves. One of the advantages of this approach is that it draws attention to the concentration issues posed by portfolios that contain asset classes with redundant factor exposures. Moreover, recasting the asset allocation problem as factor allocation is a natural framework to seek best access to the premia associated with rewarded factors, such as value and size, for example, in the equity space.

Exhibit 4 shows that 35.42% of respondents are adopting this perspective. This percentage represents less than half, but it is far from negligible, which shows that factor allocation

EXHIBIT 1

Are you familiar with the LDI paradigm?

The exhibit indicates the percentage of responses from participants on their familiarity with the LDI paradigm. The question was asked to the 104 participants and percentages have been normalized by excluding non-responses (2 out of 104).

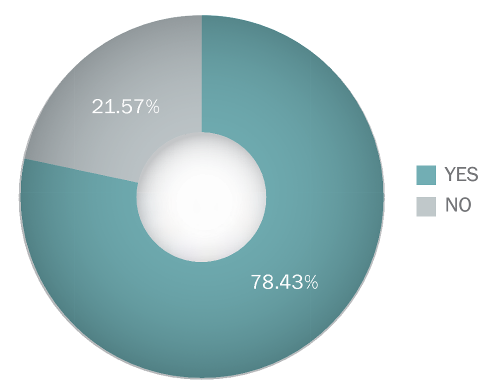
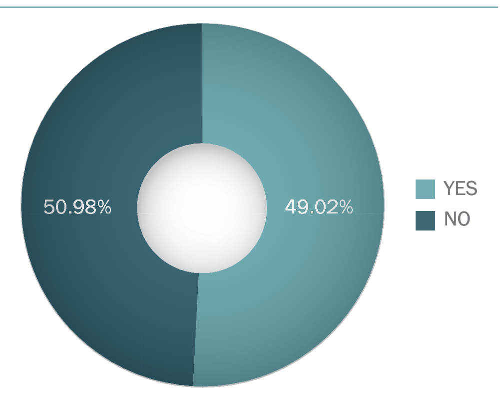


EXHIBIT 2

Do you split your portfolio into an LHP and a PSP?

The exhibit indicates the percentage of responses of participants on whether they split their portfolio into an LHP and a PSP. The question was asked to the 104 participants and percentages have been normalized by excluding non-responses (2 out of 104).



is not only gaining interest, but is also already applied in some form or another by a substantial number of pension funds.

• 50% of the pension fund respondents are hedging their liabilities, while the rest of the respondents are sitting on the sidelines.

We asked participants whether they hedge their liabilities or not. The results in Exhibit 5 are striking, given that slightly over half (54.02%) of the respondents answer that they do not hedge liabilities. Not hedging liabilities at all implies having a pure asset-only focus, which is a serious concern given that the absolute risk properties of an asset class (e.g., their volatilities) are in general very distinct from their relative risk properties (e.g., their volatility relative to liabilities, which depends to a large extent on their correlation with liabilities). It should be noted that the adoption of liability hedging is largely influenced by the discount rate applied to liabilities: 63.41% of pension funds that apply a market rate hedge their liabilities, while only 20% of those that employ a fixed rate do so.

These results show that pension funds have rather consistent behavior: those who use a variable rate recognize that fluctuations in the discount rate are reflected in liability value, which calls for liability hedging. Nevertheless, there remain a large 36.59% of funds that use a market rate but do not hedge their liabilities. Moreover, even if those that use a fixed rate are by definition not exposed to the risk of unexpected changes in the discount rate, they should still be more concerned with liability hedging, given that the value of liabilities may be impacted by other factors (e.g. by realized inflation if pension payments are indexed to inflation).

• Duration matching is only perceived as a desirable or a feasible target by about 60% of the respondents who express a focus on liability hedging, which implies that effective liability hedging is not always achieved, not even by those who express an interest in this objective.

On the specific subject of interest rate-risk hedging, we asked participants whether they match the duration of the bonds in their LHP to that of their liabilities. Duration matching is a basic form of liability hedging that aims at immunizing the funding ratio against small interest rate changes: if the LHP value and the liability value vary by the same amounts as a response to changes in interest rates, this leaves the funding ratio unchanged. The result that duration matching is only used by 40% of the respondents (see Exhibit 6) is striking.

To better understand the reasons that explain this low penetration of an otherwise standard interest rate-risk management technique, we seek to analyze jointly the responses in Exhibit 6 and the responses in Exhibit 7, related to the discount rate used in the valuation of liabilities. Somewhat unsurprisingly, we find that pension funds that discount liabilities using a market rate are much more prone to seek duration matching than those that employ a fixed rate, since there were 56.10% of "Yes" in the former category, vs. 8% in the latter, a result that suggests the existence of a strong link between the problem of pension liability valuation and pension liability hedging.

• Only about 40% implement a DLDI process, which requires periodic revisions of the portfolio policy.

We asked participants whether they are using or considering using dynamic LDI techniques. The majority of the respondents give a negative answer (see Exhibit 8), but the score of 61.46% is not overwhelming, and there remain a comfortable 38.45% of respondents who express an interest in these techniques.

• Most respondents do not translate the minimum funding requirements imposed by the regulation into floors on asset value: more than half of participants recognise that they operate under such constraints, but barely a fifth of them impose bounds.

One possible motivation for the adoption of a dynamic LDI strategy with risk budgets is the presence of regulatory minimum funding requirements: under such a regulatory regime, the floor is equal to the minimum funding ratio times the value of liabilities.

We therefore asked participants whether they do face such a regulatory constraint. While a slight majority answered "No" (Exhibit 9), this result masks important regional disparities. For instance, all participants based in the Netherlands who answered this question said, "Yes," while the quasi-totality of U.K. respondents (that is, 16 out of 19) answered "No." For Switzerland, the situation is mixed: out of 14 respondents, there were six "No" and 8 "Yes."

EXHIBIT 3

Risk framework used to measure liability risk.
The exhibit presents the rate of adoption (in percentage) of quantitative risk frameworks to measure liability risk. Respondents are able to choose more than one response; hence the category percentages add up to more than 100%. The question was asked to the 104 participants, and percentages have been normalized by excluding non-responses (2 out of 104).

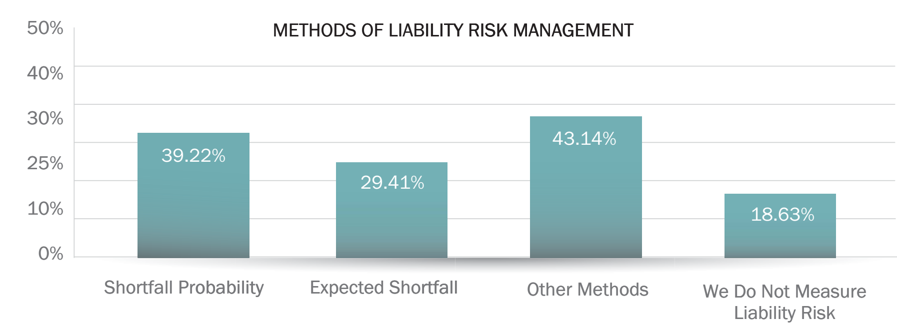


EXHIBIT 4

Do you frame your allocation decisions in terms of factors as opposed to, or in addition to, framing it in terms of asset classes?
The exhibit presents the percentage of responses on this question. The question was asked to the 104 participants, and percentages have been normalized by excluding non-responses (8 out of 104).

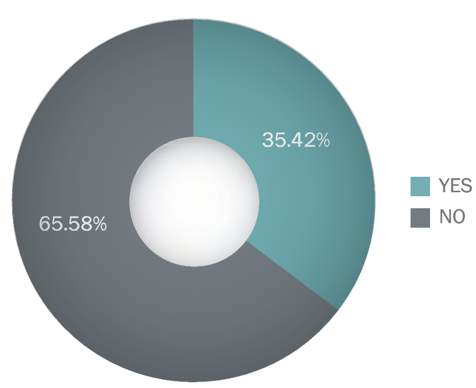


EXHIBIT 5

Do you hedge your liabilities? (Pension funds only).
The exhibit indicates the percentage of responses from pension funds on their hedging practices. The question was asked to the 91 pension funds, and percentages have been normalized by excluding non-responses (4 out of 91).

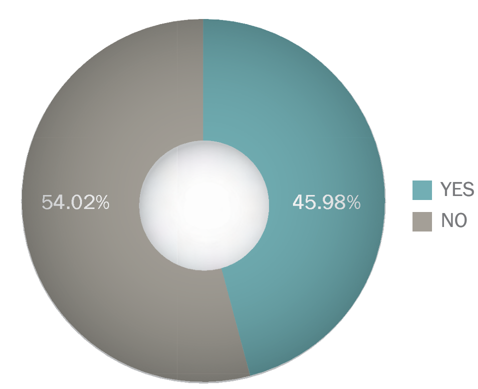


EXHIBIT 6

Do you aim to align the duration of bonds in your LHP with the duration of liabilities?
The exhibit presents the percentage of responses to this question. The question was asked to the 104 participants, and percentages have been normalized by excluding non-responses (9 out of 104).

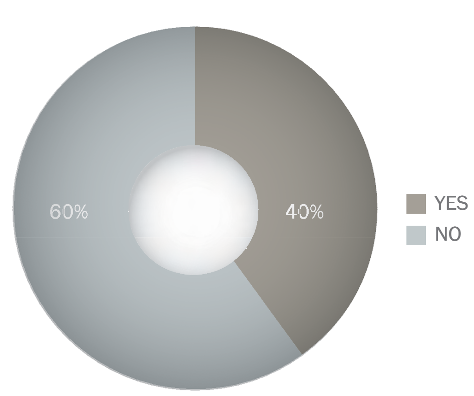
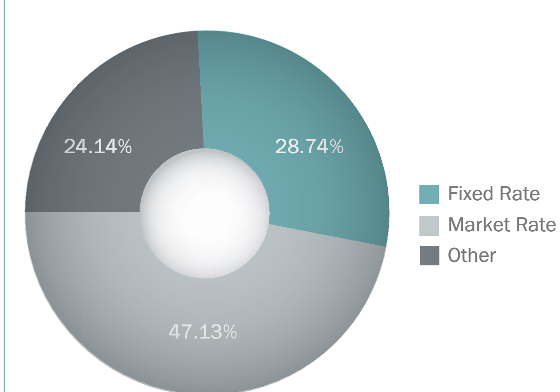


EXHIBIT 7

How are liabilities discounted? (Pension funds only).
The exhibit indicates the distribution of the different discount rates used by pension funds to discount their liabilities. The question was asked to the 91 pension funds, and percentages have been normalized by excluding non-responses (4 out of 91).



• A majority of the respondents are aware of the presence of conflicts of interest between various stakeholders, while slightly less than half of the respondents have already made some steps towards an integrated approach to pension fund asset-liability management, if only through attempts to adopt some form of hedging mechanism against sponsor risk.

For defined benefit corporate pension plans, the analysis of optimal funding and investment decisions involves a proper understanding of the possible conflicts of interest that exist between a number of stakeholders.

• Beneficiaries: employees, who spend a part of their income as contributions, and retirees, who receive pension payments;
• Pension fund managers and trustees, who are in charge of asset allocation decisions and/or the effective management of pension assets according to these decisions;
• Shareholders of the sponsor company, who may influence the productive investment choices, but not the decisions of the pension fund, and who are ultimately responsible for ensuring that promised liabilities are paid to the pensioners.

An example of a conflict of interest is the preference of shareholders for lower funding levels and more risky investment strategies, which conflicts with the preference of beneficiaries for higher funding levels and safer investment strategies, at least unless the latter group is granted some form of access to pension plan surpluses. The last section of the survey, which is open to all types of participants, including representatives of the sponsor company, aims at assessing the perception of these conflicts and at reviewing the existing approaches to sponsor risk hedging.

Exhibit 10 reviews a list of potential sources of conflicts of interest between groups of stakeholders, and participants were invited to express their views on the existence or absence of such conflicts in practice. Focusing on the items for which more than 20% of "Yes" responses were recorded, we can isolate four domains where respondents have identified the presence of conflicts of interest:

• Shareholders vs. beneficiaries (29.07% of the respondents perceive a conflict between these two groups of stakeholders)
• Employees vs. retirees (52.33%)
• Pension fund managers vs. shareholders (22.09%)
• Trustees vs. shareholders (27.91%)

The first conflict is mentioned by less than one-third of respondents. In other words, it seems that the asset-substitution problem — that is, the possibility that more risky investment decisions taking place after pension benefits have been promised to beneficiaries may lead to a wealth transfer from these beneficiaries in favor of stakeholders — is not dominant in respondents' perception of the challenges related to an integrated approach to asset-liability management.

The conflict which is most often cited, by more than half of respondents, is between employees and retirees. It is interesting to compare these responses according to the type of funds: among the 38 funds that mention this conflict, 22 are defined benefit funds, eight are hybrid funds, and six are defined contribution funds, a split which reflects the relative risks borne by workers and pensioners. In a DC fund, contributions are either fixed or at least predictable, and cannot be raised to deliver a contractual amount of pension payments: as a consequence, employees are protected against the risk of an unexpected change in their contributions. In a DB fund, on the other hand, this risk is more likely to be present: retirees may prefer a higher contribution rate, which guarantees the payment of their pensions, while employees obviously prefer to have higher available income.

We may also note that according to most respondents, the interests of trustees are well aligned with those of pension fund managers (63.95%) and beneficiaries (72.09%). Opinions are more mixed regarding the alignment with shareholders' interests, since only a small half of respondents declare that they do not perceive a conflict here. •

The research from which this article was drawn was supported by BNP Paribas Investment Partners as part of the research chair on "Asset-Liability Management and Institutional Investment Management" at EDHEC-Risk Institute.

The chair examines advanced ALM topics such as dynamic allocation strategies, rational pricing of liability schemes and formulation of an ALM model integrating the financial circumstances of pension plan sponsors.

The full version of the research is available on the EDHEC-Risk Institute website at the following address: http://www.edhec-risk.com/ALM/BNPPAM_Research_Chair

EXHIBIT 8

Do you use or are you considering using dynamic LDI strategies which require periodic revisions of the portfolio policy?
The exhibit presents the percentages of responses to this question. The question was asked to the 104 participants, and percentages have been normalized by excluding non-responses (8 out of 104).

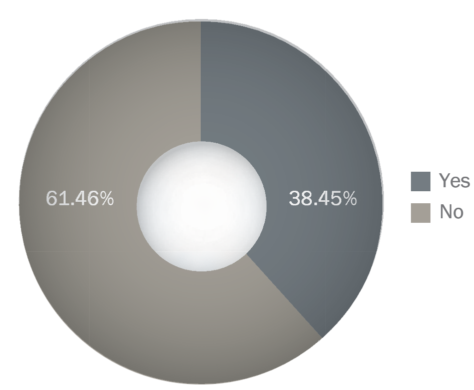


EXHIBIT 9

Do you face minimum funding ratio levels from a regulatory standpoint?
The exhibit presents the percentage of responses of participants to this question. The question was asked to the 104 participants, and percentages have been normalized by excluding non-responses (9 out of 104).

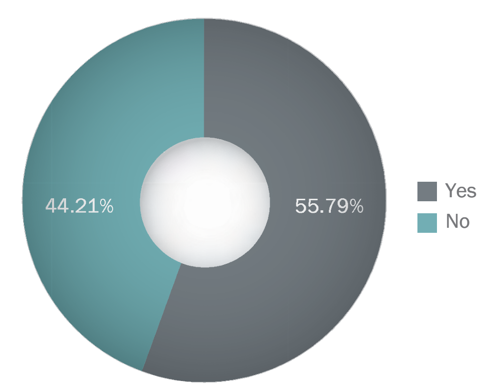
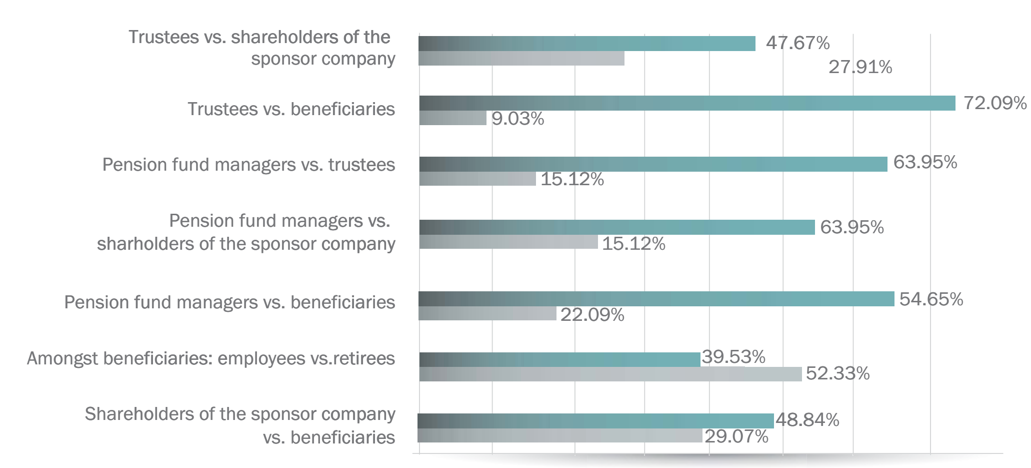


EXHIBIT 10

Do you perceive any evidence of conflict of interests between various stakeholders
The exhibit presents the percentage of responses of participants to this question. The question was asked to the 104 participants, and multiple choices were allowed, so the percentages do not add up to 100%. They have been normalized by excluding non-responses (18 out of 104). The orange (respectively blue) bars represent the answer "No" (respectively, "Yes").



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PORTFOLIO MANAGEMENT

Hedging Long-Term Inflation-Linked Liabilities under Various Inflation Regimes without Inflation-Linked Instruments

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Hedging inflation-linked liabilities without inflation-linked bonds

A long-term concern over possible enhanced inflation uncertainty has increased the need for investors to hedge against changes in price levels, a problem of particularly critical importance for pension funds that have pension payments explicitly indexed with respect to changes in consumer price levels.

The implementation of liability-hedging portfolios (or LHP), also known as liability-driven investment portfolios (or LDI portfolios) for inflation-linked liabilities has become relatively straightforward in situations where either cash instruments (inflation-linked bonds, or IL bonds, also known as Treasury Inflation Protected Securities (or TIPS), when the issuer is a sovereign state) or dedicated over-the-counter (OTC) derivatives (such as inflation swaps) can be used to achieve perfect hedging. More generally, however, the lack of capacity for inflation-linked cash instruments and the increasing concern over counterparty risk for derivatives-based solutions leaves most investors with the presence of non-hedgeable inflation risk. Another outstanding problem, even when perfect inflation hedging is possible, is that such solutions generate very modest performance given that real returns on inflation-protected securities, negatively impacted by the presence of a significant inflation risk premium, are typically very low.

In this context, the Ontario Teachers' Pension Plan (OTPP or "Teachers'") has been supporting an academic research chair at EDHEC-Risk Institute with a focus on analyzing the design of novel forms of liability-hedging portfolios that do not solely rely on inflation-linked securities. The main research question addressed in the initial stages of the research project was the following: In the presence of capacity constraints on the local inflation-linked bonds market, can one expect a suitably designed portfolio, potentially involving nominal bonds, foreign inflation-linked bonds and real assets, to be a reliable, and robust across economic regimes, substitute for real bonds in the context of hedging long-term inflation-linked liabilities?

Liability risk hedging vs. inflation risk hedging

At the risk of stating the obvious, one should first emphasize that the focus for a pension fund with inflation-indexed liabilities should not be on inflation risk hedging; but on liability risk hedging. While the two concepts coincide at liability maturity, they do not coincide otherwise because the *present value* of liabilities is exposed to interest rate risk (unexpected changes in real rates) in addition to inflation risk (unexpected changes in realized inflation).

As clear evidence of this distinction, while long-term IL bonds with corresponding duration are the best liability-matching instruments for IL liabilities, the short-term correlation between changes in inflation and the return on long-term IL bonds is close to zero.

From a quantitative standpoint, our analysis unambiguously shows that for long-term constant maturity inflation-linked liabilities, real rate risk strongly dominates realized inflation risk within liability risk (Martellini and Milhau (2012)). This is because real rate uncertainty is compounded by time-horizon, while inflation risk uncertainty is not. In other words, unexpected changes in real rates explain an overwhelming fraction of short-term changes in the funding ratio for a pension fund with inflation-linked liabilities. It is only at liability maturity that interest rate risk vanishes and inflation risk remains the only source of uncertainty.

In the end, only IL bonds allow for a perfect match for IL liabilities over both short and long horizons.

Expected inflation risk vs. realized inflation risk

In a situation where inflation-linked bonds or inflation swaps are not available, at least not at all times and not for the whole inflation exposure, natural substitute candidates would be nominal bonds, since they have well-defined interest rate exposure. One key concern, however, is that nominal bonds are exposed to changes in nominal rates, as opposed to changes in real rates. In other words, the value of a nominal bond portfolio is impacted by changes in real rates but also changes in expected inflation, while IL bonds and IL liabilities are only exposed to changes in real rates (and also changes in realized inflation, but this again is not a substantial problem when assessing the short-term risk of long-dated liabilities).

In other words, while realized inflation risk is not a serious problem for long-term constant maturity IL liabilities, expected inflation risk can be because it has no impact on IL liabilities but has a deep impact on nominal bond portfolios that are typically used as substitutes for IL bonds. This is because the expected inflation duration is equal to the real rate duration (and also equal to the standard duration) for nominal bonds, while the expected inflation duration is zero for inflation-linked bonds.[9]

The concern here would be a strong increase in expected inflation, which would lead to a drop in nominal bond prices assuming real rates stay constant, while it would leave liability values unaffected. In fact, the strong positive correlation between realized inflation and expected inflation would imply, everything else being equal, a positive performance of IL liabilities in the case of a positive shock to inflation expectations.

Our historical analysis going back to the '60s confirms that a large positive shock to expected inflation has almost always led to a drop in nominal bond prices and an increase in liability values (estimated based on a qualified proxy for expected inflation). In such situations, we have also found that commodities (proxied by the Goldman Sachs Commodity Index, or GSCI) have generated strong positive performance most of the time (see Martellini, Milhau and Tarelli (2012a) for more details). In this context, one can expect the introduction of real assets in the liability-hedging portfolio to compensate for the poor performance of nominal bonds in the case of a jump in breakeven inflation.

One key problem, however, is that while real assets such as commodities may indeed have attractive inflation hedging properties, they have poor interest rate hedging properties, since there is no well-defined duration measure for commodities or even for real estate. As a consequence, introducing substantial allocations to real assets in liability-hedging portfolios would generate high volatility in the funding ratio in most market conditions, even though it might help in extreme situations with a high increase in expected inflation. We may summarize the investment policy implications as follows.

Diversifying vs. hedging expected inflation risk in nominal bond portfolios

In a nutshell, the problem can be summarized as follows:

- Most of the time, nominal bonds would be very good substitutes for inflation-linked bonds, with which they share well-defined real interest rate risk exposure, and they should therefore dominate in liability-hedging portfolios.
- That nominal bonds cannot be used to hedge the re alised inflation risk exposure in IL liabilities is not a quantitatively meaningful problem in terms of short-term volatility of the funding ratio for long-term liabilities.
- On the other hand, since nominal bonds are exposed to expected inflation risk in addition to real rate risk, while IL liabilities are not, the liability-hedging qualities of nominal bonds would deteriorate in the case of large increases in expected inflation uncertainty, especially when shocks to expected inflation are driven by factors that are not related to interest rate risk.

From a conceptual perspective, two main approaches can be used to manage this risk of mismatch between the non-zero exposure of nominal bonds to changes in expected inflation and the zero exposure of inflation-linked bonds to the same risk factor.

Focus on diversifying away expected inflation risk

The strategy here would consist of holding at all times a dynamic mix of the liability-hedging portfolio that is optimal under different particular regimes (i.e., with more real assets or less), where the weights assigned to each liability-hedging portfolio at any point in time are taken to be a function of the estimated probability of being in each regime.

This dynamic diversification strategy contrasts with a static diversification strategy, where a static mix of the hedging portfolios that are optimal under each particular regime, which would involve holding too much of the real assets in the high expected inflation regime, and too little otherwise.

Focus on hedging away expected inflation risk

While a long-only position in nominal bonds will always have a negative exposure to unexpected inflation, long-short nominal-bond portfolio strategies can in principle be designed to achieve zero exposure to changes in unexpected inflation, while having a target exposure to changes in real rates equal to that of the liabilities. What remains to be thoroughly analyzed is whether this strategy can be effectively implemented in practice. In particular, one needs to carefully assess the out-of-sample robustness of quantitative strategies based on imperfect parameter estimates that may suffer from sample risk.

Diversifying expected inflation risk in nominal bond portfolios

Our findings can be summarized as follows (see Martellini, Milhau and Tarelli (2012a) for more details).

[9] *For a nominal bond, the real rate duration and expected inflation rate duration are equal, and equal to the standard duration; for IL bonds, the expected inflation duration is zero, but the real rate duration is not (see Siegel and Waring (2004), Fabozzi and Xu (2012) or Martellini, Milhau and Tarelli (2012b)).*

- While realized and expected inflation risks have been relatively limited in the recent past in developed economies, a long-term analysis suggests that the presence of possible regime switches needs to be taken into account. A formal statistical analysis confirms that regimes with high expected inflation and low speed of mean-reversion in expected inflation would lead to a profound mismatch between long-only nominal bond performance and liability returns.
- Given that our analysis has confirmed that real assets in general, and commodities in particular, have been formally verified to perform well in the case of large positive shocks to expected inflation, especially when these shocks are driven by increases in commodity prices, substantial benefits can be generated from a strategy dynamically weighting a nominal bond-dominated liability-hedging portfolio and a real asset-dominated nominal bond portfolio as a function of the probability to stay in the normal regime vs. entering a high expected inflation regime.
- The benefits of such a strategy are robust with respect to the introduction of a realistic lag in terms of recognising the emergence of the high expected inflation regime. This finding is particularly important because the real-time identification of regime switches is a serious statistical challenge. In particular, formal Markov regime switching models would typically identify periods of increasing inflation risk, without distinguishing clearly between increases and decreases in expected inflation, and simpler, more robust approaches would therefore be recommended.
- In addition to substantial levels of turnover generated by the switch from a bond-dominated to a commodity-dominated liability-hedging portfolio, the strategy involves holding a liability-hedging portfolio with little or no fixed-income instruments in high expected inflation regimes, which is not only at odds with standard practice but also can prove a serious problem in the case of a substantial drop in real rates.

Hedging expected inflation risk in nominal bond portfolios

Our findings can be summarized as follows (see Martellini, Milhau and Tarelli (2012b) for more details).

- Liability risk management is about matching risk exposures of assets and liabilities. Real rate exposure (also known as real rate duration) can be explicitly measured for bonds as a function of observable variables, while it has to be empirically estimated for real assets, and can therefore be made to match the real rate exposure in the liabilities.
- On the other hand, the exposure of inflation-linked liabilities to expected inflation is zero, and therefore it should also be neutralized on the asset side to avoid introducing an ALM mismatch. While a long-only position in nominal bonds will always have a negative exposure to unexpected inflation, long-short nominal-bond portfolio strategies can in principle be designed to achieve zero exposure to changes in unexpected inflation, while having a target exposure to changes in real rates equal to that of the liabilities.
- In principle, a suitably-designed long/short nominal bond portfolio would have a substantially lower (if not zero) exposure to shocks to expected inflation compared to a standard long-only bond position, while allowing for an exposure to real rate risk targeted to be similar to that of the liabilities.
- The long-duration bond has a higher sensitivity to changes in expected inflation but changes in expected inflation are less volatile for the long term; conversely, the short-duration bond has a lower sensitivity to changes in expected inflation but changes in expected inflation are more volatile for the short term. The suit able dynamic hedging strategy emerges from the quantitative analysis of this trade-off.
- In the presence of uncertainty about speed of mean reversion parameter estimates, our analysis suggest that a robust implementation of this strategy can generate substantial benefits, if not allowing for a perfect hedge, a conclusion which is supported by a detailed statistical analysis of the dynamics of the term structure of expected inflation.

Policy implications

We have three main policy implications.

- **Policy implication #1:** A static exposure to commodity (more generally real assets) in the liability-hedging portfolio would not be a good solution; whatever the exposure, it would be too high most of the time, in normal regimes where the poor interest rate hedging properties of real assets would generate substantial short-term funding ratio volatility, and too low in those rare market conditions with extreme increases in expected inflation.
- **Policy implication #2:** A dynamic exposure to commodity (more generally real assets) in the liability-hedging portfolio would be a reasonable approach to diversifying away the risk of a large positive shock to expected inflation. Using a parsimonious model based on an observable, persistent state variable such as expected inflation, one would be able to switch, even with some reasonable delay, from a bond-dominated liability-hedging portfolio to a commodities-dominated liability-hedging portfolio in the case of a strong increase in the likelihood of a regime with high expected inflation.
- **Policy implication #3:** A duration-matching strategy involving long-short allocation to nominal bonds of different maturities would be a reasonable approach to hedging away the risk of a large positive shock to expected inflation. Aiming at setting to zero the exposure of the bond portfolio to expected inflation requires parameter estimates that tend to be noisy, a robust implementation of the strategy can be performed, based on the finding that changes in expected inflation are consistently larger in absolute values for short maturities vs. long maturities. •

The research from which this article was drawn was supported by Ontario Teachers' Pension Plan as part of the research chair on "Advanced Investment Solutions for Liability Hedging for Inflation Risk" at EDHEC-Risk Institute.

The purpose of the chair is to analyze the design of novel forms of inflation-hedging portfolios that do not solely rely on inflation-linked securities but instead involve substantial investment in traditional asset classes. Overall, these novel forms of inflation-hedging solutions should be engineered to generate higher expected performance for a given inflation hedging level, which in turn will allow for a decrease in the cost of inflation hedging.

The full version of the research is available on the EDHEC-Risk Institute website at the following address: http://www.edhec-risk.com/ALM/OTPP_Research_Chair

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Constructing Improved Corporate Bond Benchmarks

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The abundance of theoretical and empirical research on the performance of portfolio optimization techniques in the equity universe stands in sharp contrast to the relative scarcity of research about how to form bond portfolios with attractive risk/reward performance from an out-of-sample basis.

For example, there is no readily available answer in the academic literature to fundamental questions such as whether an investor in sovereign or corporate bonds would be better off investing in an equally weighted combination of available bonds compared to an optimally chosen combination on the basis of careful parameter estimates.

That relatively little is known about the out-of-sample performance of bond portfolio optimization models is perhaps surprising, given the importance of fixed-income investments in institutional and private investors' portfolios, which has only been reinforced by the global trend towards de-risking. As decreasing yields have pushed investors toward higher-yielding bonds, the demand has recently been particularly strong for corporate bonds, with a record issuance of $1.111 trillion in 2013 for U.S. investment-grade companies, an amount that surpasses the previous record of $1.053 trillion of bonds set in 2012.[10] A possible explanation is that bonds are often held as part of investors' hedging portfolios, where the focus is on matching interest rate risk factor exposures on the asset side to interest rate risk factor exposures on the liability side, as opposed to risk/reward ratio maximization. This is, however, not a sufficient reason for ignoring the need to generate attractive risk-adjusted performance. After all, there are very many bond portfolios with a given target duration, and selecting the one with the highest risk/reward ratio should intuitively improve investor welfare. Besides, Treasury and corporate bonds are also natural ingredients within investors' performance-seeking portfolios, where the focus lies precisely on maximizing the risk/reward ratio.

In a previous paper (Deguest et al. (2013)), we proposed a suitable methodology to address two key challenges that are specific to bond portfolio optimization — namely, the presence of duration constraints and the presence of no-arbitrage restrictions on risk and return parameter estimates, for which no equivalent exists in the equity universe. In an empirical analysis, we found that the use of portfolio optimization techniques generates an improvement in investor welfare compared to the use of ad hoc bond benchmarks such as equally weighted or cap-weighted portfolios.

In a follow-up paper (Deguest et al. (2014)), we extended the analysis to corporate bonds, using daily data on investment-grade bond returns from the Bank of America Merrill Lynch Corporate Bond Master Index universe in the U.S., and from the Markit Iboxx EUR Liquid Corporate Index universe in the Eurozone, by providing the first formal out-of-sample comparative analysis of the performance of various bond portfolio optimization models in the presence of duration constraints.

As in the sovereign bond universe, the first challenge is to introduce a model-free methodology that can be used to generate risk-return parameter estimates for bond portfolios that are consistent with the absence of arbitrage. To do so, we first use the no-arbitrage restriction to decompose each coupon-paying bond into a sum of fictitious constant-maturity pure-discount bonds matching coupon or principal payment dates and amounts. Then we use the transition matrix from pure-discount bond prices to coupon-paying bond prices to extract a consistent covariance matrix for non-stationary coupon-paying bond returns. This procedure ensures the respect of no-arbitrage conditions, as well as the respect of the structure inherent in bond prices, e.g., the convergence of bond return volatility to zero when approaching maturity. Finally, we "robustify" the covariance matrix we obtain for coupon-paying bonds by using a factor model for the term structure. The factors are extracted from a principal component analysis, with the first few factors typically explaining a substantial large fraction of the bond return variance. While we use implicit factors for the estimation of the covariance matrix, we rely on explicit factors for the estimation of expected returns, which requires at the minimum a proper understanding of the sign of the associated risk premia.

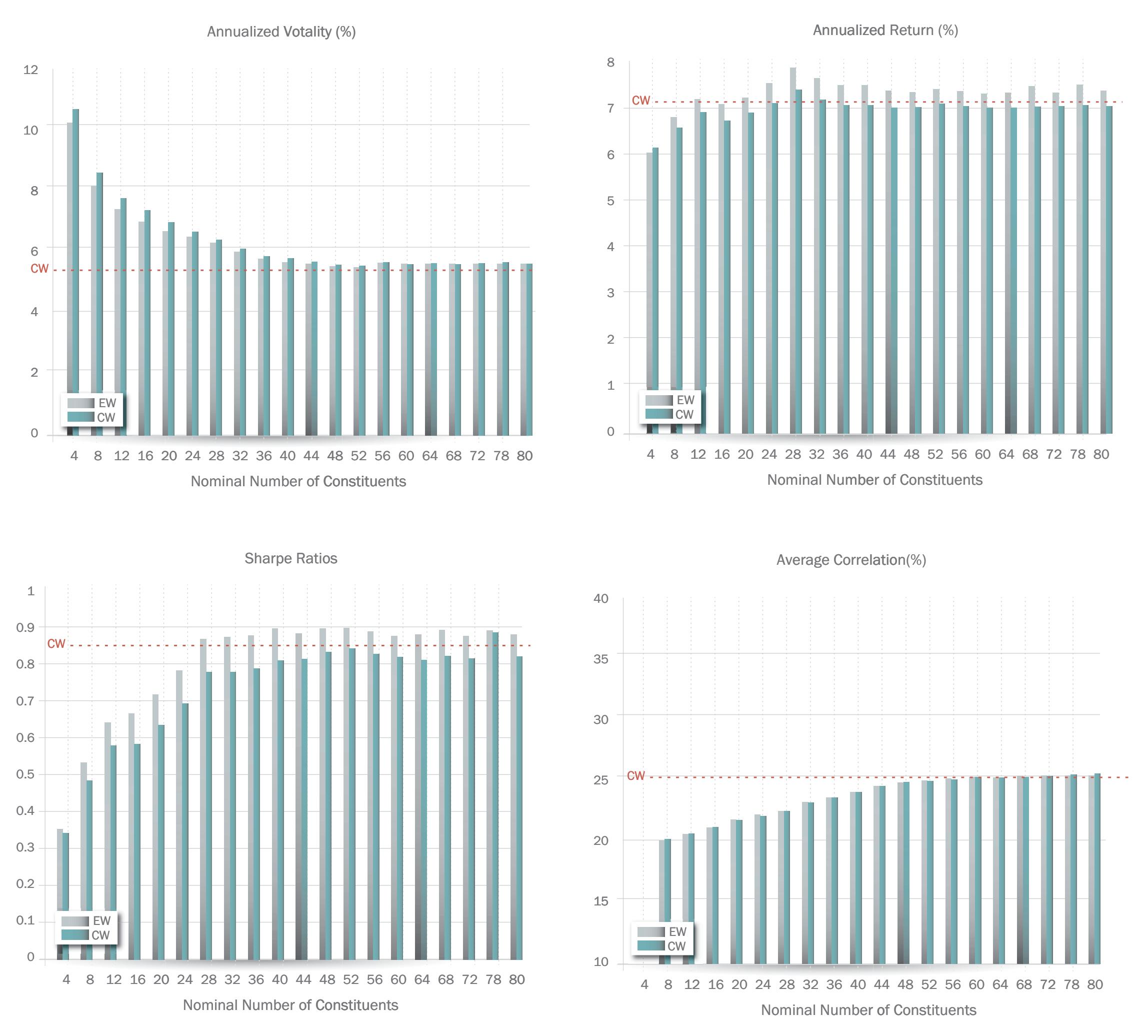
In this context, we first analyze risk-return tradeoffs in the corporate bond market universe and confirm that longer-duration, lower-credit and lower-size corporate bonds are riskier and have better performance than shorter-duration, higher-credit and higher-size bonds. We also find that the Sharpe ratio is higher for short-duration, low-quality bonds and low-size issues.[11]

Turning to an empirical analysis of the benefits of naïve diversification for corporate bonds, we find that increasing the number of constituents in a corporate bond portfolio decreases portfolio volatility with no corresponding deterioration in performance, which leads to increases in Sharpe ratio (see Figure 1). Considering the evolution of the average correlation with respect to the number of constituents, we find that the change in the correlation is not very significant (a slight increase from 20% to 25% as the number of constituents increases from four to 80) and thus cannot explain the decrease in volatility. In the U.S. universe, these benefits of diversification, however, reach a limit when the number of constituents

EXHIBIT 1

Benefits of Naïve Diversification

These figures represent the volatilities, annualized returns, and Sharpe ratios of the cap-weighted and equally weighted strategies implemented over various baskets of bonds with an increasing number of constituents (from 4 to 80) chosen among the 80 corporate bonds (adding the two bonds with the shortest and longest durations each time the number of constituents increases) that have the largest face values in the Bank of America Merrill Lynch US Corporate Bond Index (Bloomberg C0A0). We also provide the average correlations among the bonds in each of the baskets. These experiments are computed with monthly rebalanced portfolios using daily returns over the backtesting period July 1st 1997 to September 1st 2013. Correlations are measured over 3-month rolling window periods.



[10] Source: www.dealogic.com.
[11] The existence of a positive risk-return relationship is subsequently used to build scientifically-diversified maximum Sharpe ratio portfolios, an approach that we implement on the basis of the agnostic prior that all factors have the same expected excess return.

reaches approximately 40. This corresponds to one-half of the total universe (80 bonds), and we notice that after the saturation point, the volatility differences between the cap-weighted and equally weighted strategies shrink to 0.

Overall, these results suggest that diversification is useful for corporate bond portfolios. More precisely, it is possible to construct improved corporate bond benchmarks that would dominate concentrated heuristic alternative portfolio strategies, such as bullet or barbell strategies, often used in asset-liability management. On the other hand, our results suggest that it will be difficult to substantially decrease, through portfolio optimization methodologies, the volatility of a bond benchmark with an effective number of constituents that is greater than 40. In particular, the cap-weighted index with 80 bonds that we extract from the Bank of America Merrill Lynch US Corporate Bond Index lies in the saturation area, with a volatility of around 5.5%. We also find that equally weighted corporate bond portfolios have lower volatility (when the number of constituents is lower than 40), higher annualized return (whatever the number of constituents) and higher Sharpe ratio (whatever the number of constituents) than their cap-weighted counterparts.

To confirm these initial results, we turn to scientific diversification, and confirm that the use of formal portfolio optimization techniques, which allow for the proper integration of duration constraints, leads to substantial improvements in risk-adjusted performance with respect to heuristic barbell strategies for the same given duration targets (see Exhibit 2). In addition to maximum Sharpe ratio (MSR) maximization, we also use various heuristic portfolio optimization models that do not require expected return estimates, including minimum concentration (MC) portfolios (which correspond to the closest approximation to an equally weighted strategy subject to constraints such as duration or weight constraints), global minimum variance (GMV) portfolios.

We first select the 20 and 40 most liquid bonds based on their market capitalizations, a natural, albeit very imperfect, proxy for liquidity, and look at portfolios with a duration constraint equal to five years.[12] When the number of constituents is equal to 20 (see Panel (a) in Exhibit 2, then the volatility of the MC portfolio (5.56%) is higher than those obtained with scientifically diversified strategies such as the GMV or MSR (5.22% and 5.29%, respectively), leading to slightly higher Sharpe ratios for the optimal strategies (0.73 for both the GMV and MSR compared to 0.71 for the MC). However, when the number of constituents increases to 40 (see panel (b)), then the difference in risk-adjusted performance between naïve and scientific diversification vanishes. This result confirms the observation we made about the limited marginal benefits of diversification for a number of constituents greater than 40, defined as the saturation area.

Overall, our findings have important practical implications since they suggest that investors may be able to construct improved fixed-income benchmarks that would dominate standard ad hoc benchmarks from a risk-adjusted performance perspective for a given duration constraint. In the absence of a readily available alternative, traditional bond benchmarks used by investors are typically obtained by weighting individual securities by the total amount of debt outstanding, which is the exact counterpart of capitalization-weighting schemes for equity indexes.

While market-capitalization-weighted schemes have a number of advantages, including their ease of implementation, they have been criticized as being inappropriate in the bond universe. In particular, capitalization-weighted indexes are prone to the "bums problem" (Siegel (2003)), i.e., they are likely to overweight low-quality issues, and in any case, have no claim towards generating attractive risk-adjusted performance. Moreover, recent research has shown that the exposures of bond indexes to interest rate and credit risks were not stable over time (Campani and Goltz (2011)). A number of ad hoc alternative weighting schemes have been proposed (see Campani and Goltz (2011) for a review) but these initiatives are based on no academic grounds, and it is unclear whether the portfolios thus constructed would be optimal under any reasonable assumptions. Besides, the presence of duration constraints is not accounted for.

Our analysis can be extended in a number of ways. First, one could use implied probabilities of default extracted from structural defaultable bond pricing models to generate a better assessment of corporate bond exposure to credit risk compared to the use of credit ratings. Secondly, one could also extend the framework to allow for the introduction of time-varying views on factor returns. Finally, it would be desirable to extend our analysis to a dynamic long-term portfolio selection framework, in which case we expect that the introduction of intertemporal hedging demands would lead to further improvements in investor welfare compared to the use of myopic strategies such as the ones tested in this paper. •

EXHIBIT 2

Scientific versus Naive Diversification

These tables contain the main descriptive statistics of the following portfolio strategies under a 5Y-duration constraint and hard constraints ($\delta = 4$): barbell, maximum deconcentration (MC), global minimum variance (GMV), maximum Sharpe ratio (MSR) and a multi-strategy consisting of an equal mix of the previous three weighting schemes. These statistics are computed with daily returns over the back testing period July 1 1997 to September 1 2013 and semi-annual rebalancing frequency. The return, volatility, turnover, and tracking error are annualized. The turnover is one-way, and the tracking error is computed with respect to the barbell. Both Value-at-risk are computed for a holding period of one day, and with the data of the entire backtesting period. The figure represents the evolution of the wealths (the initial wealth being equal to 100) generated by following some of the strategies considered in the table.

(a) Portfolio with the 20 Most Liquid Corporate US Bonds

	Barbell	MC	GMV	MSR	Multi-Strategy
Ann. Ret. (%)	6.28	6.4	6.29	6.33	6.34
Ann. Vol. (%)	7.7	5.56	5.22	5.29	5.21
Sharpe Ratio	0.49	0.71	0.73	0.73	0.74
Max Drawdown (%)	13.58	16.76	13	12.98	14.19
VaR5% (%)	0.72	0.46	0.46	0.46	0.44
VaR1% (%)	1.28	0.91	0.83	0.86	0.84
Av. Duration	4.87	4.87	4.87	4.87	4.87
Tracking Error (%)	0	7.01	6.46	6.49	6.54
Extreme TE (95%)	0	16.34	14.15	14.16	14.68
Av. Rating	15.82	15.9	16.3	15.84	16.01
Av. Market Size	366.52	357.97	359.49	353.41	356.96
Turnover (%)	100.4	54.74	110.96	106.68	88.34
Av. Num. Constituents	2	20	20	20	20
Av. Deconcentration (%)	8.62	84.45	37.98	38.78	56.31
Min Weight (%)	0	0.97	0.82	0.82	0.99
Max Weight (%)	85.78	20.14	21.62	21.62	19.57

(b) Portfolio with the 40 Most Liquid Corporate US Bonds

	Barbell	MC	GMV	MSR	Multi-Strategy
Ann. Ret. (%)	5.88	6.76	6.51	6.56	6.61
Ann. Vol. (%)	7.76	4.67	4.51	4.63	4.49
Sharpe Ratio	0.44	0.92	0.89	0.88	0.92
Max Drawdown (%)	12.2	14.25	12.56	12.61	13.11
VaR5% (%)	0.74	0.4	0.42	0.43	0.41
VaR1% (%)	1.3	0.78	0.76	0.77	0.77
Av. Duration	4.88	4.87	4.87	4.87	4.87
Tracking Error (%)	0	7.24	7.23	7.34	7.2
Extreme TE (95%)	0	15.16	15.44	15.49	15.17
Av. Rating	15.53	15.74	15.98	15.52	15.75
Av. Market Size	303.48	302.55	307.47	295.06	301.69
Turnover (%)	124.52	47.79	112.04	109.76	87.39
Av. Num. Constituents	2	40	40	40	40
Av. Deconcentration (%)	4.29	84.32	36.64	36.7	54.94
Min Weight (%)	0	0.46	0.41	0.41	0.46
Max Weight (%)	82.55	8.1	10.91	10.89	9.69

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[12] In fact, they exhibit a duration that is slightly lower than 5 years because of the semi-annual rebalancing frequency that leads to small drifts from the perfect duration matching between each rebalancing date.