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# Research for Institutional Money Management

EDHEC-RISK Institute

## INTRODUCTION

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It is a great pleasure to introduce the latest edition of the EDHEC-Risk Institute "Research for Institutional Money Management" supplement in cooperation with *Pensions & Investments*.

With risk allocation gaining in popularity as an investment concept, we look at what risk allocation actually entails. Various interpretations exist for an approach that is presented as both a new investment paradigm and a re-interpretation of standard portfolio construction techniques. Risk allocation can be thought of both as a new investment paradigm advocating a focus on allocating to uncorrelated rewarded risk factors, as opposed to correlated asset classes, and a portfolio construction technique stipulating how to optimally allocate to these risk factors.

All smart-beta equity strategies are exposed to systematic risk factors and strategy-specific risks. Systematic risks refer to the exposure to risk factors that can be rewarded or not. They arise from the characteristics of the underlying stock universe and also from the portfolio construction. The non-rewarded or specific risk constitutes all the risks that do not have a premium in the long run, and are therefore not ultimately desired by the investor. The Smart Beta 2.0 framework allows for efficient management of exposures to rewarded risks while avoiding unrewarded (specific) risks. The smart factor indexes that result from this framework show pronounced improvements in risk-adjusted performance compared to that of cap-weighted factor-tilted indexes.

We analyze the performance of smart factor equity indexes in developed economies, at a local level, and in the global developed stock universe. Smart factor indexes show attractive performance on both an absolute and risk-adjusted basis in different developed markets. For each risk factor, the smart factor indexes outperform the tilted cap-weighted indexes in both relative returns and Sharpe ratio. Moreover, owing to international diversification, we also obtain low levels of tracking error, leading to interesting information ratios. The 5-year outperformance probability is extremely high (95%-100%), meaning the outperformance is both high and robust across these four factors.

We look at the performance and implementation benefits of multi—smart beta allocation, a new source of value added in investment management. Compared to the average stand-alone investment in a smart factor index, multibeta indexes almost always result in higher average returns net of costs owing to the turnover reduction through natural crossing effects among the component smart factor indexes.

In a further article, we provide a guide to factor investing in the equity space on the basis of the theoretical and empirical evidence in the academic literature. In order to avoid the pitfalls of non-persistent factor premia and achieve robust performance, investors should keep the following checks in mind. First, investors should require a sound economic rationale for the existence of a premium. Second, because of the risks of data mining, investors may be well advised to stick to simple factor definitions that are widely used in the literature rather than to rely on complex and proprietary factor definitions.

We verify in a separate article whether various risk controls, such as country and sector neutrality and also tracking error (TE) control, have any impact on the tail risk, or downside risk, of portfolios. We also check whether diversifying away strategy-specific risk by building a multistrategy smart-beta index affects tail risk. Generally, we do not find material improvement in tail risk when country and sector risks are controlled both for absolute and relative returns. Including a low TE target has an effect on tail risk, but the main factor behind the improvement is the reduced tracking error; the reduction in extreme risk is an expected side effect. Diversifying away strategy-specific risks by building a diversified multistrategy index diversifies the relative return tail risk. The main diversification benefits, however, arise from the related diversification benefits of the tracking error.

Finally, drawing on research supported by CACEIS as part of the New Advances in Risk Measurement and Reporting research chair at EDHEC-Risk Institute, we analyze the level of diversification of U.S. pension fund portfolios and the relationship between the level of diversification and subsequent portfolio performance. Overall, our results suggest that a pension fund that has a higher effective number of bets (and therefore holds a better-diversified portfolio) is more likely to incur lower loss levels than a pension fund that has a lower effective number of bets, assuming the policy portfolio weights remain constant.

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PORTFOLIO MANAGEMENT

# Risk Allocation Within and Across Asset Classes

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Risk allocation has gained increasing popularity among sophisticated investors. The number of papers, too numerous to be cited, recently published on the subject in the academic and practitioner literature is evidence of this trend.

What the concept exactly means, however, deserves some clarification. Indeed, various interpretations exist for what is sometimes presented as a new investment paradigm and sometimes presented as a simple re-interpretation of standard portfolio construction techniques.

To better understand the meaning of risk factor allocation, it is useful to go back to the foundations of asset pricing theory. Asset pricing theory suggests that individual securities earn their risk premium through their exposures to rewarded factors (see Merton, 1973, Intertemporal Capital Asset Pricing model for equilibrium arguments or Ross, 1976, Arbitrage Pricing Model for arbitrage arguments). Asset pricing theory also suggests that factors are rewarded if and only if they perform poorly during bad times and more than compensate during good times so as to generate a positive excess return on average across market conditions. In technical jargon, the expected excess return on a factor is proportional to the negative of the factor covariance with the pricing kernel, given by marginal utility of consumption for a representative agent (see Cochrane, 2000, for more details). Hence, if a factor generates an uncertain payoff that is uncorrelated to the pricing kernel, then the factor will earn no reward even though there is uncertainty involved in holding the payoff. On the other hand, if a factor payoff covaries positively with the pricing kernel, it means that it tends to be high when marginal utility is high, that is when economic agents are relatively poor. Because it serves as a hedge by providing income during bad times, when marginal utility of consumption is high, investors are actually willing to pay a premium for holding this payoff.

In this context, one can argue that the ultimate goal of portfolio construction is to invest in risky assets so as to ensure an efficient diversification of specific and systematic risks within the portfolio. Note that the word diversification is used with two different meanings. When the focus is on the diversification of specific risks, diversification means reduction of specific risk exposures that are not desirable because not rewarded. On the other hand, when the focus is on the diversification of systematic risks, diversification means efficient allocation to factors that bear a positive long-term reward, with modern portfolio theory suggesting that efficient allocation is in fact maximum risk/reward allocation (maximum Sharpe ratio in a mean-variance context).

This recognition provides us with a first interpretation for what risk allocation might mean. If the focus of portfolio construction is to harvest risk premia expected from holding an exposure to rewarded factors, it seems natural to express the allocation decision in terms of such risk factors. In this interpretation, the term factor allocation is a new paradigm advocating that investment decisions should be cast in terms of risk factor allocation decisions, as opposed to asset class allocation decisions, which are based on somewhat arbitrary classifications.

The second interpretation for what risk allocation might mean is to define it precisely as a portfolio construction technique that can be used to estimate what an efficient allocation to underlying components (which could be asset classes or underlying risk factors) should be. The starting point for this novel approach to portfolio construction is the recognition that a heavily concentrated set of risk exposures can be hidden behind a seemingly well-diversified allocation. In this context, the risk allocation approach, also known as a risk budgeting approach, to portfolio construction, consists in focusing on risk, as opposed to dollar, allocation. In a nutshell, the goal of risk allocation is to ensure that the contribution of each constituent to the overall risk of the portfolio is equal to a target risk budget (see Roncalli, 2013, for a comprehensive treatment of the subject). In the specific case where the allocated risk budget is identical for all constituents of the portfolio, the strategy is known as risk parity, in contrast to an equally-weighted strategy, which would recommend an equal contribution in terms of dollar budgets. To better understand the connection between this portfolio construction and standard recommendations from modern portfolio selection, it is useful to recognize that, when applied to uncorrelated factors, risk budgeting is consistent with mean-variance portfolio optimization under the assumption that Sharpe ratios are proportional to risk budgets.[1] Thus, risk parity is a specific case of risk budgeting, a natural neutral starting point that is consistent for uncorrelated factors with Sharpe ratio optimization, assuming constant Sharpe ratios at the factor level.

Such risk allocation techniques, defined as portfolio construction focusing on allocating wealth proportionally to risk budgets, can be used in two different contexts, across asset classes (for the design of a policy portfolio) or within asset classes (for the design of an asset class benchmark). In an asset allocation context, the focus of risk parity is to allocate to a variety of rewarded risk factors affecting the return on various asset classes so as to equalize (in the case of a specific focus on risk parity) the risk contribution to the policy portfolio variance.[2] Within a given asset class, for example, equities, risk parity can be applied to portfolio returns in the rare instances when there is no pre-existing benchmark, or, more often, to portfolio relative returns with respect to the investor's existing benchmark. In the latter case, it is the contribution of typical risk factors affecting the returns on the asset class (e.g., the Fama-French factors for equities) to portfolio tracking error, and not variance, that matters.

It should also be noted that the risk allocation framework can also be used to define a set of weight constraints, as opposed to being regarded as only an optimization objective. Indeed, weight constraints are known to be critically useful ingredients in scientific portfolio optimization models since they introduce a minimum level of naive diversification, which can be formally interpreted as providing an implicit form of statistical shrinkage similar to the one discussed in Ledoit and Wolf (2003). In particular, Jagannathan and Ma (2003) show that hard minimum and maximum weight constraints lead to superior out-of-sample performance, while DeMiguel, et al. (2009), obtain even better results with flexible constraints applying to the norm of the weight vector, which can be regarded as a minimum effective number of constituents constraint. In a similar spirit, Deguest, Martellini and Meucci (2013) show that mean-variance analysis with constraints on the effective number of independent bets is equivalent to a form of shrinkage towards a target portfolio that minimizes the factor exposure, as opposed to the weight vector as in DeMiguel et al. (2009).

Overall, it appears that risk allocation can be thought of as both a new investment paradigm advocating a focus on allocating to uncorrelated rewarded risk factors, as opposed to correlated asset classes, and a portfolio construction technique stipulating how to optimally allocate to these risk factors. It should be noted that the existence of factor replicating portfolios is not a necessary condition to perform risk budgeting. Indeed, one can use any set of well-diversified portfolios, as opposed to factor-replicating portfolios, as constituents, leaving to the asset allocation stage the hurdle to reach target factor exposures. We refer the interested reader to the paper by Romain Deguest and Lionel Martellini for an illustration of how risk allocation techniques can be applied to factor-tilted smart betas, which are designed to maximize risk/return efficiency based on stocks with selected attributes. ~

***References***

***Amenc, N. and L. Martellini.*** *2014. Risk Allocation: a New Investment Paradigm? Journal of Portfolio Management, winter 2014, Invited Editorial Comment.*
***Cochrane, J.*** *2000. Asset Pricing, Princeton University Press.*
***Deguest, R., L. Martellini, and A. Meucci.*** *2013. Risk Parity and Beyond— From Asset Allocation to Risk Allocation Decisions, working paper, EDHEC-Risk Institute.*
***DeMiguel, V., L. Garlappi, F. Nogales, and R. Uppal.*** *2009. A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms, Management Science, 55, 5, 798–812.*
***Jagannathan, R. and T. Ma.*** *2003. Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps, Journal of Finance, 58, 4, 1651–1683.*
***Ledoit, O., and Wolf, M.*** *2003. Improved Estimation of the Covariance Matrix of Stock Returns with an Application to Portfolio Selection, Journal of Empirical Finance, 10, 5, 603–621.*
***Meucci, A., R. Deguest and A. Santangelo.*** *2013. Measuring Portfolio Diversification Based on Optimized Uncorrelated Factors, working paper, available at ssrn.com/abstract=2276632.*
***Merton, R.*** *1973. An Intertemporal Capital Asset Pricing Model, Econometrica, 41, 5, 867–887.*
***Roncalli, T.*** *2013. Introduction to Risk Parity and Budgeting, Chapman & Hall, CRC Financial Mathematics Series.*
***Roncalli, T. and G. Weisang.*** *2012. Risk Parity Portfolios with Risk Factors, working paper, available at ssrn.com/abstract=2155159.*
***Ross, S.*** *1976. The Arbitrage Theory of Capital Asset Pricing, Journal of Economic Theory, 13, 3, 341–360.*

*[1] Orthogonalizing the factors is useful to avoid the arbitrary attribution of overlapping correlated components in the definition of risk budgets allocated to each of these factors. Principal component analysis (PCA) can be used to extract uncorrelated versions of the factors starting from correlated asset or factor returns (see for example Roncalli and Weisang, 2012, or Deguest, Martellini and Meucci, 2013). Alternatively, to avoid the difficulties related to the lack of stability and interpretability of principal components and to generate uncorrelated factors that are as close as possible to the original assets or factors, one can use the minimal linear torsion (MLT) approach recently introduced in Meucci, Deguest and Santangelo (2013).*

*[2] Risk allocation can also be defined with respect to downside risk measures such as semi-variance, Value at Risk (VaR) or expected shortfall (see Roncalli, 2013).*

PORTFOLIO MANAGEMENT

# Combining Factor Tilts and Diversification Strategies: Towards Smart Factor Indexes

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Alternative forms of equity indexes, which draw from a range of portfolio construction practices, have become increasingly popular in recent years. One approach is to use fundamental or accounting-based metrics for size, instead of market price, to weight stocks. On the other hand, scientific diversification-based approaches exist that have either a deconcentration objective (such as maximum deconcentration or maximum decorrelation) or a risk/return objective (such as maximum Sharpe ratio and minimum volatility). Whatever the weighting scheme, all alternative beta indexes directly or indirectly address either or both of the two major drawbacks of cap-weighted indexes. First, cap-weighted indexes do not efficiently diversify the unrewarded risks, as they are highly concentrated in the largest stocks. The mean effective number of stocks for the U.S.A. cap-weighted (CW) index is just 113, whereas the nominal number is 500[3] (Exhibit 3). These indexes also fail to benefit from rewarded systematic risk factors (such as size, value and momentum). Exhibit 1 shows that the CW indexes tilt toward low value (book-to-market) and large-cap stocks, and thereby do not capture value and small size risk premia (Fama and French, 1993).

Theoretically, CW indexes do not qualify as efficient benchmarks when evaluated according to the findings of two Nobel laureates, Harry Markowitz and Eugene Fama. Markowitz postulated the benefit of diversification of unrewarded risk (Modern Portfolio Theory). Fama empirically showed the existence of rewarded risk factors other than the market factor. Furthermore, it has been demonstrated that cap-weighted indexes do not provide fair compensation for the amount of risk taken (Haugen and Baker, 1991, Grinold,1992).

### Specific and systematic risks

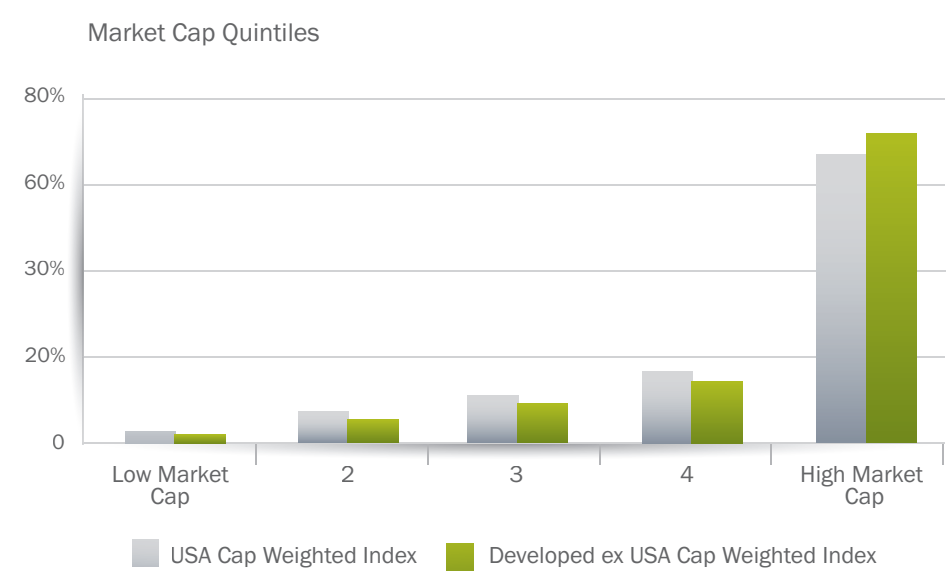
All smart-beta strategies are exposed to systematic risks and strategy-specific risks. Systematic risks refer to the exposure to risk factors that can be rewarded or not. They arise from the characteristics of the underlying stock universe and also from the portfolio construction. For example, an index formed on a selection of value stocks will have an explicit value tilt. Similarly, any weighting scheme that underweights larger-cap stocks relative to the CW index will inevitably lead to an increase in the exposure to smaller stocks, such as mid-cap stocks.

The non-rewarded, or specific, risk constitutes all the risks that do not have a premium in the long run and are therefore not desired by the investor. The first kind of specific risk is the exposure to unrewarded financial risk factors such as commodity, currency or sector. For example, minimum volatility portfolios tend to overweight certain defensive sectors. Another kind of non-rewarded financial risk is specific financial risks (also called idiosyncratic stock risks) that are related to the risks that are specific to the company itself. Similarly, all weighting schemes have specific operational risk that is specific to the implementation of the diversification model. For example, the robustness of the maximum Sharpe ratio scheme depends on an accurate

EXHIBIT 1

**Drawbacks of CW Indexes**

U.S.A. CW Index is the S&P 500 index. Developed ex-U.S. CW Index contains 1,500 stocks. For U.S. ( developed ex-U.S.), book-to-market quintiles and market-cap quintiles are formed every quarter and average values across 160 (40) quarters in the period from 12-31-1972 to 12-31-2012 (12-31-2003 to 12-31-2013) are reported. S&P® and S&P 500® are registered trademarks of Standard & Poor's Financial Services LLC ("S&P"), a subsidiary of The McGraw-Hill Companies, Inc.


[3] *The effective number of stocks (ENS) is defined as the reciprocal of the Herfindahl Index, which is defined as the sum of squared weights across portfolio constituents.*

estimate of the covariance matrix and expected returns.

Each weighting scheme, despite being smart, is exposed to these strategy-specific risks. Portfolio theory suggests that specific risks are neither predictable nor rewarded, so one is better off completely avoiding them by investing in a well-diversified portfolio. The diversified multistrategy approach, which combines the five different weighting schemes in equal proportion, is based on this specific risk diversification principle (Tu and Zhou, 2010, Kan and Zhou, 2007). Moreover, since single strategies' performance shows dependency on market conditions, a multistrategy approach can help investors smooth the overall performance across market conditions (Amenc, et al., 2012).

### Smart factor indexes: the Smart Beta 2.0 approach

Stocks earn a risk premium through their exposure to certain rewarded factors (Ross, 1976). The economic explanation for the existence of a reward for a given risk factor is that exposure to such a factor is undesirable for the average investor because it leads to losses in bad times, that is, when marginal utility is high, see, e.g., Cochrane, 2001). Fama and French have shown that value (book-to-market) and size (market cap) explain average asset returns as a complement to the market beta (Fama and French, 1993). Zhang (2005) provides a rationale for the value premium by arguing that value firms suffer more in bad times because their stock price is mainly made up of tangible assets. Similarly, the small size premium is often explained by low liquidity (Amihud and Mendelsson, 1986) and high downside risk (Chan, Chen and Hsieh, 1985). Carhart (1997) empirically proved the existence of another priced factor: the momentum factor. Momentum is explained by the sensitivity of past winner stocks to expected growth (Liu and Zhang, 2008) and by the short-term overreaction of investors (Daniel, et al., 1998). The low volatility factor, which qualifies as an anomaly rather than a risk factor, is the result of the famous "volatility puzzle," which states that low-volatility stocks tend to outperform high-volatility stocks in the long run (Ang, et al., 2006). The anomaly has been recognized as a persistent phenomenon and has been explained through leverage constraints and the lottery preferences of investors (Baker, Bradley and Wurgler, 2011).

These findings have given way to factor indexes, which fall into two major categories. The first involves selecting stocks that are most exposed to the desired risk factor and applying a weighting scheme to this selection. While this approach responds to one limitation of cap-weighted indexes, the choice of exposure to a good factor, the problem of poor diversification arising from high concentration in a small number of stocks remains unanswered. For the same underlying stocks, smart factor indexes outperform the equivalent tilted cap-weighted indexes by 68% on average.[4] The second method involves maximizing the exposure to a factor, either by weighting the whole of the universe on the basis of the exposure to this factor (score/rank weighting), or by selecting and weighting by the exposure score of the stock to that factor. Here again, the maximization of the factor exposure does not guarantee that the indexes are well diversified.

To overcome these difficulties, index providers that generally offer factor indexes on the basis of the first two approaches have recently sought to take advantage of the development of smart-beta indexes to offer investors a new framework for smart factor investing (Bender, et al., 2013). This approach recognizes that smart betas have implicit risk exposures and aims to select and combine them according to these varying exposures. The drawback of this approach is that it maximizes neither factor exposure nor diversification of the indexes.

For example, a minimum-volatility index on a broad universe does not guarantee either the highest exposure to low

EXHIBIT 2

**Sharpe ratio and Information ratio of Competing Factor Indexes**

The table shows Sharpe ratios and Information ratios of Russell, S&P, and MSCI indices marketed as factor indices with the same performance metric for the corresponding Scientific Beta US Diversified Multi-Strategy and CW indices with stock selection based on mid cap, momentum, low volatility, and value, as well as the SciBeta Broad CW. All statistics are annualized and the analysis is based on daily total returns. Data is always taken for the ten-year period 01/2004 to 12/2013 as available on Bloomberg; Indices which have shorter than 10-year data available are compared for their respective period of data availability to broad CW, the corresponding tilted CW, and Smart Factor Index for the same period. MSCI® is a registered trademark of MSCI Inc. S&P® and S&P 500® are registered trademarks of Standard & Poor's Financial Services LLC ("S&P"), a subsidiary of The McGraw-Hill Companies, Inc. Russell 1000® and Russell® are registered trademarks of Russell Investments.

| | | Sharpe Ratio | | | | Information Ratio | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PROVIDER | Tilt | Broad CW | Competitive Index | Tilted CW | SciBeta Diversified Multi-strategy Index | Competitive Index | Tilted CW | SciBeta Diversified Multi-strategy Index | From | To | Full Competitive Index Name |
| RUSSELL | Low Vol | 0.28 | 0.37 | 0.32 | 0.44 | 0.05 | 0.03 | 0.36 | 03/01/2005 | 31/12/2013 | Russell 1000 Low Volatility |
| | Mid Cap | 0.30 | 0.38 | 0.40 | 0.45 | 0.51 | 0.54 | 0.74 | 01/01/2004 | 31/12/2013 | Russell Mid Cap |
| | Value | 0.30 | 0.27 | 0.27 | 0.43 | -0.03 | -0.03 | 0.84 | 01/01/2004 | 31/12/2013 | Russell 1000 Value |
| | Mom. | 0.28 | 0.31 | 0.33 | 0.34 | 0.12 | 0.29 | 0.24 | 03/01/2005 | 31/12/2013 | Russell US LC High Momentum |
| S&P | Low Vol | 0.26 | 0.26 | 0.31 | 0.40 | -0.06 | 0.07 | 0.31 | 31/03/2006 | 31/12/2013 | S&P 1500 Reduced Volatility Tilt |
| | Mid Cap | 0.30 | 0.38 | 0.40 | 0.45 | 0.39 | 0.54 | 0.74 | 01/01/2004 | 31/12/2013 | S&P Mid Cap 400 |
| | Value | 0.26 | 0.27 | 0.18 | 0.29 | 0.19 | -0.30 | 0.27 | 31/03/2006 | 31/12/2013 | S&P 1500 Low Valuation Tilt |
| | Mom. | 0.26 | 0.25 | 0.28 | 0.26 | -0.10 | 0.10 | -0.03 | 31/03/2006 | 31/12/2013 | S&P 1500 Positive Momentum Tilt |
| MSCI | Low Vol | 0.30 | 0.39 | 0.36 | 0.50 | 0.10 | 0.06 | 0.47 | 01/01/2004 | 31/12/2013 | MSCI USA Minimum Volatility |
| | Mid Cap | 0.30 | 0.35 | 0.40 | 0.45 | 0.41 | 0.54 | 0.74 | 01/01/2004 | 31/12/2013 | MSCI USA Equal Weighted |
| | Value | 0.30 | 0.27 | 0.27 | 0.43 | -0.04 | -0.03 | 0.84 | 01/01/2004 | 31/12/2013 | MSCI USA Value Weighted |
| | Mom. | 0.30 | 0.38 | 0.35 | 0.39 | 0.23 | 0.22 | 0.34 | 01/01/2004 | 31/12/2013 | MSCI USA Momentum |

[4] *Average percent increase in Sharpe ratio observed between the 12/31/1972 and the 12/31/2012 (40 years) for all long-term track record smart factor indexes over their cap-weighted factor equivalent calculated on a universe of the 500 largest capitalization U.S. stocks.*

volatility stocks or the best diversification of this low volatility portfolio. Similarly, seeking exposure to the size factor through equal weighting of a broad universe is certainly less effective than selecting the smallest size stocks in the universe and then diversifying them and including an equal weighting scheme. Finally, seeking to be exposed to the value factor through a value-weighted index will not produce a well-diversified index, simply because the integration of the attributes characterizing the value exposure into the weighting does not take the correlations between these stocks into account. Exhibit 2 shows that smart factor indexes that use diversification-based weighting schemes for each factor outperform competitors' factor indexes.

In view of these problems, EDHEC-Risk Institute has promoted the concept of smart factor investing using the Smart Beta 2.0 approach. The idea is to construct a factor-tilted portfolio to extract the factor premia most efficiently and is based on: 1) explicitly selecting appropriate stocks for the desired beta and 2) using a diversification-based weighting scheme (Amenc, et al., 2013). ERI Scientific Beta constructs smart-factor indexes by using diversified multistrategy weighting on characteristics-based half universes: small size, high momentum, low volatility, and value.[5] The Smart Beta 2.0 approach allows investors not only to manage systematic risks but also to diversify strategy-specific risk by combining different strategies.

### Comparing the performance of smart factor indexes with tilted cap-weighted indexes

Tilted indexes outperform the broad CW index because they are exposed to the rewarded factors. However, it is useful to note that smart factor indexes outperform tilted cap-weighted indexes on both absolute and risk-adjusted bases. In both U.S. and developed ex-U.S., the Sharpe ratio of smart factor indexes is considerably higher than that of tilted CW indexes. The robustness of outperformance is apparent by looking at the outperformance probability (5Y), which measures how often the strategy has managed to outperform the broad CW reference index in the past for a five-year investment horizon. With U.S. long term, the low volatility diversified multistrategy index has much higher outperformance probability compared to the low-volatility CW index.

A similar observation can be made for the value factor in the developed ex-U.S. universe. The GLR measure,[6] a measure of diversification benefits, shows that smart factor indexes benefit from risk reduction by exploiting imperfect stock correlations, which tilted CW fails to do, as it ignores any information on correlations. With the exception of the developed ex-U.S. mid-cap universe,[7] smart factor indexes are more deconcentrated, as suggested by their higher ENS numbers.

### Assessing the investability of smart factor indexes

ERI Scientific Beta applies weight adjustments to limit liquidity issues that may arise upon investing and upon rebalancing. The indexes are also governed by an optimal turnover control technique based on rebalancing thresholds, the object of which is to reduce turnover and associated transaction costs.[8] To meet the needs of investors who have high liquidity constraints, high-liquidity smart factor indexes can be constructed. These indexes make a highly liquid stock selection on top of the existing factor-tilted selection and then use the appropriate weighting scheme. This selection of liquid stocks further increases the liquidity and capacity of the indexes proposed, whether it involves measuring the capacity effect or, to an even greater degree, the impact in terms of trading days to enter (or exit) the investment. On the basis of a theoretical investment of $1bn, the number of trading days required to carry out the investment, given the liquidity of each of the underlying stocks, remains very low. With the exception of the momentum tilt,[9] all smart factor indexes have manageable levels of one-way annual turnover. Transaction cost of 20 bps per 100% 1-W (100% one-way) turnover represents the worst case observed historically and 100 bps represents an 80% reduction in market liquidity. The excess returns net of

EXHIBIT 3

**Performance and Risk Analysis**

Broad cap-weighted index, containing 500 (1,500) stocks for US Long Term (Developed ex-US) is used as the benchmark. 95% tracking error is the 95th percentile of the tracking error computed using a rolling window of one year and step size of one week. Maximum relative drawdown is the maximum drawdown of the long-short index, the return of which is given by the fractional change in the ratio of the strategy index to the benchmark index. Probability of outperformance (5Y) is the historical empirical probability of outperforming the benchmark over a typical investment horizon of 5 years and is computed using a rolling window analysis with 1-week step size. The Secondary Market US Treasury Bill (3M) is the risk-free rate. All statistics are annualized. The GLR measure is the ratio of the portfolio variance to the weighted variance of its constituents. The effective number of stocks (ENS) is defined as the reciprocal of the Herfindahl Index, which in turn is defined as the sum of squared weights of portfolio constituents. For US Long Term (Developed ex-US), the analysis is based on daily total returns and quarterly weights from 31-12-1972 to 31-12-2012 (31-12-2003 to 31-12-2013).

| USA Long Term Track Records (Dec 1972 – Dec 2012) | | Mid Cap | | High Momentum | | Low Volatility | | Value | |
|---|---|---|---|---|---|---|---|---|---|
| | USA Long Term Cap Weighted | CW | Diversified Multi-strategy | CW | Diversified Multi-strategy | CW | Diversified Multi-strategy | CW | Diversified Multi-strategy |
| Ann Returns | 9.74% | 12.54% | 14.19% | 10.85% | 13.30% | 10.09% | 12.64% | 11.78% | 14.44% |
| Ann Volatility | 17.47% | 17.83% | 16.73% | 17.60% | 16.30% | 15.89% | 14.39% | 18.02% | 16.55% |
| Sharpe Ratio | 0.24 | 0.39 | 0.52 | 0.30 | 0.48 | 0.29 | 0.50 | 0.35 | 0.54 |
| Ann Excess Returns | - | 2.80% | 4.45% | 1.10% | 3.56% | 0.35% | 2.90% | 2.04% | 4.70% |
| Ann Tracking Error | - | 5.99% | 6.80% | 3.50% | 4.88% | 4.44% | 6.17% | 4.74% | 5.82% |
| 95% Tracking Error | - | 9.38% | 11.55% | 6.83% | 8.56% | 9.20% | 11.51% | 8.70% | 10.15% |
| Information Ratio | - | 0.47 | 0.66 | 0.32 | 0.73 | 0.08 | 0.47 | 0.43 | 0.81 |
| Max Rel Drawdown | - | 35.94% | 42.06% | 14.44% | 17.28% | 33.82% | 43.46% | 20.31% | 32.68% |
| Outperf. prob (5Y) | - | 75.3% | 78.9% | 86.8% | 91.2% | 54.3% | 85.0% | 72.0% | 88.3% |
| GLR Measure | 26.51% | 19.12% | 16.72% | 28.52% | 21.08% | 29.60% | 22.20% | 26.46% | 19.51% |
| Mean ENS | 113 | 181 | 191 | 65 | 199 | 64 | 201 | 69 | 190 |

| Developed ex US (Dec 2003 – Dec 2013) | | Mid Cap | | High Momentum | | Low Volatility | | Value | |
|---|---|---|---|---|---|---|---|---|---|
| | Developed ex US Cap Weighted | CW | Diversified Multi-strategy | CW | Diversified Multi-strategy | CW | Diversified Multi-strategy | CW | Diversified Multi-strategy |
| Ann Returns | 7.91% | 9.83% | 10.01% | 9.26% | 11.16% | 9.38% | 11.06% | 8.02% | 10.03% |
| Ann Volatility | 19.03% | 18.22% | 16.57% | 19.16% | 17.11% | 17.18% | 15.26% | 20.40% | 18.74% |
| Sharpe Ratio | 0.33 | 0.45 | 0.51 | 0.40 | 0.56 | 0.45 | 0.62 | 0.32 | 0.45 |
| Ann Excess Returns | - | 1.93% | 2.10% | 1.35% | 3.25% | 1.48% | 3.15% | 0.12% | 2.13% |
| Ann Tracking Error | - | 3.65% | 4.36% | 3.40% | 4.33% | 3.27% | 5.10% | 2.96% | 2.98% |
| 95% Tracking Error | - | 7.05% | 9.01% | 6.25% | 8.97% | 5.57% | 9.82% | 4.95% | 5.50% |
| Information Ratio | - | 0.53 | 0.48 | 0.40 | 0.75 | 0.45 | 0.62 | 0.04 | 0.71 |
| Max Rel Drawdown | - | 11.25% | 8.51% | 8.80% | 12.91% | 6.95% | 10.31% | 9.34% | 9.09% |
| Outperf. prob (5Y) | - | 100.0% | 100.0% | 84.7% | 100.0% | 100.0% | 100.0% | 34.7% | 83.6% |
| GLR Measure | 27.64% | 21.91% | 20.43% | 28.18% | 22.23% | 30.49% | 24.26% | 30.76% | 25.53% |
| Mean ENS | 300 | 435 | 382 | 159 | 428 | 147 | 415 | 149 | 375 |

[5] *The smart factor indexes are available for nine geographical universes: U.S.A., U.K., Eurozone, Europe ex-U.K., Japan, Asia Pacific ex-Japan, Developed, Developed ex-U.S., and Developed ex-U.K.*
[6] *GLR measure, which can be used to measure the diversification benefit, is the ratio of portfolio variance to the weighted variance of its constituents (Goetzmann, Li and Rouwenhorst, 2001).*
[7] *The developed exU.S. midcap universe has quite a flat market cap profile. Therefore in this selection cap weights do not pose the problem of high concentration in few stocks.*
[8] *For more information on turnover and liquidity rules, refer to the white paper "Overview of Diversification Strategies" by Gonzalez and Thabault (2013).*
[9] *Persistence in price movement is a short-term phenomenon and mean reversion is observed in longer horizons. Therefore, to extract the momentum premium, the momentum score assignment is done semi-annually, which results in higher turnovers (Chan, et al., 1999).*

EXHIBIT 4

**Investability**

Weighted average market capitalization of index is in million $. Turnover is mean annual 1-way. For US Long Term (Developed ex US), they are average values across 160 (40) quarters in the period from 31-12-1972 to 31-12-2012 (31-12-2003 to 31-12-2013).The net returns are the relative returns over the cap-weighted benchmark net of transaction costs. Two levels of transaction costs are used - 20 bps per 100% 1-W turnover and 100 bps per 100% 1-W turnover. Days To Trade is the number of days necessary to trade the total stock positions, assuming a USD 1bln AUM and that 100% of the Average Daily Dollar Traded Volume can be traded every day. (*)Due to data availability, the period is restricted to last 10 years of the sample for Scientific Beta US indices.

| USA Long Term Track Records (Dec 1972 – Dec 2012) | | Diversified Multi-Strategy | | | | High Liquidity Diversified Multi-Strategy | | | |
|---|---|---|---|---|---|---|---|---|---|
| | USA Long Term Cap Weighted | Mid Cap | High Momentum | Low Volatility | Value | Mid Cap | High Momentum | Low Volatility | Value |
| Wgt Avg Mkt Cap (m$) | 45 171 | 2 750 | 12 853 | 13 738 | 8 373 | 3 312 | 18 610 | 19 795 | 11 749 |
| Days to trade for $1bn Initial Investment (Quantile 95%)(*) | 0.03 | 0.24 | 0.18 | 0.20 | 0.19 | 0.24 | 0.12 | 0.15 | 0.13 |
| 1-Way Annual Turnover | 2.66% | 23.79% | 63.81% | 25.80% | 23.93% | 31.00% | 66.76% | 27.40% | 27.55% |
| Ann Excess Returns | - | 4.45% | 3.56% | 2.90% | 4.70% | 4.28% | 2.80% | 2.09% | 4.10% |
| Net Ret (20 bps) | - | 4.41% | 3.43% | 2.85% | 4.65% | 4.22% | 2.67% | 2.03% | 4.04% |
| Net Ret (100 bps) | - | 4.22% | 2.92% | 2.65% | 4.46% | 3.97% | 2.13% | 1.81% | 3.82% |

| Developed ex US (Dec 2003 – Dec 2013) | | Diversified Multi-Strategy | | | | High Liquidity Diversified Multi-Strategy | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Developed ex US Cap Weighted | Mid Cap | High Momentum | Low Volatility | Value | Mid Cap | High Momentum | Low Volatility | Value |
| Wgt Avg Mkt Cap (m$) | 42 776 | 3 572 | 12 308 | 14 911 | 14 115 | 4 001 | 17 306 | 21 895 | 19 613 |
| Days to trade for $1bn Initial Investment (Quantile 95%) | 0.06 | 1.75 | 1.14 | 1.27 | 1.13 | 0.76 | 0.33 | 0.38 | 0.36 |
| 1-Way Annual Turnover | 4.49% | 40.50% | 80.62% | 30.59% | 32.95% | 43.41% | 82.45% | 30.79% | 31.52% |
| Ann Excess Returns | - | 2.10% | 3.25% | 3.15% | 2.13% | 2.41% | 3.06% | 2.97% | 1.84% |
| Net Ret (20 bps) | - | 2.02% | 3.09% | 3.09% | 2.06% | 2.32% | 2.90% | 2.90% | 1.78% |
| Net Ret (100 bps) | - | 1.70% | 2.45% | 2.84% | 1.80% | 1.97% | 2.24% | 2.66% | 1.52% |

unrealistically high transaction costs, even for high momentum indexes, remains significantly high.

The Smart Beta 2.0 framework allows efficient management of exposures to rewarded risks while avoiding unrewarded (specific) risks. The smart factor indexes that result from this framework show pronounced improvements in risk-adjusted performance compared to cap-weighted factor tilted indexes. Such smart factor indexes provide suitable building blocks for the implementation of static or dynamic factor allocation decisions. ~

***References***

***Amenc, N., F. Goltz and L. Martellini.*** *2013. Smart Beta 2.0. EDHEC-Risk Institute Position Paper.*

***Amenc, N., F. Goltz, A. Lodh and L. Martellini.*** *2012. Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance. Journal of Portfolio Management 38 (3): 72–88.*

***Amihud, Y., and H. Mendelson.*** *1986. Asset Pricing and the Bid-Ask Spread, Journal of Financial Economics 17, 223–249.*

***Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang.*** *2006. The Cross-Section of Volatility and Expected Returns, Journal of Finance, 61 (1), 259–99.*

***Baker, M. P., B. Bradley, and J. Wurgler.*** *2011. Benchmarks as Limits to Arbitrage: Understanding the Low Volatility Anomaly, Financial Analysts Journal, 67 (1), 1–15.*

***Bender, J., R. Briand, D. Melas, R. Subramanian and M. Subramanian.*** *December 2013. Deploying Multi-Factor Index Allocations in Institutional Portfolios. MSCI White Paper (www.msci.com/resources/pdfs/Foundations_of_Factor_Investing.pdf).*

***Carhart, M. M.*** *1997. On Persistence in Mutual Fund Performance. Journal of Finance, 52: 57–82.*

***Chan K.C., J. Karceski and J. Lakonishok.*** *1999. On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model, Review of Financial Studies.*

***Chan, K. C., Chen, N. F., and D. A. Hsieh.*** *1985. An Exploratory Investigation of the Firm Size Effect. Journal of Financial Economics 14(3), 451–471.*

***Cochrane, J.*** *2001. Asset Pricing, Princeton University Press*

***Daniel, K., D. Hirshleifer and A. Subrahmanyam.*** *1998. Investor Psychology and Security Market under- and Overreactions, Journal of Finance Vol. 53, No. 6 (Dec., 1998), pp. 1839–1885.*

***Fama, E. F., and K. R. French.*** *1993. Common Risk Factors in the Returns on Stocks and Bonds, Journal of Financial Economics, 33 (1), 3–56.*

***Goetzmann, W., L. Li and K. G. Rouwenhorst.*** *2001. Long-Term Global Market Correlations. NBER Working Paper*

***Gonzalez, N. and A. Thabault.*** *2013. Overview of Diversification Strategies. ERI Scientific Beta White Paper.*

***Grinold, R.*** *1992. Are Benchmark Portfolios Efficient? Journal of Portfolio Management 19(1): 34–40.*

***Haugen, R. and N. Baker.*** *1991. The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios. Journal of Portfolio Management 17(3): 35–40.*

***Kan, R. and G. Zhou.*** *2007. Optimal Portfolio Choice with Parameter Uncertainty. Journal of Financial and Quantitative Analysis, Cambridge University Press, vol. 42(03), pages 621–656.*

***Liu, L. X., and L. Zhang.*** *2008. Momentum Profits, Factor Pricing, and Macroeconomic Risk, Review of Financial Studies, 21(6), 2417–2448.*

***Ross, S.*** *1976. The Arbitrage Theory of Capital Asset Pricing. Journal of Economic Theory 13 (3): 341–360.*

***Tu, J. and G. Zhou.*** *2010. Incorporating Economic Objectives into Bayesian Priors: Portfolio Choice under Parameter Uncertainty. Journal of Financial and Quantitative Analysis, Cambridge University Press, vol. 45(04), pages 959–986.*

***Zhang, L.****, 2005, The Value Premium, Journal of Finance.*

PORTFOLIO MANAGEMENT

# The Performance of Smart Factor Indexes in Developed Economies

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Exposure to rewarded factors is a key source of long-term performance for smart-beta strategies. In addition to harvesting the risk premia associated with the rewarded long-term factors (such as value, momentum, etc.), factor indexes need to avoid taking unrewarded risks, such as stock-specific risk or the model risks inherent in a particular index weighting scheme. Important in smart factor investing are i) the choice of the right risk factor and ii) the choice of a diversification-based weighting scheme to best extract the risk premium.

ERI Scientific Beta offers five robust diversification schemes: maximum deconcentration, diversified risk weighted, maximum decorrelation, efficient minimum volatility and efficient maximum Sharpe ratio. However, these schemes are exposed to non-rewarded operational risks that are specific to the implementation of the diversification model. For example, the robustness of the maximum Sharpe ratio scheme depends on an accurate estimate of the covariance matrix and expected returns. Also, these weighting schemes have residual exposure to other financial risk factors (e.g. commodity, currency, sector risks) and specific financial risks (company-level idiosyncratic risks) which can be reduced by diversification. The diversified multistrategy approach combines the five different weighting schemes to reduce the non-rewarded strategy-specific risks (Amenc, et al., 2012) and is thus used for the construction of smart factor indexes.[10] We assess the performance of smart factor indexes[11] on four well-known factors: mid cap (as a proxy for small cap), high momentum, low volatility, and value.[12]

In this article, we analyze the performance of smart factor indexes in developed economies (at a local level) and in the global developed stock universe. Our results thus provide an assessment of the consistency of the performance of the smart factor indexing approach when applied to different stock universes.

### Assessing the performance of local smart factor indexes

The Scientific Beta global universe consists of 2,700 securities and is divided into 12 non-overlapping basic geographic blocks, each comprising a fixed number of securities. Eligibility of securities for each basic geographic block is determined by various criteria such as country classification, exchange on which it is traded, and issue date. The eligible securities are subject to a free-float, market-cap screen and liquidity screen to select the largest and most liquid securities available to non-domestic investors—the basic geographic blocks. Eight out of 12 basic geographic blocks are developed blocks and contain 2,000 securities in total. Each Scientific Beta investable universe is an aggregate of one or more of these geographic blocks.[13] The local developed universes analyzed and their respective stock universe sizes are: U.S.A. (500), Eurozone (300), U.K. (100), Japan (500), and Asia Pacific ex-Japan (400).

Exhibit 1 presents performance statistics for smart factor indexes in different locations. The benchmark used is the cap-weighted (CW) index constructed using all stocks in the respective region (broad cap-weighted index). Tilted cap-weighted indexes are portfolios which are based on the

EXHIBIT 1

**Absolute Performance and Risk of Local Scientific Beta Diversified Multi-Strategy Factor Indexes**
Benchmark is the Cap-Weighted index on the full universe for each region. Risk-free rate used for these regions is Secondary Market US T-bill (3M), Euribor (3M), UK T-bill (3M), Japan Gensaki T-bill (1M) and Secondary Market US T-bill (3M) respectively. All statistics are annualized. The analysis is based on daily total returns from 31-Dec-2003 to 31-Dec-2013 (10 years).

| | | Mid Cap | | High Momentum | | Low Volatility | | Value | |
|---|---|---|---|---|---|---|---|---|---|
| | Broad Cap Weighted | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy |
| USA | | | | | | | | | |
| Ann Returns | 7.68% | 10.41% | 10.80% | 8.64% | 9.40% | 7.94% | 10.08% | 7.56% | 10.54% |
| Ann Volatility | 20.23% | 22.33% | 20.29% | 20.38% | 20.07% | 17.82% | 16.99% | 22.46% | 20.63% |
| Sharpe Ratio | 0.30 | 0.40 | 0.45 | 0.35 | 0.39 | 0.36 | 0.50 | 0.27 | 0.43 |
| Eurozone | | | | | | | | | |
| Ann Returns | 6.35% | 7.99% | 8.41% | 9.09% | 10.60% | 8.39% | 9.19% | 6.09% | 7.68% |
| Ann Volatility | 20.58% | 18.63% | 16.69% | 19.74% | 16.66% | 18.35% | 14.96% | 22.81% | 20.26% |
| Sharpe Ratio | 0.21 | 0.32 | 0.38 | 0.35 | 0.51 | 0.34 | 0.47 | 0.18 | 0.28 |
| UK | | | | | | | | | |
| Ann Returns | 8.32% | 11.76% | 11.10% | 9.46% | 12.71% | 8.19% | 11.86% | 6.04% | 10.09% |
| Ann Volatility | 19.18% | 19.67% | 17.95% | 20.57% | 17.99% | 16.57% | 15.33% | 21.35% | 19.43% |
| Sharpe Ratio | 0.30 | 0.46 | 0.47 | 0.33 | 0.56 | 0.34 | 0.60 | 0.16 | 0.38 |
| Japan | | | | | | | | | |
| Ann Returns | 4.09% | 4.97% | 5.72% | 3.64% | 5.31% | 5.34% | 7.15% | 5.65% | 6.86% |
| Ann Volatility | 22.62% | 21.21% | 19.26% | 22.39% | 19.95% | 19.50% | 17.42% | 22.60% | 20.15% |
| Sharpe Ratio | 0.17 | 0.23 | 0.29 | 0.15 | 0.26 | 0.26 | 0.40 | 0.24 | 0.33 |
| Asia Pacific ex Japan | | | | | | | | | |
| Ann Returns | 12.91% | 15.31% | 15.91% | 16.12% | 18.01% | 13.86% | 14.24% | 14.92% | 16.82% |
| Ann Volatility | 23.93% | 23.08% | 20.72% | 25.45% | 22.13% | 22.85% | 17.74% | 24.36% | 21.93% |
| Sharpe Ratio | 0.47 | 0.60 | 0.69 | 0.57 | 0.74 | 0.54 | 0.71 | 0.55 | 0.70 |

[10] *For more details on the weighting scheme, refer to the ERI Scientific Beta white paper, "Scientific Beta Diversified Multistrategy Index," by Badaoui and Lodh (2013).*
[11] *The term "smart factor indexes" is used to refer to diversified multistrategy indexes.*
[12] *Refer to the article "A Guide to Factor Investing in the Equity Space," in the current P&I supplement for information on the choice of factors, the economic rationale behind these factors, and empirical evidence of their risk premia.*
[13] *More information on the Scientific Beta stock universe can be found in the ERI Scientific Beta white paper, "ERI Scientific Beta Universe Construction Rules."*

same characteristics-based stock selection as respective smart factor indexes, and are cap weighted. They represent poorly diversified factor-tilted portfolios. All smart factor indexes in all regions outperform the broad cap-weighted index. For example, in the U.K. the improvement in returns over the cap-weighted reference index ranges from 1.77% for the value factor to 4.39% for the momentum factor. In Japan, smart factor indexes outperform the broad cap-weighted index with margins of 1.22% ( momentum) to 3.06% (low volatility).

Moreover, all smart factor indexes exhibit superior Sharpe ratios and have higher returns than tilted cap-weighted indexes (with the exception of mid cap U.K.). For example, Eurozone high momentum and U.K. value smart factor indexes outperform their respective tilted cap-weighted indexes by 1.52% and 4.04%, respectively. The results show that choosing the right factor tilt adds benefit when using a diversification-based weighting scheme rather than simple cap-weighting. Using the diversified multistrategy weighting scheme, which diversifies not only stock-specific risk but also strategy-specific risk, makes it possible to extract the risk premium from each factor at low levels of portfolio risk.

Since broad cap-weighted indexes are default benchmarks for most active and passive managers, the relative risk of smart factor indexes becomes important. smart-beta offerings are sold on the basis of outperformance over cap-weighted indexes, thus generating reputation risk for index providers and passive managers in the event of underperformance. The importance of such reputation risk makes the risk-adjusted performance in relative terms (i.e. the information ratio) a key performance measure for such strategies. Exhibit 2 shows that the information ratios of the four smart factor indexes are usually higher than those of tilted cap-weighted indexes and often reach impressive levels, such as 0.84 for U.S.A. value and 0.69 for U.K. momentum. Outperformance probability measures how often the strategy has managed to outperform the cap-weighted reference index in the past. It is reported for investment horizons of 5 years by using a rolling window analysis with 1-week step size. The smart factor indexes achieve high levels of outperformance probability and in general deliver more robust outperformance than tilted CW indexes.

Market conditions such as bullish or bearish markets have a considerable impact on how different equity strategies perform (Ferson and Qian, 2004). Certain subperiods and/or market conditions favor some strategies but prove detrimental to others because risk factors carry time varying risk premia (Asness, 1992, Cohen, Polk, and Vuolteenaho, 2003). Exhibit 3 shows that low volatility indexes underperform in bull markets with relatively high outperformance in bear markets. Owing to their strong factor-tilted nature, the performance of all smart factor indexes shows dependency on market conditions. This observation also makes the case for diversification across several smart factor indexes, an issue discussed further in the dedicated article on multifactor allocation in the present supplement.

### Global smart factor indexes

Smart factor indexes show attractive performance on both an absolute and a risk-adjusted basis in different developed markets. How does this outperformance translate when investors construct global portfolios? It is interesting that none of the smart factor indexes for the global developed universe posted excess returns of fewer than 200 bps. This is because the results of relatively poorly performing local smart factor indexes can be offset by the value added that is generated in other regions. Sharpe ratios of developed smart factor indexes lie in the range of 0.50 to 0.65 compared to a mere 0.36 for the global developed broad cap-weighted index. For each risk factor, the smart factor indexes outperform the tilted cap-weighted indexes in both relative returns and Sharpe ratio. Moreover, because of international diversification, we also obtain low levels of tracking error, leading to information ratios ranging from 0.62 for the low volatility smart factor index to 1.03 for the value smart factor index. The 5-year outperformance probability is extremely high (95%-100%) meaning the outperformance is both high and robust across these four factors. ~

EXHIBIT 2

**Relative Performance and Risk of Scientific Beta Diversified Multi-Strategy Factor Indexes**

Benchmark is the Cap-Weighted index on the full universe for each region. Outperformance Probability is defined as the historical probability of outperforming the cap-weighted reference index over a 5-year investment horizon and is computed using a rolling window analysis with 5-year length and 1-week step size. All statistics are annualized. The analysis is based on daily total returns from 31-Dec-2003 to 31-Dec-2013 (10 years).

| | Mid Cap | | High Momentum | | Low Volatility | | Value | |
|---|---|---|---|---|---|---|---|---|
| | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy |
| USA | | | | | | | | |
| Excess Returns | 2.73% | 3.12% | 0.96% | 1.72% | 0.26% | 2.40% | -0.12% | 2.86% |
| Tracking Error | 5.07% | 4.23% | 4.29% | 5.07% | 4.05% | 5.15% | 4.04% | 3.41% |
| Information Ratio | 0.54 | 0.74 | 0.22 | 0.34 | 0.06 | 0.47 | -0.03 | 0.84 |
| Outperformance Probability (5Y) | 99.6% | 100.0% | 75.6% | 59.2% | 53.4% | 100.0% | 5.0% | 100.0% |
| Eurozone | | | | | | | | |
| Excess Returns | 1.64% | 2.05% | 2.73% | 4.25% | 2.04% | 2.84% | -0.26% | 1.33% |
| Tracking Error | 6.28% | 7.07% | 4.82% | 7.05% | 4.48% | 7.27% | 3.96% | 4.55% |
| Information Ratio | 0.26 | 0.29 | 0.57 | 0.60 | 0.45 | 0.39 | -0.07 | 0.29 |
| Outperformance Probability (5Y) | 70.2% | 95.0% | 100.0% | 100.0% | 100.0% | 100.0% | 30.5% | 42.4% |
| UK | | | | | | | | |
| Excess Returns | 3.44% | 2.78% | 1.14% | 4.39% | -0.13% | 3.54% | -2.27% | 1.77% |
| Tracking Error | 7.17% | 7.29% | 5.95% | 6.37% | 5.53% | 7.60% | 4.93% | 5.78% |
| Information Ratio | 0.48 | 0.38 | 0.19 | 0.69 | -0.02 | 0.47 | -0.46 | 0.31 |
| Outperformance Probability (5Y) | 88.5% | 67.2% | 69.1% | 95.0% | 38.9% | 100.0% | 0.0% | 29.0% |
| Japan | | | | | | | | |
| Excess Returns | 0.89% | 1.64% | -0.45% | 1.22% | 1.26% | 3.06% | 1.56% | 2.77% |
| Tracking Error | 6.62% | 7.73% | 5.28% | 7.48% | 5.95% | 8.65% | 3.84% | 6.22% |
| Information Ratio | 0.13 | 0.21 | -0.09 | 0.16 | 0.21 | 0.35 | 0.41 | 0.45 |
| Outperformance Probability (5Y) | 62.6% | 96.9% | 4.2% | 92.0% | 94.3% | 97.3% | 92.4% | 97.3% |
| Asia Pacific ex-Japan | | | | | | | | |
| Excess Returns | 2.40% | 2.99% | 3.21% | 5.10% | 0.94% | 1.33% | 2.01% | 3.91% |
| Tracking Error | 6.98% | 7.55% | 4.73% | 6.85% | 4.05% | 8.21% | 5.70% | 6.77% |
| Information Ratio | 0.34 | 0.40 | 0.68 | 0.74 | 0.23 | 0.16 | 0.35 | 0.58 |
| Outperformance Probability (5Y) | 72.1% | 85.5% | 95.8% | 99.2% | 84.0% | 78.6% | 100.0% | 92.0% |

EXHIBIT 3

**Conditional Performance of Local Scientific Beta Diversified Multi-Strategy Factor Indexes**
Benchmark is the cap-weighted index on the full universe for each region. Calendar quarters with positive benchmark returns comprise bull markets and the rest constitute bear markets. All statistics are annualized. The analysis is based on daily total returns from Dec. 31, 2003 to Dec. 31, 2013 (10 years).

| | Mid Cap | | High Momentum | | Low Volatility | | Value | |
|---|---|---|---|---|---|---|---|---|
| | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy |
| USA | | | | | | | | |
| Excess Returns in Bull Markets | 5.06% | 2.87% | 0.28% | 1.02% | -3.35% | -2.37% | 0.97% | 1.77% |
| Excess Returns in Bear Markets | -0.24% | 3.20% | 1.70% | 2.46% | 4.78% | 8.37% | -1.40% | 3.99% |
| Eurozone | | | | | | | | |
| Excess Returns in Bull Markets | 3.58% | 1.55% | 3.03% | 3.64% | -1.86% | -1.54% | 2.11% | 4.93% |
| Excess Returns in Bear Markets | -0.84% | 2.46% | 2.09% | 4.56% | 6.81% | 8.22% | -3.00% | -2.99% |
| UK | | | | | | | | |
| Excess Returns in Bull Markets | 2.34% | -0.30% | 2.15% | 3.54% | -3.90% | -1.13% | -0.31% | 2.45% |
| Excess Returns in Bear Markets | 5.04% | 7.81% | -0.55% | 5.51% | 6.27% | 11.42% | -5.14% | 0.57% |
| Japan | | | | | | | | |
| Excess Returns in Bull Markets | -4.46% | -8.18% | -1.79% | -6.78% | -7.73% | -11.62% | 1.77% | -3.16% |
| Excess Returns in Bear Markets | 4.24% | 8.04% | 0.42% | 6.37% | 7.09% | 13.09% | 1.29% | 6.38% |
| Asia Pacific ex Japan | | | | | | | | |
| Excess Returns s in Bull Market | 6.37% | 3.53% | 4.98% | 5.64% | -1.14% | -5.04% | 2.59% | 4.97% |
| Excess Returns in Bear Markets | -2.73% | 1.80% | 0.47% | 3.52% | 3.42% | 9.75% | 0.94% | 1.91% |

EXHIBIT 4

**Scientific Beta Developed Diversified Multi-Strategy Factor Indexes**
Number of stocks in the Scientific Beta Developed universe is 2,000. Benchmark is the Cap-Weighted index on the full universe. Risk-free rate is Secondary Market US T-bill (3M). Outperformance Probability is defined as the historical probability of outperforming the cap-weighted reference index over a 5-year investment horizon and is computed using a rolling window analysis with 5-year length and 1-week step size. All statistics are annualized. The analysis is based on daily total returns from 31-Dec-2003 to 31-Dec-2013 (10 years).

| | | Mid Cap | | High Momentum | | Low Volatility | | Value | |
|---|---|---|---|---|---|---|---|---|---|
| | Broad Cap Weighted | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy | CW | Diversified Multi-Strategy |
| Ann Returns | 7.80% | 10.18% | 10.45% | 8.90% | 10.30% | 8.67% | 10.54% | 7.82% | 10.21% |
| Ann Volatility | 17.09% | 17.80% | 16.12% | 17.23% | 16.09% | 15.07% | 13.79% | 18.80% | 17.23% |
| Sharpe Ratio | 0.36 | 0.48 | 0.55 | 0.43 | 0.54 | 0.47 | 0.65 | 0.33 | 0.50 |
| Excess Returns | - | 2.38% | 2.65% | 1.09% | 2.49% | 0.86% | 2.73% | 0.01% | 2.40% |
| Tracking Error | - | 3.46% | 3.33% | 3.16% | 3.70% | 3.14% | 4.40% | 2.82% | 2.34% |
| Information Ratio | - | 0.69 | 0.79 | 0.35 | 0.67 | 0.27 | 0.62 | 0.00 | 1.03 |
| Outperformance Probability (5Y) | - | 100.0% | 100.0% | 84.0% | 100.0% | 87.8% | 100.0% | 19.8% | 95.0% |

*Since broad cap-weighted indexes are default benchmarks for most active and passive managers, the relative risk of smart factor indexes becomes important.*

***References***

***Amenc, N., F. Goltz, A. Lodh and L. Martellini.*** *2012. Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance. Journal of Portfolio Management 38 (3): 72–88.*
***Badaoui, S. and A. Lodh.*** *2013. Scientific Beta Diversified Multi-strategy Index. ERI Scientific Beta White Paper.*
***Asness, C.*** *1992. Changing Equity Risk Premia and Changing Betas over the Business Cycle and January. University of Chicago Working Paper.*
***Cohen, R.B., C. Polk and T. Vuolteenaho.*** *2003. The Value Spread. Journal of Finance 58(2): 609–642.*
***Ferson, W. H. and M. Qian.*** *2004. Conditional Performance Evaluation Revisited. Research Foundation*

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# Multi Smart-Beta Allocation: Performance and Implementation Benefits

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**Multi-smart–beta allocation: a new source of value added in investment management**

Many investors are seeking to improve the performance of their equity portfolios by capturing exposure to rewarded factors. In this article, we analyze the potential benefit of combining factor tilts. Combinations of tilts to different factors may be of interest for two reasons. First, multifactor allocations should result in improved risk-adjusted performance. In fact, even if the factors to which the factor indexes are exposed are all positively rewarded over the long term, there is extensive evidence that each may encounter prolonged periods of underperformance. More generally, the reward for exposure to these factors has been shown to vary over time (see Harvey, 1989; Asness, 1992; Cohen, Polk and Vuolteenaho, 2003). If this time variation in returns is not completely in sync for different factors, allocating across factors allows investors to diversify the sources of their outperformance and smooth their performance across market conditions. Exhibit 1 provides an illustration of the time-varying premia of Carhart Factors: it shows that the cyclicality of returns differs from one factor to the other.

Pronounced allocation benefits should occur across factors that have low correlation with each other. As shown in Exhibit 2, the correlation of the relative returns of the four smart factor indexes over the cap-weighted benchmark is below one. A combination of these indexes would lower the overall tracking error of the portfolio significantly. The same analysis done conditionally for either bull or bear markets leads to similar results.

Second, investors may benefit from allocating across factors in terms of implementation. Some of the trades necessary to pursue exposure to different factors may actually cancel each other out. Consider the example of an investor who pursues an allocation across a value and a momentum tilt. If some of the low valuation stocks with high weights in the value strategy start to rally, their weight in the momentum-tilted portfolio will tend to increase at the same time that their weight in the value-tilted portfolio will tend to decrease. The effects will not cancel out completely, but some reduction in turnover can be expected through such natural crossing effects.

We now turn to a detailed analysis of the two key benefits of multifactor allocations, notably the performance benefits and the implementation benefits. We provide practical illustrations of multifactor allocations drawing on Scientific Beta smart factor indexes (see the articles on smart factor investing in this supplement for more information), representing a set of four main risk factors: value, momentum, low volatility and size (see the article on the principles of equity factor investing in this supplement for a detailed discussion). In a nutshell, our results suggest that multibeta indexes present new opportunities for active managers and multimanagers to enhance their performance at very low marginal cost.

EXHIBIT 1

**Conditional Returns of the Factors**

Calendar quarters with positive factor returns and negative factor returns are indicated in green and red respectively. Factors are from SciBeta US Long-Term Track Records. The Market factor is the daily return of cap-weighted index of all stocks that constitute the index portfolio in excess of the risk free rate. Small size factor is the daily return series of a cap-weighted portfolio that is long a cap-weighted portfolio of market cap deciles 6 to 8 (NYSE, Nasdaq, AMEX) and short 30% largest market cap stocks from the complete long-term universe (500 largest market cap US stocks and smaller stocks as well). Value factor is the daily return series of a cap-weighted portfolio that is long 30% highest and short 30% lowest B/M ratio stocks of the 500 largest market cap US stocks. Momentum factor is the daily return series of a cap-weighted portfolio that is long the 30% highest and short the 30% lowest 52 weeks (minus most recent 4 weeks) past return stocks of the 500 largest market cap US stocks. The "Secondary Market US Treasury Bills (3M)" is the risk-free rate in US Dollar. All statistics are annualized. The analysis is based on daily total returns from 01/01/1973 to 31/12/2012.


Postive factor returns (in green)/Negative factor returns (in red) for each quarter through 40-year history. Alternatively, the second plot shows this information indicating the top 25% of quarters in green, the bottom 25% in red, and the 50% of quarters with medium factor return realisations in white.

**Performance benefits of allocating across factors**

Investors may use allocation across factor tilts to target an absolute (Sharpe ratio, volatility) or relative (information ratio, tracking error with respect to broad cap-weighted index) risk objective. We show in Exhibit 3 the performance and risk characteristics of two multibeta allocations in the US stock market over a 40-year track record and in the developed excluding U.S. universe over the last 10 years. The first is an equal-weight allocation of the four smart factor indexes (low volatility, mid-cap, value and momentum). This is an example of a simple and robust allocation to smart factors, which is efficient in terms of absolute risk. The second combines the four smart factor indexes so as to obtain equal contributions (see Maillard, et al., 2010) to the tracking error risk from each component index. This approach is an example of allocation with a relative risk objective. Both multibeta allocations are rebalanced quarterly. Of course, the multibeta multistrategy equal weight (EW) and equal risk contribution (ERC) indexes are starting points in smart factor allocation. More sophisticated allocation approaches (e.g., conditional strategies or strategies that may depend on the rewards of the different smart factor indexes) can be deployed using smart factor indexes as ingredients to reach more specific investment objectives (see Amenc, Deguest, Martellini, 2013b).

Exhibit 3 shows that both the multibeta multistrategy EW and ERC indexes present returns that are close to the average performance of the constituents but have lower absolute and relative risk than the average constituent index. Both allocations thus deliver improvements in the Sharpe ratio compared to the average constituent index. The EW allocation delivers the highest Sharpe ratio (0.52 in the U.S., 0.54 in the developed ex U.S. universe) which, compared to the broad cap-weighted reference (0.24 in the U.S., and 0.33 in developed ex U.S. universe), represents a relative Sharpe ratio gain of 116% in U.S. and 64% in the developed ex U.S. universe.

However, the most impressive gains compared to the average of components are in relative risk, where both in the U.S. and in the developed ex U.S. universe, the reduction in the tracking error is around 0.70% for the EW allocation and 1% for the ERC allocation (representing a risk reduction of about 11.5% for the EW allocation and more than 16% for the ERC allocation relative to the average tracking error of the component indexes in the U.S. case). This TE reduction yields an increase in the information ratios to levels of 0.75 and 0.77 from an average information ratio for the constituent indexes of 0.67 in the U.S., while in the developed ex U.S. the average constituent information ratio is 0.64 and the multibeta indexes deliver even higher information ratios at 0.76 and 0.82, respectively, for the EW and ERC allocations. Such improvements in the information ratio, of 18.75% and 28.1% for the EW and ERC allocations, respectively, in the developed ex U.S. universe, are significant and support the idea of diversification among smart factors. Moreover, compared to the average of their constituent indexes, the multibeta multistrategy indexes also exhibit significantly lower extreme relative risk (95% Tracking Error) and maximum relative drawdown. The maximum relative drawdown of the multibeta indexes is even lower than that of any of the constituent indexes in the developed ex U.S. universe. Because of its focus on balancing relative risk contributions of constituents, the ERC allocation provides greater reductions in the relative risk measures such as the tracking error and the extreme tracking error risk.

Additionally, the benefits of allocation across different factors can be seen in the probability of outperformance, which is the historical frequency with which the index will outperform its cap-weighted reference index for a given investment horizon. The probability of outperformance increases considerably for the multibeta indexes compared to the component indexes, especially at short horizons. The higher probabilities of outperformance reflect the smoother and more robust outperformance resulting from the combination of different rewarded factors within a multibeta index.

### Implementation benefits of allocating across factors

The multibeta indexes analyzed in Exhibit 3 were designed not only to provide efficient management of risk and return but also for genuine investability. Each of the smart factor indexes has a target of 30% annual one-way turnover, which is set through optimal control of rebalancing (with the notable exception of the momentum tilt, which allows for 60%

EXHIBIT 2

**Correlation of Relative Returns across Factor-Tilted Multi-Strategy Indices**

The table shows the correlation of the relative returns of four Scientific Beta Factor-Tilted Multi-Strategy Indices (mid-cap, momentum, low volatility, and value) over the full period (Panel A), or conditional to the market regime (25% Most Bullish in Panel B, 25% Most Bearish in Panel C). Calendar quarters with returns in the top 25% (respectively bottom 25%) of the quarterly returns of the Broad CW reference index are classified as 25% Most Bullish (resp. 25% Most Bearish). Daily total returns from 01-January-1973 to 31-December-2012 are used for the analysis. S&P-500 index is used as the cap-weighted benchmark (Broad CW). S&P® and S&P 500® are registered trademarks of Standard & Poor's Financial Services LLC ("S&P"), a subsidiary of The McGraw-Hill Companies, Inc.

**Panel A – Unconditional Relative Returns Correlation Matrix**

| US Long Term (1973-2012) | | Diversified Multi-Strategy | | | |
|---|---|---|---|---|---|
| | | Low Volatility | Mid Cap | Value | Momentum |
| Diversified Multi-Strategy | Low Volatility | 100% | 65% | 71% | 64% |
| | Mid Cap | | 100% | 86% | 69% |
| | Value | | | 100% | 66% |
| | Momentum | | | | 100% |

**Panel B – Bull Market Relative Returns Correlation Matrix**

| US Long Term (1973-2012) 25% Most Bullish Market | | Diversified Multi-Strategy | | | |
|---|---|---|---|---|---|
| | | Low Volatility | Mid Cap | Value | Momentum |
| Diversified Multi-Strategy | Low Volatility | 100% | 60% | 67% | 71% |
| | Mid Cap | | 100% | 87% | 64% |
| | Value | | | 100% | 62% |
| | Momentum | | | | 100% |

**Panel C – Bear Market Relative Returns Correlation Matrix**

| US Long Term (1973-2012) 25% Most Bearish Markets | | Diversified Multi-Strategy | | | |
|---|---|---|---|---|---|
| | | Low Volatility | Mid Cap | Value | Momentum |
| Diversified Multi-Strategy | Low Volatility | 100% | 66% | 72% | 66% |
| | Mid Cap | | 100% | 87% | 69% |
| | Value | | | 100% | 65% |
| | Momentum | | | | 100% |

EXHIBIT 3

**Performances and Risks of Multi-Beta Multi-Strategy Allocations vs Single Factor Tilts**

The table compares performance and risk of Scientific Beta Diversified Multi-Strategy indices on US Long Term Track Records (Panel A) and SciBeta Dev. Ex. US Indices. The Multi-Beta Multi-Strategy EW Allocation is the equal combination of the four Factor-Tilted Diversified Multi-Strategies (low volatility, mid cap, value, and momentum). The Multi-Beta Multi-Strategy ERC Allocation is an optimised combination of the four tilted indices in which beginning of quarter optimal allocations to the component indices are determined from the covariance of the daily relative returns of the component indices over the last 6 quarters (18 months), so as to obtain (in-sample) equal contributions to the (tracking error) risk. The analysis is based on daily total return data from 31 December 1972 to 31 December 2012 (40 years) in panel A and from 31 December 2003 to 31 December 2013 (10 years) in panel B. The S&P 500 index and SciBeta Dev. Ex. US CW index are used respectively as the cap-weighted reference for US Long-Term Track Records and SciBeta Dev. Ex. US Investable Indices. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate.

| US Long-Term Track Records (Dec 1972 – Dec 2012) | Scientific Beta Diversified Multi-Strategy | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Smart Factor Indices | | | | | Multi-Beta Allocations | |
| | USA Long Term Cap Weighted | Low Vol | Mid Cap | Value | Momentum | Average of 4 Smart Factor Indices | Equal Weight | ERC |
| Annual Returns | 9.74% | 12.64% | 14.19% | 14.44% | 13.30% | 13.64% | 13.69% | 13.49% |
| Annual Volatility | 17.47% | 14.39% | 16.73% | 16.55% | 16.30% | 15.99% | 15.76% | 15.70% |
| Sharpe Ratio | 0.24 | 0.50 | 0.52 | 0.54 | 0.48 | 0.51 | 0.52 | 0.51 |
| Max DrawDown | 54.53% | 50.13% | 58.11% | 58.41% | 49.00% | 53.91% | 53.90% | 53.34% |
| Excess Returns | | 2.90% | 4.45% | 4.70% | 3.56% | 3.90% | 3.95% | 3.75% |
| Tracking Error | | 6.17% | 6.80% | 5.82% | 4.88% | 5.92% | 5.23% | 4.90% |
| 95% Tracking Error | | 11.53% | 11.55% | 10.14% | 8.57% | 10.45% | 8.92% | 8.11% |
| Information Ratio | | 0.47 | 0.66 | 0.81 | 0.73 | 0.67 | 0.75 | 0.77 |
| Outperf Prob (1Y) | | 67.83% | 67.88% | 70.87% | 68.42% | 68.75% | 73.77% | 74.02% |
| Outperf Prob (3Y) | | 76.35% | 74.12% | 78.83% | 84.42% | 78.43% | 80.28% | 80.64% |
| Max Rel DrawDown | | 43.46% | 42.06% | 32.68% | 17.28% | 33.87% | 33.77% | 28.89% |

| SciBeta Investable Developed ex US Indices (Dec 2003 – Dec 2013) | Scientific Beta Diversified Multi-Strategy | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Smart Factor Indices | | | | | Multi-Beta Allocations | |
| | Devel-oped ex US Cap Weighted | Low Vol | Mid Cap | Value | Momentum | Average of 4 Smart Factor Indices | Equal Weight | ERC |
| Annual Returns | 7.91% | 11.06% | 10.01% | 10.03% | 11.16% | 10.56% | 10.60% | 10.55% |
| Annual Volatility | 19.03% | 15.26% | 16.57% | 18.74% | 17.11% | 16.92% | 16.80% | 17.20% |
| Sharpe Ratio | 0.33 | 0.62 | 0.51 | 0.45 | 0.56 | 0.54 | 0.54 | 0.52 |
| Max DrawDown | 59.23% | 51.15% | 56.29% | 60.10% | 55.43% | 55.74% | 55.82% | 56.15% |
| Excess Returns | | 3.15% | 2.10% | 2.13% | 3.25% | 2.66% | 2.69% | 2.64% |
| Tracking Error | | 5.10% | 4.36% | 2.98% | 4.33% | 4.19% | 3.52% | 3.22% |
| 95% Tracking Error | | 9.77% | 9.02% | 5.51% | 8.99% | 8.32% | 7.38% | 6.91% |
| Information Ratio | | 0.62 | 0.48 | 0.71 | 0.75 | 0.64 | 0.76 | 0.82 |
| Outperf Prob (1Y) | | 66.81% | 73.62% | 71.28% | 78.30% | 72.50% | 78.09% | 81.28% |
| Outperf Prob (3Y) | | 94.54% | 73.50% | 79.78% | 85.25% | 83.27% | 91.53% | 93.44% |
| Max Rel DrawDown | | 10.31% | 8.51% | 9.09% | 12.91% | 10.20% | 8.02% | 7.18% |

turnover). In addition, the stock selections used to tilt the indexes implement buffer rules in order to reduce unproductive turnover resulting from small changes in stock characteristics. The component indexes also apply weight and trading constraints relative to market-cap weights so as to ensure high capacity. Finally, these indexes offer an optional high liquidity feature that allows investors to reduce the application of the smart factor index to the most liquid stocks in the reference universe.

In addition to these implementation rules, which are applied at the level of each smart factor index, the multibeta allocations provide a reduction in turnover (and hence of transaction costs) compared with separate investment in each of the smart factor indexes. This reduction in turnover arises from different sources. First, when the renewal of the underlying stock selections takes place, it can happen that a stock being dropped from the universe of one smart factor index is being simultaneously added to the universe of another smart factor index. Second, for constituents that are common to several smart factor indexes, the trades to rebalance the weight of a stock in the different indexes to the respective target weight may partly offset each other.

Exhibit 4 displays statistics relative to the investability of the multibeta equal-weight and relative ERC allocations along with the average of the mid-cap, momentum, low volatility and value smart factor indexes. For comparison, we also show the same analytics for their highly liquid counterparts. We see that the turnover of multibeta indexes is very reasonable. In fact, managing a mandate on each smart factor index separately would yield a turnover higher than the average turnover across the smart factor indexes. This is because rebalancing each component index to the allocation target would induce extra turnover. However, implementing the multibeta index in a single mandate exploits the benefits of natural crossing arising among the different component indexes and actually reduces the turnover below the average level observed for component indexes. We provide in the table for each multibeta allocation the amount of turnover that is internally crossed in multibeta indexes as compared to managing the same allocations separately. We see that about 6% turnover is internally crossed by the EW allocation.

In addition to turnover, the exhibit also shows the average capacity of the indexes in terms of the weighted average market cap of stocks in the portfolio. This index capacity measure indicates very decent levels with an average market cap of around US$ 10bn for the multibeta index, while the highly liquid version further increases capacity to levels exceeding US$ 15bn in the case of the U.S. long-term track records. In the case of the developed ex U.S. region, the weighted average market caps are higher since the period under scrutiny is more recent (last 10 years), around US$ 11.4bn for the standard indexes and US$ 16bn for the highly liquid ones. In both regions, we provide an estimate of the time that would be necessary to set up an initial investment (i.e. full weights) of US$ 1bn AUM in the indexes, assuming that the average daily dollar traded volume can be traded (100% participation rate) and that the number of days required grows linearly with the fund size.

Overall this highlights the ease of implementation of the multibeta indexes and the effectiveness of the high liquidity option. Indeed, the Days to Trade required for the initial investment on US indexes are very manageable (about 0.12 days for the standard multibeta indexes, and 0.07 days with the highly liquid feature). Even in the developed ex U.S. universe, the highly-liquid multibeta indexes would require about 0.2 days of trading. In addition, one should keep in mind that the number of days needed to rebalance the indexes (i.e. trade the weight change rather than the full weight on each stock) would be much lower (about 0.1 day of trading for the global developed ex U.S. case). It should be noted that the highly-liquid multibeta indexes also maintain the level of performance (information ratio) of the standard multibeta indexes in the U.S. case and provide even stronger information ratios in the developed ex U.S. universe.

Finally, even when assuming unrealistically high levels of transaction costs, all the smart factor indexes deliver significant outperformance net of costs in both regions. Compared to the average stand-alone investment in a smart factor index, the multibeta indexes almost always result in higher average returns net of costs because of the turnover reduction through natural crossing effects among the component smart factor indexes. ~

EXHIBIT 4

**Implementation of EW Allocation across Standard or Highly Liquid Factor-Tilted Indices**

The analysis is based on daily total return data from 31 December 1972 to 31 December 2012 (40 years) in panel A and from 31 December 2003 to 31 December 2013 (10 years) in panel B. The S&P 500 index and SciBeta Dev. Ex. US CW index are used respectively as the cap-weighted reference for US Long-Term Track Records and SciBeta Dev. Ex. US Investable Indices. Days to Trade is the number of days necessary to trade the total stock positions, assuming USD1bn AUM and that 100% of the Average Daily Dollar Traded Volume can be traded every day. The weighted average market capitalization of the index is in $million and averaged over the 40-year period. All statistics are average values across 160 quarters (40 years). The net returns are the relative returns over the cap-weighted benchmark net of transaction costs. Two levels of transaction costs are used - 20 bps per 100% 1-Way turnover and 100 bps per 100% 1-Way turnover. The first case corresponds to the worst case observed historically for the large and mid-cap universe of our indices while the second case assumes 80% reduction in market liquidity and a corresponding increase in transaction costs. The risk-free rate is the return of the 3-month US Treasury Bill. (*)Due to data availability, the period is restricted to the last 10 years of the sample for Scientific Beta US indices. S&P® and S&P 500® are registered trademarks of Standard & Poor's Financial Services LLC ("S&P"), a subsidiary of The McGraw-Hill Companies, Inc.

| PANEL A | Diversified Multi-Strategy | | | | | |
|---|---|---|---|---|---|---|
| US Long-Term Track Records | All Stocks | | | High Liquidity Stocks | | |
| (Dec 1972 – Dec 2012) | Average of 4 Smart Factor Indices | EW Multi Beta | ERC Multi Beta | Average of 4 Smart Factor Indices | EW Multi Beta | ERC Multi Beta |
| 1-Way Turnover | 34.19% | 28.91% | 31.49% | 38.00% | 33.17% | 36.77% |
| Internally Crossed Turnover | - | 5.65% | 7.55% | - | 5.53% | 7.63% |
| Days to Trade for $1bn Initial Investment (Quantile 95%)(*) | 0.20 | 0.12 | 0.12 | 0.16 | 0.07 | 0.0.7 |
| Weighted Avg. Market Cap ($m) | 9 381 | 9 381 | 10 283 | 13 299 | 13 299 | 15 288 |
| Information Ratio | 0.67 | 0.75 | 0.77 | 0.59 | 0.78 | 0.79 |
| Relative Returns | 3.90% | 3.95% | 3.75% | 3.32% | 3.39% | 3.00% |
| Relative Returns net of 20 bps transaction costs (historical worst case) | 3.84% | 3.89% | 3.69% | 3.24% | 3.32% | 2.92% |
| Relative Returns net of 100 bps transaction costs (extreme liquidity stress scenario) | 3.56% | 3.66% | 3.44% | 2.94% | 3.06% | 2.63% |

| PANEL B | Diversified Multi-Strategy | | | | | |
|---|---|---|---|---|---|---|
| SciBeta Investable Developed ex-US Indices | All Stocks | | | High Liquidity Stocks | | |
| (Dec 2003 – Dec 2013) | Average of 4 Smart Factor Indices | EW Multi Beta | ERC Multi Beta | Average of 4 Smart Factor Indices | EW Multi Beta | ERC Multi Beta |
| 1-Way Turnover | 45.61% | 39.42% | 39.55% | 46.23% | 40.01% | 39.61% |
| Internally Crossed Turnover | - | 6.38% | 8.00% | - | 6.42% | 8.38% |
| Days to Trade for $1bn Initial Investment (Quantile 95%)(*) | 1.32 | 0.72 | 0.71 | 0.46 | 0.22 | 0.21 |
| Weighted Avg. Market Cap ($m) | 11 112 | 11 112 | 11 687 | 15 547 | 15 547 | 16 523 |
| Information Ratio | 0.64 | 0.76 | 0.82 | 0.65 | 0.91 | 0.98 |
| Relative Returns | 2.66% | 2.69% | 2.64% | 2.57% | 2.61% | 2.53% |
| Relative Returns net of 20 bps transaction costs (historical worst case) | 2.57% | 2.61% | 2.56% | 2.48% | 2.53% | 2.45% |
| Relative Returns net of 100 bps transaction costs (extreme liquidity stress scenario) | 2.20% | 2.30% | 2.25% | 2.11% | 2.21% | 2.13% |

***References***

***Amenc, N., F. Goltz and L. Martellini.*** *2013a. Smart Beta 2.0. EDHEC-Risk Institute Position Paper.*
***Amenc, N., R. Deguest, and L. Martellini.*** *2013b. Investing in Multi Smart Beta Portfolios: Reconciling Risk Factor Allocation and Smart Beta Allocation. EDHEC-Risk Institute Working Paper.*
***Amenc, N., F. Goltz, A. Lodh and L.*** *Martellini. 2012b. Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance. Journal of Portfolio Management 38 (3): 72–88.*
***Asness, C.*** *1992. Changing Equity Risk Premia and Changing Betas over the Business Cycle and January. University of Chicago Working Paper.*
***Cohen, R. B., C. Polk and T. Vuolteenaho.*** *2003. The Value Spread. Journal of Finance 58(2): 609–642.*
***Harvey, C. R.*** *1989. Time-Varying Conditional Covariances in Tests of Asset Pricing Models. Journal of Financial Economics 24: 289–317.*
***Gonzalez, N. and A. Thabault.*** *2013. Overview of Diversification Strategies. ERI Scientific Beta White Paper.*
***Maillard, S., T. Roncalli and J. Teiletche.*** *2010. The Properties of Equally Weighted Risk Contributions Portfolios. Journal of Portfolio Management 36(4): 60–70.*

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# A Guide to Factor Investing in the Equity Space

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Sophisticated institutional investors have started to review factor-based investment strategies. For example, the Parliament of Norway, which acts as a trustee for the Norwegian Oil Fund, commissioned a report on the investment returns of the fund. This report was requested after the fund's performance fell short of the performance of popular equity market benchmarks. The resulting report (Ang, Goetzmann and Schaefer, 2009) showed that the returns relative to a cap-weighted benchmark of the fund's actively-managed portfolio can be explained by exposure to a set of well-documented alternative risk factors. After taking into account such exposures, active management did not have any meaningful impact on the risk and return of the portfolio. The authors argue that such exposures can be obtained through purely systematic strategies without a need to rely on active management. Therefore, rather than simply observing the factor tilts brought by active managers ex-post, investors may consider which factors they wish to tilt towards and make explicit decisions on these tilts. This discussion of active managers' sources of outperformance has naturally led to factor indexes being considered as a more cost efficient and transparent way of implementing such factor tilts. As institutional investors' discussions have gone on, providers of exchange-traded products have rolled out a series of factor-based equity investment products. For example, the Wall Street Journal reported in 2011 on the launch of numerous exchange-traded funds following factor-based equity strategies.[14]

That such factor-based equity strategies may deliver outperformance over standard cap-weighted indexes is supported by asset pricing theory, which postulates that multiple sources of systematic risk are priced in securities markets. In particular, both equilibrium models, such as Merton's (1973) intertemporal capital asset pricing model, and noarbitrage models such as Ross's (1976) arbitrage pricing theory, allow for the existence of multiple priced risk factors. The economic explanation for the existence of a reward for a given risk factor is that exposure to such a factor is undesirable for the average investor because it leads to losses in bad times[15] (i.e. when marginal utility is high, see Cochrane, 2001). This can be illustrated, for example, with liquidity risk. While investors may gain a payoff from exposure to illiquid securities as opposed to their more liquid counterparts, such illiquidity may lead to losses in times when liquidity dries up and a flight to quality occurs, such as during the 1998 Russian default crisis and the 2008 financial crisis. In such conditions, hard-to-sell (illiquid securities) may post heavy losses.

### Which factors matter

While asset pricing theory provides a sound rationale for the existence of multiple factors, theory provides little guidance on which factors should be expected to be rewarded. Empirical research however has come up with a range of factors that have led to significant risk premia in typical samples of data from U.S. and international equity markets.[16] For investors to accept factors as relevant in their investment process, however, a key requirement is that there be a clear economic explanation for why the exposure to this factor constitutes a systematic risk that requires a reward and is

**Empirical evidence for selected factor premia: key references**

| | Factor Definition | Within US Equities | International Equities | Other Asset Classes |
|---|---|---|---|---|
| Value | Stocks with high book-to-market versus stocks with low book-to-market | Basu (1977)<br>Rosenberg, Reid, Lahnstein (1985)<br>Fama and French (1993) | Fama and French (2012) | Asness, Moskowitz, Pedersen (2013) |
| Momentum | Stocks with high returns over the past 12 months omitting the last month versus stocks with low returns | Jegadeesh and Titman (1993), Carhart (1997) | Rouwenhorst (1998) | Asness, Moskowitz, Pedersen (2013) |
| Low Risk | Stocks with low risk (beta, volatility or idiosyncratic volatility) versus stock with high risk | Ang, Hodrick, Xing, Zhang (2006), Frazzini and Pedersen (2014) | Ang, Hodrick, Xing, Zhang (2009), Frazzini and Pedersen (2014) | Frazzini and Pedersen (2014) |
| Size | Stocks with low market cap versus stocks with high market cap | Banz (1981)<br>Fama and French (1993) | Rouwenhorst, Heston, Wessels (1999)<br>Fama and French (2012) | N.A. ) |
| Liquidity | Stocks with low trading volume or high sensitivity to changes in market liquidity | Pastor and Stambaugh (2003), Acharya and Pedersen (2005) | Lee (2011) | Lin, Wang, Wu (2011), Sadka (2010) |

**Economic explanations for selected factor premia: overview**

| | Risk-Based Explanation | Behavioural Explanation |
|---|---|---|
| Value | Costly reversibility of assets in place leads to high sensitivity to economic shocks in bad times. | Overreaction to bad news and extrapolation of the recent past leads to subsequent return reversal. |
| Momentum | High expected growth firms are more sensitive to shocks to expected growth. | Investor overconfidence and self-attribution bias leads to returns continuation in the short term. |
| Low Risk | Liquidity-constrained investors hold leveraged positions in low-risk assets which they may have to sell in bad times when liquidity constraints become binding. | Disagreement of investors about high-risk stocks leads to overpricing in the presence of short sales constraints. |
| Size | Low profitability leads to high distress risk and downside risk. Low liquidity and high cost of investment needs to be compensated by higher returns. | N.A. |
| Liquidity | Assets with low returns in times of funding liquidity constraints or low market liquidity require a risk premium. | N.A. |

[14] *"Here's What's Really Driving Your Returns", Wall Street Journal, December 24, 2011, available at: online.wsj.com/news/articles/SB10001424052970204058404577110523854626572 >*

[15] *It is worth emphasizing that asset pricing theory suggests that factors are (positively) rewarded if, and only if, they perform poorly during bad times, and more than compensate during good times so as to generate a positive excess return on average across all possible market conditions. In technical jargon, the expected excess return on a factor is proportional to the negative of the factor covariance with the pricing kernel, given by marginal utility of consumption for a representative agent. Hence, if a factor generates an uncertain payoff that is uncorrelated to the pricing kernel, then the factor will earn no reward even though there is uncertainty involved in holding the payoff. On the other hand, if a factor payoff covaries positively with the pricing kernel, it means that it tends to be high when marginal utility is high, that is when economic agents are relatively poor. Because it serves as a hedge by providing income during bad times, when marginal utility of consumption is high, investors are actually willing to pay a premium for holding this payoff.*

[16] *These effects are often referred to as anomalies in the academic literature as they contradict the CAPM prediction that the cross section of expected returns depends only on stocks' market betas and should be void of any other patterns. However, when using a more general theoretical framework, such as the intertemporal CAPM or arbitrage pricing theory, there is no reason to qualify such patterns as anomalies.*

likely to continue producing a positive risk premium (see Ang, 2013; also compare Cochrane, 2001, who refers to the practice of identifying merely empirical factors as "factor fishing").

Harvey, Liu and Zhu (2013) review the empirical literature that has identified factors that affect the cross section of expected stock returns and count a total of 314 factors for which results have been published. The table on page 13 provides an overview of the main factors used in common multifactor models of expected returns and the references to seminal work providing empirical evidence. It is interesting to note that these factors have been found to explain expected returns across stocks not only in U.S. markets, but also in international equity markets, and, in many cases, even in other asset classes including fixed income, currencies and commodities.

The debate about the existence of positive premia for these factors is far from closed. For example, debate is ongoing on the low risk premium. Early empirical evidence suggests that the relation between systematic risk (stock beta) and return is flatter than predicted by the CAPM (Black, Jensen, and Scholes, 1972). More recently, Ang, Hodrick, Xing, and Zhang (2006, 2009) find that stocks with high idiosyncratic volatility have had low returns. Other papers have documented a flat or negative relation between total volatility and expected return. However, a number of recent papers have questioned the robustness of such results and show that the findings are not always robust to changes to portfolio formation (Bali and Cakici, 2008) or to adjusting for short-term return reversals (Huang, et al., 2010).

**Economic explanations for selected factor premia: overview**

| | Risk-Based Explanation | Behavioural Explanation |
|---|---|---|
| Value | Costly reversibility of assets in place leads to high sensitivity to economic shocks in bad times. | Overreaction to bad news and extrapolation of the recent past leads to subsequent return reversal. |
| Momentum | High expected growth firms are more sensitive to shocks to expected growth. | Investor overconfidence and self-attribution bias leads to returns continuation in the short term. |
| Low Risk | Liquidity-constrained investors hold leveraged positions in low-risk assets which they may have to sell in bad times when liquidity constraints become binding. | Disagreement of investors about high-risk stocks leads to overpricing in the presence of short sales constraints. |
| Size | Low profitability leads to high distress risk and downside risk. Low liquidity and high cost of investment needs to be compensated by higher returns. | N.A. |
| Liquidity | Assets with low returns in times of funding liquidity constraints or low market liquidity require a risk premium. | N.A. |

### Why factors matter

In addition to an empirical assessment of factor premia, it is important to find a compelling economic rationale for why the premium would persist. Such persistence can be expected if the premium is related to risk taking. In an efficient market with rational investors, systematic differences in expected returns should result from differences in risk. Kogan and Tian (2013) argue that to determine meaningful factors "we should place less weight on the [data] the models are able to match, and instead closely scrutinise the theoretical plausibility and empirical evidence in favour or against their main economic mechanisms." This point is best illustrated with the example of the equity risk premium. Given the wide fluctuation in equity returns, the equity risk premium can be statistically indistinguishable from zero even for relatively long sample periods. However, it is reasonable to expect that stocks have higher reward than bonds because investors are reluctant to hold too much equity because of its risks. For other equity risk factors, such as value, momentum, low risk and size, similar explanations that interpret the factor premia as compensation for risk have been suggested in the literature.

It is worth noting that the existence of the factor premia can also result from investors making systematic errors based on behavioral biases such as over- or under-reaction to news on a stock. However, whether such behavioural biases persistently affect asset prices in the presence of some smart investors who do not suffer from these biases is a point of contention. In fact, even if the average investor makes systematic errors because of behavioral biases, it could still be possible that some rational investors who are not subject to such biases exploit any small opportunity resulting from the irrationality of the average investor. The trading activity of such smart investors may then make the return opportunities disappear. Therefore, behavioral explanations of persistent factor premia often introduce so called limits to arbitrage, which prevent smart investors from fully exploiting the opportunities arising from the irrational behaviour of other investors. The most commonly-mentioned limits to arbitrage are short-sales constraints and funding-liquidity constraints. The table above summarizes the main economic explanations for common factor premia.

### Value

Zhang (2005) provides a rationale for the value premium based on costly reversibility of investments. The stock price of value firms is mainly made up of tangible assets that are hard to reduce, while growth firms' stock price is mainly driven by growth options. Therefore value firms are much more affected by bad times. Choi (2013) shows that value firms have increasing betas in down markets ( owing to rising asset betas and rising leverage) while growth firms have more stable betas. The value premium can thus be interpreted as compensation for the risk of suffering from losses in bad times.

In an influential paper, Lakonishok, Shleifer and Vishny (1994) argue that "value strategies exploit the suboptimal behavior of the typical investor." Their explanation of the value premium focuses on the psychological tendency of investors to extrapolate recent developments into the future and to ignore evidence that is contrary to the extrapolation. Glamour firms with high recent growth thus tend to obtain valuations that correspond to overly optimistic forecasts, while distressed firms obtain stock market valuations that are overly pessimistic.

### Momentum

Momentum stocks are exposed to macroeconomic risk. In particular, Liu and Zhang (2008) provide empirical evidence that past winners have temporarily higher loadings on the growth rate of industrial production. This higher sensitivity of firms with higher expected growth rates is a natural result of firm valuation and is similar to the higher interest rate sensitivity (duration) of bonds at high levels of interest rate (see Johnson, 2002). Low momentum stocks, on the other hand, have low expected growth and are less sensitive to changes in expected growth.

Behavioral explanations for momentum profits focus on the short-term over-reaction of investors. Daniel, et al., (1998) show that two cognitive biases, overconfidence and self-attribution, can generate momentum effects. In particular, they show that investors will attribute the recent performance of the winning stocks they have selected to their stock picking skill and thus further bid up the prices for these stocks, thus generating a momentum effect in the short term, with stock prices only reverting to their fundamental values at longer horizons.

### Low Risk

Frazzini and Pedersen (2014) provide a model in which liquidity-constrained investors are able to invest in leveraged positions of low-beta assets but are forced to liquidate these assets in bad times when their liquidity constraints mean they can no longer sustain the leverage. Thus low-risk assets are exposed to a risk of liquidity shocks and investors are compensated for this risk when holding low-beta assets. High-beta assets, on the other hand, expose investors to lower liquidity risk and rational investors may thus require less expected return from these stocks than what would be in line with their higher market beta.

Behavioral explanations for the low-risk premium suggest that high-risk stocks tend to have low returns because irrational investors bid up prices beyond their rational value. For example, Hong and Sraer (2012) show that when there is disagreement among investors on the future cash flow of firms, short sales constraints will lead to overpricing of stocks when investor disagreement is high. As disagreement increases with a stock's beta, high-beta stocks are more likely to be overpriced.

### Size and Liquidity

Small stocks tend to have lower profitability (in terms of return on equity) and greater uncertainty of earnings (see Fama and French, 1995), even when adjusting for book-to-market effects. Therefore, small stocks are more sensitive to economic shocks, such as recessions. It has also been argued that stocks of small firms are less liquid and expected returns of smaller firms have to be large in order to compensate for their low liquidity (Amihud and Mendelsson, 1986). It has also been argued that smaller stocks have higher downside risk (Chan, Chen and Hsieh, 1985). For liquidity as a risk factor, the need for investors to be compensated for taking on liquidity risk is straightforward, as investors are naturally averse to assets with evaporating liquidity in times of market stress (see Nagel, 2012).

### Robust factors versus data mining

Investors who wish to exploit factor premia need to address robustness when selecting a set of factors. Indeed, an important issue is that the premium may decrease if investors are increasingly investing to capture it. Another issue is that the discovery of the premium in the first place may have been a result of data mining. In order to avoid the pitfalls of nonpersistent factor premia and achieve robust performance, investors should keep the following checks in mind. First, investors should require a sound economic rationale

for the existence of a premium. Second, the risks of data mining mean that investors should stick to simple factor definitions that are widely used in the literature rather than relying on complex and proprietary factor definitions.

The empirical literature uses different proxies to capture a given factor exposure, and practical implementations of factor exposures may deviate considerably from factor definitions in the literature. For example, when capturing the value premium one may use extensive fundamental data including not only valuation ratios but also information on, for example, sales growth of the firm. Moreover, many value-tilted indexes include a large set of ad hoc choices, opening the door to data mining. Consider the impact of strategy specification for fundamental equity indexation strategies, which are commonly employed as a way to harvest the value premium. As shown in the table below, the outperformance of a fundamental equity indexation strategy is highly sensitive to strategy specification choices. The table summarizes the maximum calendar year difference between any two variants of fundamental indexes that make different choices for one of five methods, such as variable selection, leverage adjustment, turnover control, stock selection and rebalancing date.

We see that variable selection and rebalancing date have the highest influence on short-term performance. Different leverage adjustment methods can also lead to large differences in yearly performance. The differences of as much as 10% in annual return between strategies that make different choices clearly show that histories depend heavily on strategy specifications.

In contrast to the wide variety of factor definitions used by index providers and asset managers, most empirical asset pricing studies resort to simple and consensual factor definitions. For example, the most widely used definition of value is based on a single variable: the book-to-market ratio of a stock. This simple factor definition may be an important guard against data mining risks.

In addition to selecting robust factors, investors need to address the question of capturing the factor premium in a cost-effective and risk/return efficient way through appropriately designed factor.

**Maximum calendar year performance difference of fundamental equity indexation strategies with different strategy specifications**

Returns of best and worst performing variants for each specification of the fundamental weighting scheme on the universe of top 1,000 US stocks. Portfolios are formed using fundamental data from the period January 1982 to December 2010. Data is obtained from Datastream and Worldscope

| | Best performing | | Worst performing | | Max difference | Year |
|---|---|---|---|---|---|---|
| Variable selection | Earnings | -12.2% | Dividends | -23.0% | 10.8% | 1999 |
| Leverage adjustment | Total leverage adj. | 5.3% | Operating leverage adj. | -4.0% | 9.3% | 2008 |
| Turnover control | Optimal control | 9.0% | No control | 4.6% | 4.5% | 2002 |
| Selection effect | Fundamental | 4.6% | Cap selection | 2.3% | 2.3% | 2003 |
| Rebalancing | March | 11.3% | September | 0.2% | 11.1% | 2009 |

*It is worth noting that the existence of the factor premia can also result from investors making systematic errors based on behavioral biases such as over-or under-reaction to news on a stock.*

***References***

***Acharya, V., and L. H. Pedersen.*** *2005. Asset Pricing with Liquidity Risk, Journal of Financial Economics 77, 375–410*
***Amihud, Y., and H. Mendelson.*** *1986. Asset Pricing and the Bid-Ask Spread, Journal of Financial Economics 17, 223–249.*
***Ang, A.*** *2013. "Asset Management: A Systematic Approach to Factor Investing", book draft, available at < www.columbia.edu/~aa610/>*
***Ang, A., W. Goetzmann and S. Schaefer.*** *2009. Evaluation of Active Management of the Norwegian Government Pension Fund – Global*
***Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang.*** *2006. The cross-section of volatility and expected returns. Journal of Finance, 61 (1): 259–99.*
***Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang.*** *2009. High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence. Journal of Financial Economics, 61, 1, 259–299*
***Asness, C., T. Moskowitz, and L.H. Pedersen.*** *2013. Value and Momentum Everywhere. Journal of Finance. 68(3), 929–985.*
***Basu, S.*** *1977. Investment Performance of Common Stocks in relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis, Journal of Finance. 32, 663–682*
***Bali, T., and N. Cakici.*** *2008. Idiosyncratic Volatility and the Cross-Section of Expected Returns. Journal of Financial and Quantitative Analysis 43:29–58.*
***Banz, R.W.*** *1981. The Relationship between Return and Market Value of Common Stock, Journal of Financial Economics, 9, 3–18*
***Black, F., M.C. Jensen, and M. Scholes.*** *1972. The Capital Asset Pricing Model: Some Empirical Tests. In Michael C. Jensen (ed.), Studies in the Theory of Capital Markets, New York, pp. 79–121.*
***Carhart, M.*** *1997. On Mutual Fund Performance, Journal of Finance*
***Chan, K. C., Chen, N. F., and D. A. Hsieh.*** *1985. An Exploratory Investigation of the Firm Size Effect. Journal of Financial Economics 14(3), 451–471.*
***Choi, J.*** *2013. What Drives the Value Premium? The Role of Asset Risk and Leverage, Review of Financial Studies, 26 (11), 2845–2875.*
***Cochrane, J.*** *2001. Asset Pricing, Princeton University Press*
***Daniel, K., D. Hirshleifer and A. Subrahmanyam.*** *1998. Investor Psychology and Security Market Under- and Overreactions, Journal of Finance Vol. 53, No. 6 (Dec., 1998), pp. 1839–1885*
***Fama, E. F. and K. R. French.*** *1993. Common Risk Factors in the Returns on Stocks and Bonds, Journal of Financial Economics, 33, 3–56.*
***Fama, E. F., and K. R. French.*** *1995. Size and Book-to-Market Factors in Earnings and Returns, Journal of Finance, 50 (1), 131–155*
***Fama, E. F. and K. R. French.*** *2012. Size, Value, and Momentum in International Stock Returns, Journal of Financial Economics, 105, 457–472.*
***Frazzini, A., and L.H. Pedersen.*** *2014. Betting Against Beta. Journal of Financial Economics. 111 (1), 1–25*
***Harvey, C., Y. Liu and H. Zhu.*** *2013. …and the Cross-Section of Expected Returns, working paper, Duke University*
***Hong, H., and D. Sraer.*** *2012. Speculative Betas, working paper, Princeton University*
***Huang, W., Q. Liu, S. G. Rhee, and L. Zhang.*** *2010. Return Reversals, Idiosyncratic Risk, and Expected Returns. Review of Financial Studies 23 (1): 147–68.*
***Jegadeesh, N., and S. Titman.*** *1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency, Journal of Finance, 48 (1), 65–91.*
***Johnson, T. C.*** *2002. Rational Momentum Effects, Journal of Finance, 57(2), 585–608*
***Kogan, L., and M. Tian.*** *2013. Firm Characteristics and Empirical Factor Models: a Data-Mining Experiment, working paper, MIT*
***Lakonishok, J., A. Shleifer and R. W. Vishny.*** *1994. Contrarian Investment, Extrapolation, and Risk, Journal of Finance, 49(5), 1541–1577*
***Lee, K.H.*** *2011. The World Price of Liquidity Risk, Journal of Financial Economics, 99(1), 136–161*
***Lin, H., J. Wang and C. Wu.*** *2011. Liquidity Risk and Expected Corporate Bond Returns. Journal of Financial Economics. 99(3), 628–650*
***Liu, L. X., and L. Zhang.*** *2008. Momentum Profits, Factor Pricing, and Macroeconomic Risk, Review of Financial Studies, 21(6), 2417–2448*
***Merton, R.*** *1973. An Intertemporal Capital Asset Pricing Model, Econometrica, 41, 5, 867–887.*
***Nagel, S.*** *2012. Evaporating Liquidity, Review of Financial Studies, 25(7), 2005–2039.*
***Pastor, L., and R. Stambaugh.*** *2003. Liquidity Risk and Expected Stock Returns, Journal of Political Economy*
***Rosenberg B., K. Reid, and R. Lanstein.*** *1985. Persuasive Evidence of Market Inefficiency. Journal of Portfolio Management. Vol 11, 9–17*
***Ross, S.*** *1976. The Arbitrage Theory of Capital Asset Pricing, Journal of Economic Theory, 13, 3, 341–360.*
***Rouwenhorst, G.K.*** *1998. International Momentum Strategies, Journal of Finance, 53*
***Rouwenhorst, G.K., S.L. Heston and R. Wessels.*** *1999. The Role of Beta and Size in the Cross-Section of European Stock Returns, European Financial Management*
***Sadka, R.*** *2010. Liquidity Risk and the Cross Section of Hedge Fund Returns, Journal of Financial Economics, 98(1), 54–71*
***Zhang, L.*** *2005. The Value Premium, Journal of Finance*

INDEXES

# The Impact of Risk Controls and Strategy-Specific Risk Diversification on Extreme Risk

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Since its introduction in the 1990s, value at risk (VaR) has become a standard measure of risk in the practice of finance. Another measure of risk that is more informative than VaR, conditional value at risk (CVaR), computes the average losses beyond VaR. Both measures are sensitive to large portfolio losses, whose frequency of occurrence is described by what is known as the tail of the distribution. They are thus the standard measure for estimating tail risk, or downside risk, of portfolio losses. In practice, VaR provides a loss threshold exceeded with some small predefined probability such as 1% or 5%, while CVaR measures the average loss higher than VaR and is, therefore, more informative about extreme losses.

Loh and Stoyanov (2013) analyze tail risk as measured by VaR and CVaR across different stock markets. The analysis is performed by dividing tail risk into a volatility component and a residual component through a two-step process. First, the clustering of volatility is explained through application of the standard econometric framework of the Generalized Autoregressive Conditional Heteroskedastic model (GARCH) and, second, the remaining tail risk is estimated from the residual process using extreme value theory (EVT). From a riskmanagement perspective, it is important to segregate the two components because the dynamics of volatility contributes to the unconditional tail thickness phenomenon. Generally, the GARCH part is responsible for capturing the dynamics of volatility while EVT provides a model for the behavior of the extreme tail of the distribution. Finally, Loh and Stoyanov (2014) discuss the important question of whether smart beta is exposed to more tail risk.

In this article, we complement the analysis by Loh and Stoyanov (2014) by focusing on two new aspects. First, we verify whether various risk controls, such as country and sector neutrality but also tracking error (TE) control, have any impact on tail risk. Secondly, we check if diversifying away strategy-specific risk by building a multistrategy smart-beta index affects tail risk. Both questions are explored through in-sample analysis as in Loh and Stoyanov (2014).

### The impact of risk control on tail risk

To carry out a comparison of tail risk, we calculate annualized averages of several statistics. We provide annualized averages of volatility,[17] constant scale tail risk (CVaR with constant volatility of 17% for absolute returns and constant tracking error of 3% for relative returns) and the total tail risk computed through the GARCH-based model (Total CVaR) for the diversified portfolios. The decomposition provides insight into what underlies the differences in total CVaR among different risk controls across different strategies: whether it is the average volatility (or TE) or the residual tail risk, having explained the clustering of volatility effect.

CVaR is computed at 1% tail probability and is interpreted as the average loss, provided that the loss exceeds VaR at 1% tail probability. The sample period is June 2003 – December 2013. We use data from the Scientific Beta platform, which provides indexes constructed using stocks from different geographical regions with different strategies and stock-selection criteria.

EXHIBIT 1

**Relative return tail risk of diversified portfolios with controlled tracking error on World Developed Market from June 2003 to December 2013. All statistics are annualized.**

| | Strategy | Relative Returns | | | |
|---|---|---|---|---|---|
| | | Realized Returns | Average TE | CVaR Constant TE at 3% | Total CVaR |
| SciBeta Developed | Maximum Deconcentration | 1.71% | 2.04% | 8.71% | 5.91% |
| | Maximum Deconcentration (5% TE/CW) | 1.40% | 1.72% | 9.29% | 5.34% |
| | Maximum Deconcentration (3% TE/CW) | 0.86% | 1.06% | 9.33% | 3.30% |
| | Maximum Deconcentration (2% TE/CW) | 0.59% | 0.74% | 9.33% | 2.31% |
| | Maximum Decorrelation | 1.81% | 2.31% | 9.39% | 7.23% |
| | Maximum Decorrelation (5% TE/CW) | 1.49% | 1.93% | 9.66% | 6.22% |
| | Maximum Decorrelation (3% TE/CW) | 0.89% | 1.20% | 9.65% | 3.87% |
| | Maximum Decorrelation (2% TE/CW) | 0.60% | 0.84% | 9.56% | 2.68% |
| | Efficient Minimum Volatility | 2.34% | 3.56% | 9.08% | 10.79% |
| | Efficient Minimum Volatility (5% TE/CW) | 1.22% | 2.32% | 8.90% | 6.89% |
| | Efficient Minimum Volatility (3% TE/CW) | 0.77% | 1.47% | 8.98% | 4.40% |
| | Efficient Minimum Volatility (2% TE/CW) | 0.54% | 1.02% | 8.89% | 3.02% |
| | Efficient Maximum Sharpe Ratio | 1.95% | 2.43% | 9.71% | 7.87% |
| | Efficient Maximum Sharpe Ratio (5% TE/CW) | 1.02% | 1.78% | 9.17% | 5.45% |
| | Efficient Maximum Sharpe Ratio (3% TE/CW) | 0.57% | 1.11% | 9.05% | 3.33% |
| | Efficient Maximum Sharpe Ratio (2% TE/CW) | 0.38% | 0.77% | 8.94% | 2.31% |

[17] *The average volatility is simply the average of the estimated volatility through the GARCH model.*

First, we examined the effect of adding country or sector neutrality constraints with different strategies. We considered the following strategies: maximum deconcentration (MDecon), maximum decorrelation (MDecor), efficient minimum volatility (MVol) and efficient maximum Sharpe ratio (MSR).[18] We looked at both absolute and relative returns, where relative return is defined as the portfolio excess return over the corresponding cap-weighted market index return. We found that country and sector neutrality do not have a material effect on tail risk for either absolute or relative returns.[19]

As a next step, we considered the effect of TE control on tail risk. Because of the use of the core-satellite technique to achieve TE control, the risk profile of the TE-controlled index should increasingly resemble that of the cap-weighted index as the TE target is reduced from 5% to 2%. The results provided in Exhibit 1 concern the relative returns of four strategies constructed from the Scientific Beta developed universe and are consistent with this expectation. We notice that the total CVaR decreases with the decrease in the TE target and since there are no significant differences in the constant TE CVaR, it follows that this decrease in total CVaR is driven by the decrease in average TE. In other words, reducing the TE target leads to lower TE which in turn leads to a lower total CVaR.

The case of absolute returns is very similar in that the total CVaR of the TE-controlled strategy converges to the total CVaR of the cap-weighted index, which is driven by the convergence of the corresponding volatilities. The same effects are present in the other single-market or broader regional indexes; the conclusion is not specific to a given universe.

### The impact of strategy-specific risk diversification on tail risk

The weighting schemes considered by the industry as alternatives to cap-weighting are exposed to risks specific to each. For example, since each weighting scheme has a specific objective, it is exposed to the sample risk inherent in the adopted estimators used to calculate parameters from data. Such strategy-specific risks can be diversified away by exploiting the imperfect correlations between the arising estimation errors. An approach offered on the Scientific Beta platform is the diversified multistrategy index, which is an equally-weighted combination of the five available weighting schemes on the platform. Not only does this approach diversify strategy-specific risks but it also improves performance. It is, therefore, important to compare the tail risk of the diversified multistrategy index to that of its constituent subindexes.

CVaR, like any other risk measure, is supposed to be able to identify diversification opportunities if they exist; that is, the CVaR of any portfolio must not exceed the weighted average of the stand-alone CVaRs of the constituents. Thus, to explore the effect of the diversified multistrategy index on tail risk, we calculate a tail-risk diversification ratio (TDR) similar to the GLR measure[20] for variance:

$$TDR = \frac{CVaR(r_p)}{\sum w_i CVaR(r_i)}$$

EXHIBIT 2

**Relative return tail risk of diversified portfolios based on regions and strategies for the period June 2003–December 2013. All statistics are annualized.**

| Strategy | | Relative Returns | | | | TDR | VDR |
|---|---|---|---|---|---|---|---|
| | | Realized Returns | Average TE | CVaR Constant TE at 3% | Total CVaR | | |
| SciBeta United States | Maximum Deconcentration | 1.70% | 2.99% | 9.01% | 8.97% | | |
| | Maximum Decorrelation | 1.60% | 3.09% | 9.68% | 9.96% | | |
| | Efficient Minimum Volatility | 2.09% | 3.88% | 9.17% | 11.84% | | |
| | Efficient Maximum Sharpe Ratio | 1.66% | 2.90% | 9.63% | 9.30% | | |
| | Diversified Risk Weighted | 1.77% | 2.58% | 8.81% | 7.59% | | |
| | Diversified Multi-Strategy | 1.78% | 2.62% | 9.31% | 8.13% | 0.8531 | 0.8495 |
| SciBeta Eurozone | Maximum Deconcentration | 1.27% | 4.24% | 9.03% | 12.76% | | |
| | Maximum Decorrelation | 1.86% | 5.60% | 9.14% | 17.04% | | |
| | Efficient Minimum Volatility | 2.33% | 6.98% | 8.90% | 20.71% | | |
| | Efficient Maximum Sharpe Ratio | 2.05% | 5.83% | 9.05% | 17.60% | | |
| | Diversified Risk Weighted | 1.65% | 4.43% | 9.10% | 13.43% | | |
| | Diversified Multi-Strategy | 1.86% | 5.18% | 9.01% | 15.55% | 0.9535 | 0.9565 |
| SciBeta United Kingdom | Maximum Deconcentration | 1.96% | 4.73% | 9.45% | 14.91% | | |
| | Maximum Decorrelation | 1.79% | 4.83% | 9.12% | 14.69% | | |
| | Efficient Minimum Volatility | 2.73% | 6.21% | 8.92% | 18.48% | | |
| | Efficient Maximum Sharpe Ratio | 2.75% | 4.81% | 9.07% | 14.54% | | |
| | Diversified Risk Weighted | 2.00% | 4.65% | 9.42% | 14.59% | | |
| | Diversified Multi-Strategy | 2.27% | 4.63% | 9.25% | 14.27% | 0.9239 | 0.9165 |
| SciBeta Japan | Maximum Deconcentration | 1.96% | 4.09% | 9.68% | 13.18% | | |
| | Maximum Decorrelation | 1.71% | 5.06% | 8.94% | 15.10% | | |
| | Efficient Minimum Volatility | 2.22% | 7.38% | 8.72% | 21.45% | | |
| | Efficient Maximum Sharpe Ratio | 1.97% | 5.39% | 8.87% | 15.95% | | |
| | Diversified Risk Weighted | 2.16% | 4.46% | 9.20% | 13.70% | | |
| | Diversified Multi-Strategy | 2.03% | 5.04% | 8.81% | 14.80% | 0.9322 | 0.9547 |
| SciBeta Developed Asia-Pacific ex-Japan | Maximum Deconcentration | 2.80% | 4.83% | 9.13% | 14.70% | | |
| | Maximum Decorrelation | 4.14% | 5.83% | 8.78% | 17.06% | | |
| | Efficient Minimum Volatility | 4.95% | 6.88% | 8.17% | 18.73% | | |
| | Efficient Maximum Sharpe Ratio | 3.94% | 6.04% | 8.44% | 16.97% | | |
| | Diversified Risk Weighted | 3.11% | 4.78% | 8.60% | 13.70% | | |
| | Diversified Multi-Strategy | 3.81% | 5.36% | 8.47% | 15.13% | 0.9320 | 0.9446 |
| SciBeta Developed | Maximum Deconcentration | 1.71% | 2.04% | 8.71% | 5.91% | | |
| | Maximum Decorrelation | 1.81% | 2.31% | 9.39% | 7.23% | | |
| | Efficient Minimum Volatility | 2.34% | 3.56% | 9.08% | 10.79% | | |
| | Efficient Maximum Sharpe Ratio | 1.95% | 2.43% | 9.71% | 7.87% | | |
| | Diversified Risk Weighted | 1.85% | 1.92% | 8.95% | 5.72% | | |
| | Diversified Multi-Strategy | 1.95% | 2.15% | 9.60% | 6.88% | 0.9164 | 0.8767 |

[18] *Details on index construction are available at www.scientificbeta.com.*
[19] *To save space, these results are not reported in a tabular format in this article. All results will be published in a forthcoming EDHEC Risk Institute Publication.*
[20] *The GLR measure is defined as the ratio of the portfolio variance to the weighted variance of its constituents.*

in which CVaR($r_p$) denotes the total CVaR of the portfolio (the diversified multistrategy index), CVaR($r_i$) denotes the stand-alone total CVaR of the portfolio constituents (the subindexes) and $w_i$ denotes the weights of the constituents (equal weights). Since volatility is a component in tail risk, we compare TDR to the volatility diversification ratio (VDR):[21]

$$VDR = \frac{\sigma(r_p)}{\sum w_i \sigma(r_i)}$$

where $\sigma$ denotes volatility. It can be demonstrated that neither ratio can exceed 1 and the lower they are, the bigger the diversification effect. Comparing the two to one another is useful because if the main driver of diversification is the volatility component of CVaR, then the two measures take similar values.

Exhibit 2 provides the averaged relative return risk statistics together with the two ratios computed for the diversified multistrategy indexes for the main geographical regions. Regardless of region, the total CVaR of the diversified multistrategy is always smaller than the average of the stand-alone total CVaRs, which indicates that there are diversification benefits for tail risk. All TDRs are, however, very similar to the corresponding VDRs which, together with the fact that the constant scale CVaRs of the multistrategy and the constituents are very similar, indicates that the main source of the diversification is in fact the tracking error.

It is curious that the constant scale CVaR does not seem to change significantly in the multistrategy index. This result is consistent with the finding reported by Loh and Stoyanov (2014) that within a given universe, the constant scale CVaR is not sensitive to the weighting scheme. It is also consistent with a theoretical result in probability theory that, in the presence of independent risks, the tail behavior of any portfolio of these risks is dominated by the heaviest tail of the stand-alone risks; in other words, diversification is not expected to be an efficient technique for managing tail thickness. This is indirectly confirmed in another finding by Loh and Stoyanov (2014), that in contrast to the weighting scheme, stock selection criteria can have an impact on residual tail risk.

**Does risk control and strategy-specific risk diversification impact extreme risk in smart beta?**

Generally, we do not find material improvement in tail risk when country and sector risks are controlled for both absolute and relative returns. As far as TE control is concerned, including a low TE target has an effect on tail risk as measured by the total CVaR of the relative return. The main factor behind the improvement in total CVaR, however, is the reduced tracking error; that is, the reduction in extreme risk is an expected side effect. Finally, diversifying away strategy-specific risks by building a diversified multistrategy index diversifies the relative return tail risk. The main diversification benefits, however, arise from the related diversification benefits of the tracking error. ~

*An approach offered on the Scientific Beta platform is the diversified multistrategy index, which is an equally-weighted combination of the five available weighting schemes on the platform. Not only does this approach diversify strategy-specific risks but it also improves performance. It is, therefore, important to compare the tail risk of the diversified multistrategy index to that of its constituent subindexes*

***References***

***Loh, L, and S. Stoyanov.*** *August 2013. Tail Risk of Asian Markets: An Extreme Value Theory Approach, EDHEC-Risk Institute Publication.*

***Loh, L, and S. Stoyanov.*** *2014. The Extreme Risk across Strategies and Stock Selection Criteria, Investment & Pensions Europe Research Insights Supplement Spring 2014, pp 13–17.*

[21] *We use VDR instead of the standard GLR measure because the scale of CVaR is defined in terms of volatility rather than variance.*

# Are U.S. Pension Fund Allocations Well Diversified?

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In a recent research paper (Carli, Deguest and Martellini, 2014), supported by CACEIS as part of the New Frontiers in Risk Assessment and Performance Reporting research chair at EDHEC-Risk Institute, we analyze the diversification of the portfolio held by U.S. pension funds and its relationship with subsequent portfolio performance.

## Introducing a formal measure of diversification for pension fund portfolios

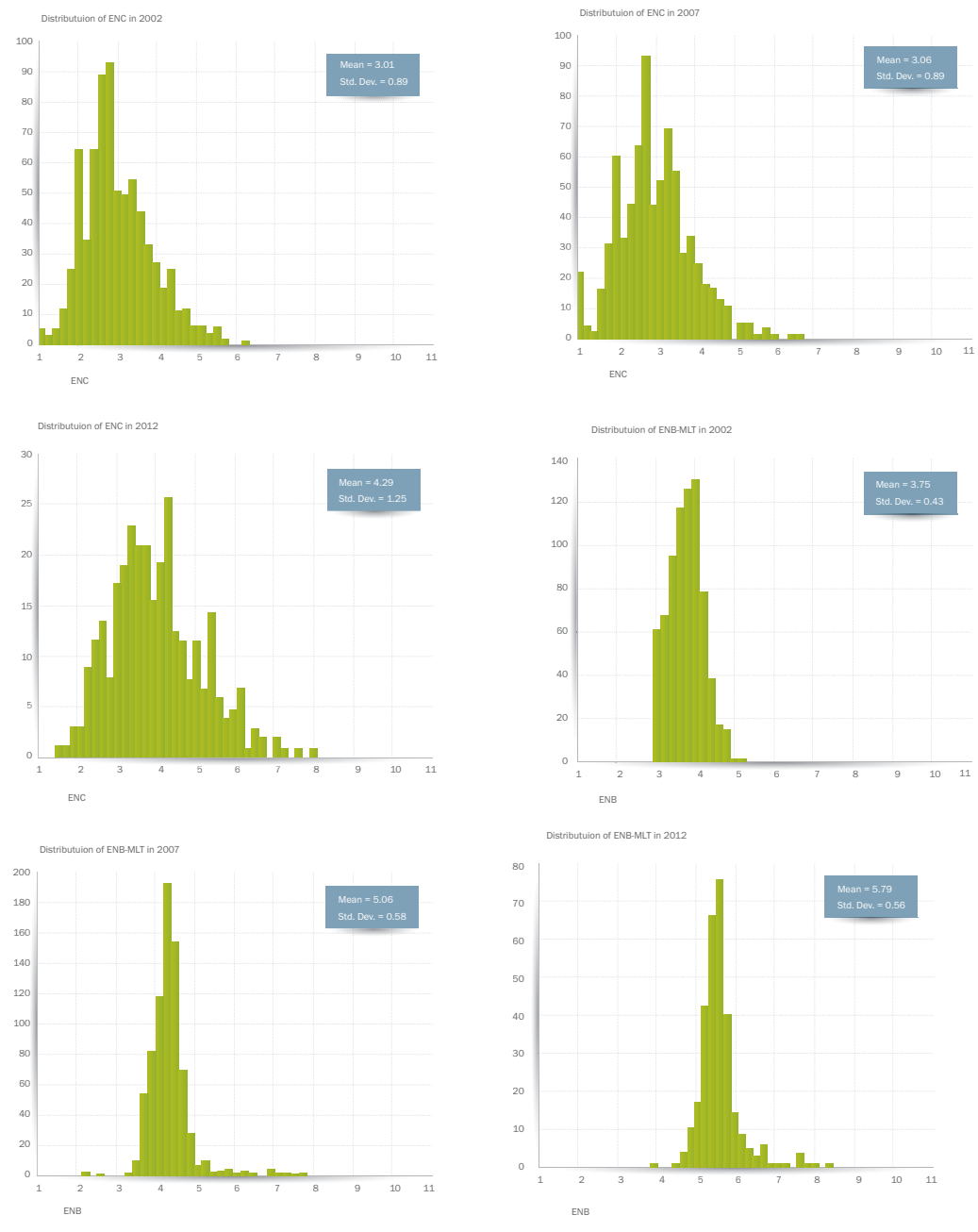
Common sense and portfolio theory both suggest that the degree of diversification of a pension fund portfolio is a key ingredient for the generation of attractive performance across various market conditions. The benefits of diversification are clear: efficient diversification generates a reduction in unrewarded risks that leads to an enhancement of the portfolio's risk-adjusted performance. On the other hand, providing a quantitative measure of how well or poorly diversified a portfolio is, is not as straightforward a task as it might seem in the absence of a formal definition for diversification. The usual definition of diversification is that it is the practice of not "putting all your eggs in one basket." Having eggs (dollars) spread across many baskets is, however, a rather loose prescription in the absence of a formal definition for what is the true meaning of many and baskets.

One first approach to measuring pension fund portfolio diversification would consist of a simple count of the number of constituents the portfolio is invested in. One key problem with this approach is that what matters from a risk perspective is not the nominal number of constituents in a portfolio, but instead its effective number of constituents (ENC). To understand the nuance, let us consider the example of a fictitious pension fund facing 10 asset classes and allocating 99% of the assets to one asset class and spreading the remaining 1% of the wealth among the 9 remaining asset classes. While the nominal number of asset class constituents in that portfolio, defined as the number of classes that receive some non-zero allocation, is 10, it is clear that the effective number of constituents in the portfolio is hardly greater than one, and that this poorly diversified portfolio will behave essentially like a fully concentrated portfolio from a risk perspective. In this context, it appears that a natural and meaningful measure of the effective number of constituents (ENC) in a portfolio is given by the entropy of the portfolio weight distribution. This quantity, a dispersion measure for probability distributions commonly used in statistics and information theory, is indeed equal to the nominal number N for a well-balanced equally-weighted portfolio, but would converge to 1 if the allocation to all assets but one converges to zero as in the example above, thus confirming the extreme concentration in this portfolio.

At this stage, the need remains for a critical assessment of what the proper interpretation for the baskets should be in this proverbial definition of diversification. The straightforward approach, which suggests that baskets are asset classes, is in fact misleading or at least severely incomplete. For example, a seemingly well-diversified allocation to many asset classes that essentially load on the same risk factor (e.g., equity risk) can eventually generate a portfolio with a

EXHIBIT 1

**Distribution of Diversification Measures of U.S. Pension Funds**
These figures display the distribution of the effective number of constituents (ENC) and the effective number of bets (ENB) for the U.S. pension funds in the P&I database in years 2002, 2007 and 2012.



very concentrated set of risk exposures. Going back to the eggs-and-baskets analogy, having a well-balanced allocation of eggs to many different baskets that are tied together can hardly be regarded as an astute way of ensuring proper diversification of the risks involved in carrying eggs to the market. In other words, baskets should be interpreted as uncorrelated risk factors, as opposed to correlated asset classes, and it is only if the distribution of the contributions of various factors to the risk of the portfolio is well balanced that the investor's portfolio can truly be regarded as well diversified. Putting all these elements together, we propose to use in our empirical analysis the effective number of bets (ENB), formally defined as the entropy of the distribution of contributions of uncorrelated factors to the risk of the portfolio, as a meaningful measure of diversification for investors' portfolios (see Meucci, 2009 and Deguest, Martellini and Meucci, 2013 for more details).

### Analyzing the relationship between pension fund portfolio diversification and portfolio performance in various market conditions

We use the P&I Top 1,000 database to obtain information on the asset allocation of each of the largest 1,000 U.S. pension funds as of September 30, 2002, September 30, 2007 and September 30, 2012. We focus exclusively on the portion allocated to their defined benefit plan, and divide their portfolio according to the following partition: domestic fixed income, international fixed income, high-yield bond, inflation-linked bond, domestic equity, international equity, global equity, private equity, real estate, commodity, mortgage, and cash. Once the partition is completed, we choose appropriate benchmarks for each asset class and use the minimal linear torsion approach (Meucci, et al., 2013) to turn correlated asset class returns into uncorrelated factor returns.[22] We then estimate the ENB diversification measure for each pension fund in the database at the end of September 2002, at the end of September 2007 and at the end of September 2012. We also compute the effective number of constituents (ENC) at the same dates (see Exhibit 1).

When looking at the evolution of each diversification measure, it seems that a change occurred between 2007 and 2012, as most U.S. pension funds seem to have increased the diversification level in their portfolio between these two dates, at least when using ENC as a diversification measure. For instance, between 2002 and 2007, the mean of the distribution of the ENC measures increases by 1.3%, while between 2007 and 2012, it increases by 40.7%. However, the average increase in terms of ENB is only 14.4% over the same period.

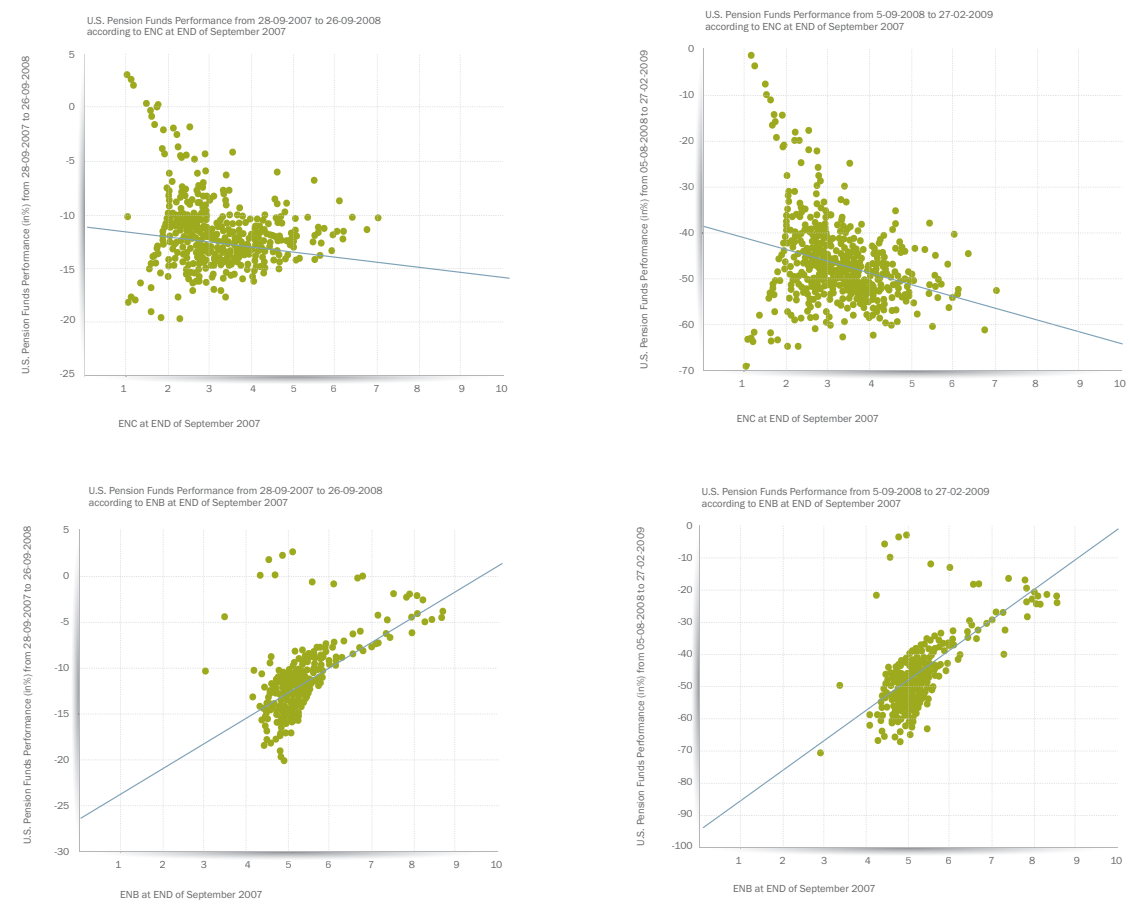
We then analyze whether the diversification measures computed over these pension funds at the end of September 2007 can give insights on the returns of U.S. pension fund performance in subsequent months.[23] In our test, we compute the fund returns over two different periods: over the year directly following the date of computation of the diversification measures (from 09/28/2007 to 09/26/2008), and over the worst period of the subprime crisis for the financial sector (from 09/05/2008 to 02/27/2009). For each diversification measure, we first plot the relationship between the U.S. pension funds' annualized performances at date t+n months according to their level of diversification measure at date t (end of September 2007). Then, we statistically test the degree of significance of our results. We replicate this test for each diversification measure and for the two periods of time considered, and report the results in Exhibit 2.

It is first striking to see that the relationship between U.S. pension fund performances and their level of ENB is positive, and this relationship is statistically significant.[24] This result holds true for the two periods of performance computation.

Overall, these results suggest that, at the end of September 2007, a pension fund that had a higher ENB (hence holding a better diversified portfolio) was more likely to incur lower loss levels during 09/28/2007–09/26/2008 and 09/05/2008–02/27/2009 than a pension fund that had a lower ENB, assuming the policy portfolio weights remain constant. On the other hand, higher levels of ENC for a pension fund at the end of September are likely to have a zero if not negative impact on its performances during 09/28/2007–09/26/2008 and 09/05/2008–02/27/2009 compared to another pension fund with lower levels of ENC. These results imply that better diversification in the sense of a more balanced exposure to uncorrelated risk factors can help pension funds in bear market conditions, but a similar result is not obtained when diversification is naively measured in terms of balanced allocation to correlated asset classes.~

EXHIBIT 2

**Performances of U.S. Pension Funds with respect to their Diversification Measures at the End of September 2007**
These figures display the annualized performances of the U.S. pension funds in the P&I database computed for two different periods with respect to their diversification measures at the end of September 2007. The annualized performances are calculated for the year immediately following the date of computation of the diversification measures (from 09/28/2007 to 09/26/2008) and during the worst of the subprime crisis (from 09/05/2008 to 02/27/2009). We consider that pension funds' asset allocations have not changed since the end of September 2007; therefore, the performances displayed here are only estimates.




## RESEARCH

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*This chair looks at improved risk reporting, integrating the shift from asset allocation to factor allocation, improved geographic segmentation for equity investing, and improved risk measurement for diversified equity portfolios.*
*The full version of the research is available on the EDHEC-Risk Institute website at the following address: www.edhec-risk.com/performance_and_style_analysis/CACEIS_Research_Chair*

***References***

***Carli, T., R. Deguest and L. Martellini.*** *2014. Improved Risk Reporting with Factor-Based Diversification Measures, EDHEC-Risk Institute publication supported by CACEIS as part of the research chair on New Frontiers in Risk Assessment and Performance Reporting at EDHEC-Risk Institute.*
***Deguest, R., L. Martellini, and A., Meucci.*** *2013. Risk Parity and Beyond — From Asset Allocation to Risk Allocation Decisions, EDHEC-Risk Institute publication.*
***Meucci, A.*** *2009. Managing Diversification, Risk, 22, 5, 74–79.*
***Meucci, A., A. Santangelo, and R. Deguest.*** *2013. Measuring Portfolio Diversification based on Optimized Uncorrelated Factors. SSRN: ssrn.com/abstract=2276632.*

[22] *The minimal linear torsion approach (MLT) focuses on extracted uncorrelated factors that are as close as possible to the original constituents.*
[23] *Because of data availability constraints, we do not use actual pension fund performance in our analysis and assume instead that the fund asset allocation remains constant over the months following the computation of the diversification measures at date t.*
[24] *It should be noted that not all pension fund managers seek to hold a well-diversified portfolio. In particular, the liability-driven investing paradigm implies that pension fund managers interested in minimizing the volatility of their funding ratio would hold a concentrated fixed-income portfolio with interest rate exposures similar to the interest rate risk exposures in the pension liabilities. We expect such an extremely safe strategy to offer, by construction, good downside protection in bear equity markets, and we did find that, in spite of the positive relationship between ENB and performance in 2008, the very top performers were the pension funds holding only sovereign bonds. The opportunity cost of this exceedingly cautious strategy is of course prohibitive in terms of renouncing access of the risk premia on risky asset classes that is allowed by a well-diversified portfolio.*