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Research for Institutional Money Management

EDHEC-RISK Institute

INTRODUCTION

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It is my great pleasure to introduce this inaugural issue of the EDHEC-Risk Institute supplement in Pensions & Investments. Since it was founded twelve years ago, EDHEC-Risk Institute has been endeavoring to produce industry-relevant research of the highest academic standards. The present supplement is an inherent part of that mission and we hope that you will find the insights both useful and original.

Our first article shows that failing to disentangle long-term risk aversion and short-term loss aversion may lead to poor investment decisions. As our authors show, relatively simple solutions exist that can be implemented as dynamic asset—allocation strategies in order to control short-term risk levels while maintaining access to long-term sources of performance. These solutions are a substantial improvement over traditional strategies without dynamic risk control. We then examine the question of low-volatility—equity investing. We think that the correct question for investors to ask themselves is how a low-volatility strategy fits into their risk budget, so that they can design the investment strategy that best serves their risk objectives. Low-volatility portfolios can be constructed in many different ways and the risks and performance of a particular strategy will depend on the choices that are made about the critical components of the strategy.

We look at optimizing the investment policy of a corporate pension fund when the pension plan beneficiaries, trustees and managers, not to mention the equity holders, bondholders and managers of the sponsor firm, have different preferences with respect to the risk/return profile of the fund strategy. Our research finds that a constant proportion portfolio insurance strategy adapted to the asset-liability management context reduces the conflicts of interest between pensioners and shareholders by allowing the pension fund to invest more in risky assets, which generates surpluses from which equity holders will benefit, while protecting the funding ratio, which is in the pensioners' interest.

Smart-beta investing in equities is becoming more and more popular as reservations about the suitability of cap-weighted equity indexes for investment, long expressed in the academic literature, have increasingly come to the fore. In our article on the subject, we discuss the "Smart Beta 2.0" approach developed by EDHEC-Risk Institute. At a time when it is universally recognized that the ability to manage risks effectively is of paramount importance for asset owners, this approach allows investors to invest in these advanced forms of benchmarks with full knowledge and control of the risks of their choice.

In a further article, we examine alternative equity-diversification strategies. We explain the conceptual groundings behind five alternative weighting schemes identified by ERI Scientific Beta, an EDHEC-Risk Institute venture, as being credible vehicles for diversification. We emphasize their intrinsic specific and systematic risks, as well as the impacts of these risks on their individual conditional and unconditional performance and risk profiles, and we introduce a multistrategy approach which aims at diversifying away these risks and provides a solution for gaining robust exposure to smart beta.

We then turn to the question of alternative bond indexes. Research conducted by EDHEC-Risk Institute has clearly demonstrated the limits of traditional debt-weighted index offerings, which are not justified by any genuine economic thinking or model. To date, the weighting schemes available on the market or documented in practitioner journals have not convinced EDHEC-Risk Institute's scientific management team of their robustness and pertinence. Whether for sovereign or corporate debt, EDHEC-Risk Institute considers that there is room for improvement.

Finally, in light of the dramatic bankruptcy of the city of Detroit, we look at the important question of the proper discount rate for liabilities. There are at least three different types of discount rates that can be used to value pension liabilities: arbitrary discount rates, market discount rates and endogenous discount rates. In our article, we review some of the key academic insights about the use of various discount rates for liabilities, and the pros and cons associated with such choices. Also, we discuss the importance of the perspective of the key stakeholders in settling these issues.

We would like to thank our partners at P&I for their help in preparing and producing this first edition of the EDHEC-Risk Institute Supplement. We look forward to a long and fruitful partnership and hope that all readers will gain useful insights from the results of our research.

CONTENTS

PORTFOLIO MANAGEMENT

INDEXES

DEFINED BENEFIT PLAN

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PORTFOLIO MANAGEMENT

Long-Term Investing with Short-Term Constraints

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Failing to separate long-term risk aversion and short-term loss aversion may lead to poor investment decisions. One of the main findings in the academic literature on long-term allocation decisions with mean-reverting equity returns is the fact that equities serve as a hedge against unfavorable equity returns in the presence of mean-reverting equity returns. As a result, the optimal allocation to stocks is higher compared to the myopic case, and investors with longer time-horizon hold more stocks compared to investors with shorter horizon. This prescription has very often been taken at face value by target-date funds or life-cycle funds, an investment solution advocating a deterministic decrease of equity allocations (also known as glidepath) when approaching retirement date.

One key problem, however, is that this prescription can lead to extremely difficult situations when risk is assessed from a shorter-term perspective, in particular in the context of a severe bear equity market such as the one experienced in 2008. In fact, it appears that most, if not all, investors, even those (such as pension funds or sovereign wealth funds) with the longest possible horizons, inevitably face a number of short-term performance constraints imposed by accounting and/or regulatory pressure, political pressure, peer pressure, etc. In a private wealth—management context, there is also strong evidence that investors typically face (mostly self-imposed) short-term constraints, e.g., maximum drawdown constraints.

While it is widely perceived that tension exists between a focus on hedging long-term risk and a focus on insurance with respect to short-term constraints, we cast new light on this debate by arguing that long-term objectives and short-term constraints need not be mutually exclusive.[1] In fact, our analysis shows that both motives may naturally coexist within the context of a long-term investing strategy consistent with short-term performance constraints.

An increasing, anti-cyclical allocation to equities can be justified in the absence of short-term–risk constraints, if a mean-reverting equity risk premium is present. Investors endowed with consumption/liability objectives need to invest in two distinct portfolios, in addition to cash: one performance-seeking portfolio (PSP) and one liability-hedging portfolio (LHP); this is the liability-driven investing paradigm (LDI). The allocation to the "risky" performance-seeking portfolio (PSP) vs. "safe" liability-hedging portfolio (LHP) is found to be increasing in the PSP Sharpe ratio.

The optimal strategy displays a state-dependent component, suggesting that the allocation to equity should be increased (respectively, decreased) when equity has become cheap (respectively, expensive), as measured through a proxy for the equity risk premium. In the context of a model with a stochastic mean-reverting equity risk premium, one can also show that the optimal allocation involves hedging demand against unexpected changes in the PSP Sharpe ratio, known as the risk premium hedging portfolio (RPHP), which indeed implies a deterministic decrease of the allocation to equity as the investor gets closer to the time horizon. One key element that is missing in the analysis presented so far is the integration of short-term constraints into the design of the optimal allocation strategy.

The allocation to equities should also be an increasing function of the current value of the risk budget, in addition to being an increasing function of time-horizon and of the current value of the equity-risk premium. This can be justified in the presence of short-term constraints. These short-term constraints are not managed through hedging strategies, which focus on immunizing the portfolio value with respect to changes in risk factors affecting asset and liability values, but instead through dedicated insurance strategies. The practical implication of the introduction of short-term constraints is that optimal investment in a performance-seeking satellite portfolio (PSP) is not only a function of risk aversion, but also becomes a function of risk budgets (margin for error measured in terms of distance with respect to minimum acceptable wealth levels), as well as probability of the risk budget to be spent before horizon. In a nutshell, a pre-commitment to risk management allows one to adjust risk exposure in an optimal state-dependent manner, and therefore to generate the highest exposure to the upside potential of the PSP while respecting risk constraints.

It is widely perceived that tension exists between a focus on hedging long-term risk and a focus on insurance with respect to short-term constraints: dynamic risk-controlled strategies, which imply a reduction to equity allocation when a drop of equity prices has led to a substantial diminution of the risk budget, have often been blamed for their pro-cyclical nature, and long-term investors are often reluctant to sell equity holdings in those states of the world where equity markets have become particularly attractive in the presence of mean reversion in the equity risk premium.

Our research actually suggests that long-term objectives and short-term constraints need not be mutually exclusive, and can be integrated in a comprehensive asset allocation framework. Depending on market conditions and parameter values the pro-cyclical risk-controlled motivation may outweigh the revision of strategic asset—allocation motivation, or vice versa, with risk management always prevailing ultimately. In other words, the risk-control methodology can be made entirely consistent with internal or external processes aiming at generating active asset allocation views. In fact, casting the active view generation process within the formal framework of a dynamic risk-control strategy appears to be the only way to successfully implement active asset allocation while ensuring the respect of risk limits.

In practice, a number of key improvements can be used in implementation. While the original approach was developed in a simple framework, it can be extended in a number of important directions, allowing for the introduction of more complex floors. A large variety of floors can in fact be introduced (simultaneously if necessary) so as to accommodate the needs of different kinds of investors. Among the possible floors, the following possibly stand out in terms of their relevance for various kinds of investors: capital guarantee floors allowing the protection of a fraction of the initial capital; benchmark protection floors allowing the protection of a fraction of the value of any given stochastic benchmark (with the liability portfolio being the most natural benchmark for investors facing liabilities); maximum drawdown floors allowing limits on maximum consecutive losses; trailing performance floors allowing the protection of a fraction of the prior value of the portfolio on a rolling basis, etc.

In addition to accounting for the presence of floors, the dynamic risk-controlled strategies can also accommodate the presence of various forms of caps or ceilings. These strategies recognize that the investor has no utility over a *cap target* level of wealth, which represents the investor's goal (actually a cap), which can be a constant, a deterministic or a stochastic function of time. From a conceptual standpoint, it is not clear a *priori* why any investor should want to impose a strict limit on upside potential. The assumption is that by forgiving performance beyond a certain threshold, where they have relatively lower utility from higher wealth, investors benefit from a decrease in the cost of the downside protection (short position) in a convex payoff in addition to the long position—collar flavor.

Putting it differently, without the performance cap, investors have a greater chance of failing to achieve an almost-eached goal when their wealth level is very high, and we show that the presence of upper (in addition to lower) bounds on performance, consistent with the kind of utility satiation often exhibited by long-term investors, is another, independent, reason why a fall in equity prices should not always lead to a decrease in equity allocation, even without the mean-reverting equity risk premium.

> The opportunity costs implied by the short-term constraints are significantly lower when these constraints are optimally addressed through insurance strategies, as opposed to being inefficiently addressed through an unconditional decrease in the equity allocation.

Our analysis suggests that asset allocation and portfolio construction decisions are intimately related to risk management.

Investment management is essentially about finding optimal ways to spend risk budgets that investors are reluctant to set, with a focus on allowing the highest possible access to performance potential while respecting such risk budgets. Risk diversification, risk hedging and risk insurance are three useful approaches to optimal spending of investors' risk budgets. In this context, improved forms of investment solutions rely on a sophisticated exploitation of the benefits of the three competing approaches to risk management, namely risk diversification (key ingredient in the design of better benchmarks for performance-seeking portfolios), risk hedging (key ingredient in the design of better benchmarks for hedging portfolios) and risk insurance (key ingredient in the design of better dynamic asset-allocation benchmarks for long-term investors facing short-term constraints). In the end, risk management, which focuses on maximizing the probability of achieving investors' long-term objectives while respecting the short-term constraints they face, appears to be the key source of added value in investment management.

The results we obtain confirm that dynamic asset-allocation benchmarks can be designed so as to allow more efficient spending of investors' risk budgets. A commitment to reduce the allocation to equity in times and market conditions that require such a reduction so as to avoid over-spending risk budgets would seem to allow a greater average investment

[1]*For more details, see Deguest, R., L. Martellini and V. Milhau, 2013, Hedging vs. Insurance: Long-Horizon Investing with Short-Term Constraints, EDHEC-Risk publication.*

in equities compared to a simple static strategy that is calibrated to respect the same risk budget constraints. The welfare gains involved in this higher allocation to equities are found to be substantial for reasonable parameter values, especially for long-term horizons and in the presence of a mean-reverting equity risk premium.

As a numerical illustration of the benefits of risk-controlled strategies, we first simulate the performance of unconstrained strategies, taking the time-horizon to be equal to 20 years, while the risk-aversion parameter, which is not observable, is calibrated in such a way that the average allocation to equity over the 20-year life of the strategy is equal to a target of 10%, 20% or 30%. The three corresponding long-term unconstrained strategies will be referred to as defensive (leading to an average stock weight of 10%), moderate (leading to an average stock weight of 20%) and aggressive (leading to an average stock weight of 30%) respectively.

Exhibit 1 shows the resulting distribution of unconstrained terminal wealth for various risk-aversion levels. We find the usual risk/return trade-off: strategies implemented by less risk-averse investors will contain a higher allocation to equities, which will result in a higher average wealth level as well as higher uncertainty around the terminal wealth level.

While long-term strategies are engineered to achieve optimal risk/return trade-offs over the long term, short-term losses and drawdown levels can remain extremely large, especially for the aggressive investor, with a maximum drawdown at 24.4%, as can be seen from Exhibit 2.

In this context, an investor wishing (or obliged) to maintain the maximum drawdown around say 15% would have to opt for the defensive strategy, even if the level of wealth achieved with this strategy is much less attractive than what is allowed by the aggressive strategy. In this context, the objective measure of the opportunity cost associated with a 15% maximum drawdown can be formally defined as the additional initial contribution needed to reach with the defensive strategy the same average wealth level as with the aggressive strategy, a cost that turns out to be a prohibitive 32.51% in this particular example.

A less costly solution is to use insurance, as opposed to hedging, to manage downside risk. As an alternative to the defensive strategy, the investor can choose the aggressive strategy, which allows for much higher access to the equity risk premium, and implement a dynamic risk-controlled investing overlay designed to ensure that the maximum drawdown will be kept below 15% (see Exhibit 3 for the resulting distribution of terminal wealth, where we have also tested maximum drawdown levels at 10% and 20%).

Exhibit 3 shows that the average wealth of the aggressive strategy with a 15% maximum drawdown constraint is substantially higher than the unconstrained defensive one, for essentially the same level of extreme losses. This result makes a strong case for the management of short-term constraints through dynamic risk budgeting rather than through the choice of unnecessarily conservative investment policies. So as to provide an objective assessment of the opportunity cost of imposing stricter drawdown constraints, when these constraints are optimally managed through insurance techniques, we find that a mere 5.38% additional investment is needed to reach the aggressive benchmark with a maximum drawdown constraint of 15% the same average wealth level as with the aggressive strategy without maximum drawdown constraints. This value very favorably compares to the aforementioned 32.51% opportunity cost involved in managing maximum drawdown constraints inefficiently through excessive hedging.

Overall, these results illustrate that not disentangling long-term risk aversion and short-term loss aversion may lead to poor investment decisions. Relatively simple solutions exist that can be implemented as dynamic asset allocation strategies in order to control short-term risk levels while maintaining access to long-term sources of performance. These solutions are a substantial improvement over traditional strategies without dynamic risk control, which inevitably lead to under[solid]spending of investors' risk budgets in normal market conditions, with a strong associated opportunity cost, and overspending of investors' risk budget in extreme market conditions. ~

EXHIBIT 1

Distributions of Terminal Wealth Generated by Long-term Investment Strategies

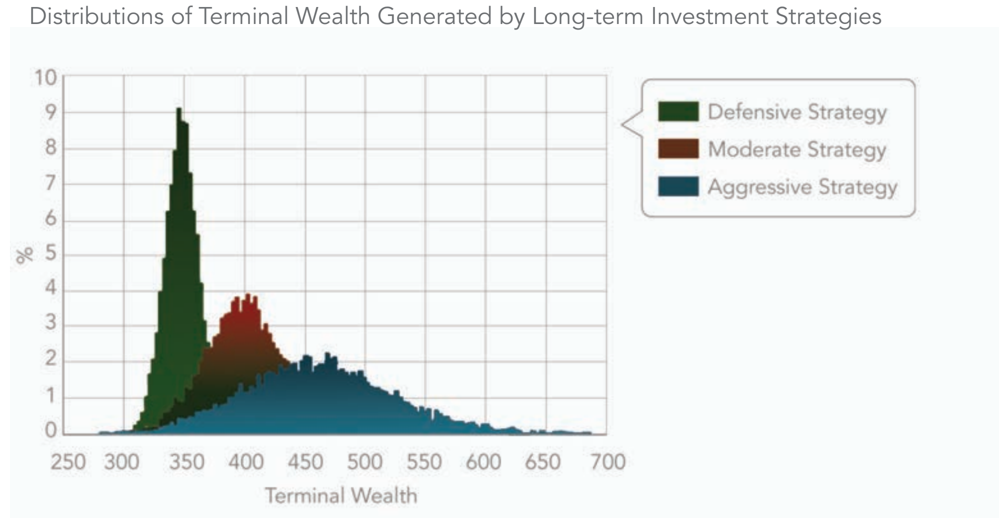


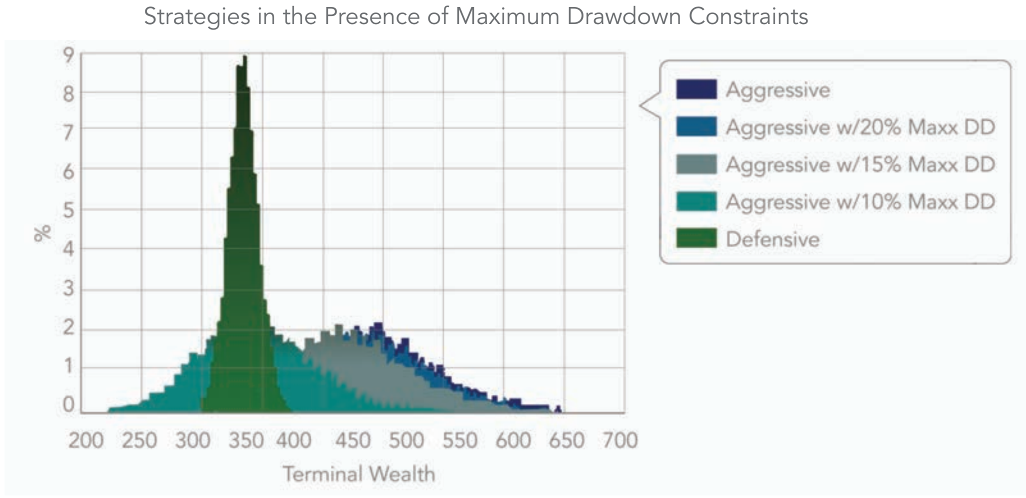
EXHIBIT 2

Risk and Performance Indicators for Long-Term–[en dash] Investment Strategies

	Aggressive	Moderate	Defensive
Min Wealth	251.71	280.10	293.75
Q5	362.39	345.16	324.78
Low Target Wealth (Q25)	419.64	378.00	338.89
Medium Target Wealth (Q50)	459.49	399.71	348.11
High Target Wealth (Q75)	500.33	421.75	357.40
Q95	567.38	456.22	371.75
Max Wealth	739.16	540.35	403.29
Average Wealth	461.37	400.15	348.18
High minus Low	80.69	43.75	18.51
(High minus Low)/(2 x Medium)	8.78%	5.47%	2.66%
Max 3Y-Loss	15.89%	9.26%	7.75%
Max DD	24.40%	17.78%	15.16%

EXHIBIT 3

Distributions of Terminal Wealth Generated by Long-Term Investment Strategies in the Presence of Maximum Drawdown Constraints



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Deguest, R., L. Martellini and V. Milhau. *2013. Hedging vs. Insurance: Long-Horizon Investing with Short-Term Constraints, EDHEC-Risk publication produced as part of the BNP Paribas Investment Partners research chair on "Asset-Liability Management and Institutional Investment*

The research from which this article was drawn was supported by BNP Paribas Investment Partners as part of the research chair on "Asset-Liability Management and Institutional Investment Management" at EDHEC-Risk Institute.The chair examines advanced ALM topics such as dynamic allocation strategies, rational pricing of liability schemes, and formulation of an ALM model integrating the financial circumstances of pension plan sponsors.The full version of the research is available on the EDHEC-Risk Institute website at the following address:www.edhec-risk.com/ALM/BNPPAM_Research_Chair

PORTFOLIO MANAGEMENT

Low-Volatility Investing and the Low-Volatility Anomaly

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Interest in low-volatility equity strategies has grown in recent years and its proponents have received an additional boost from the spate of new index offerings in this area. However, this investment approach is sub-optimal from the viewpoint of traditional asset-pricing theory. Indeed, a global minimum variance (GMV) portfolio can be regarded as a maximum Sharpe ratio portfolio subject to the assumption that all stocks have the same expected returns, whatever their factor exposures. This contrasts with the prediction of a positive relationship between the systematic risks (measured by market betas) and the expected return of stocks. This prediction is supported by the single-factor CAPM (Sharpe, 1964) and multifactor (Merton, 1973) equilibrium models as well as arbitrage pricing arguments (Ross, 1976). Empirical confirmation is provided by Fama and French (1993) and Carhart (1997), among others. Although the market efficiency arguments invoked in these models predict that investors should not expect a reward (earn a risk premium) for specific risk because it can be diversified away, there is also some evidence indicating the existence of a positive relationship between idiosyncratic (as opposed to systematic) risk and expected returns. For instance, Merton (1987) shows that total risk will matter to investors who are unable to hold the market portfolio, thus forcing them to pay attention to firm-specific risk. Therefore firms with larger specific risk will offer higher average returns to compensate investors for holding imperfectly diversified portfolios. The empirical results in Malkiel and Xu (2006) and Barberis and Huang (2001) support this notion.

In this context, one would expect that overweighting low-volatility stocks, implemented explicitly through weighting schemes using the inverse of volatility as a weighting factor or implicitly with GMV portfolios that do not penalize low-risk stocks for having low returns, will result in poor performance. On the other hand, advocates of low-volatility investing point to a growing body of empirical evidence of a negative relationship between risk and return in equity markets, which should result in attractive performance of low-volatility investing strategies. For example, Black (1972), and Black, Jensen and Scholes (1972) document a "low beta anomaly" that states that the relationship between systematic risk as measured by a stock's beta and its return is much flatter than predicted by the CAPM. Haugen and Heinz (1975) go as far as to claim that the relationship was not merely flat in their sample period, but actually inverted. Ang, Hodrick, Xing and Zhang (2006, 2009) focus on the specific risk component and report that high idiosyncratic volatility stocks have yielded "abysmally low returns" based on longer time series of data from U.S. and international markets (the "idiosyncratic volatility puzzle"). Others (Blitz and van Vliet, 2007) have documented a negative relationship between total (as opposed to specific) volatility and expected return, an anomaly that has come to be known as the "total volatility puzzle."

Early papers (Haugen and Heinz, 1972, 1975) and Haugen and Baker (1991) question both the methodology used for testing cross-sectional risk-return relationships and the effect of assumptions underlying standard asset pricing models. Their empirical findings cast doubts on the existence and significance of the risk premia implied by these models. In a more recent study, Blitz and van Vliet (2007) analyze a 20-year period and show results in which portfolios of low-volatility stocks have higher returns than portfolios of high-volatility stocks, but do not disclose significance levels. Evidence continues to mount, with findings from Baker, Bradley and Wurgler (2011), indicating that portfolios formed by sorting stocks by past volatility display higher returns for the low-volatility quintile over the subsequent month than for the high-volatility quintile. Results from Baker and Haugen (2012) and Blitz, Pang and van Vliet (2012) are similar and include emerging market stocks.

Given the critical importance of the risk/return relationship to both asset pricing theory and investment practice, the conclusion that there is no consensus about whether it is positive, flat or negative is rather disconcerting. As the debate between the two sides has intensified, attention has turned to providing justification for the empirical results discussed above. Among recent attempts to explain the low-volatility anomaly (i.e., attempts to find economic reasons that explain why a negative relationship might exist between some risk measure and expected returns) Baker, Bradley and Wurgler (2011) offer two behavioral arguments. First, some irrational market participants show a preference for lotteries and heuristic biases such as representativeness and overconfidence resulting in a demand for higher volatility stocks that is not justified by fundamentals. Second, the typical institutional investor's mandate to maximize the ratio of excess returns to tracking error relative to a fixed benchmark (i.e. the information ratio) without using leverage induces a demand for higher beta stocks. The increased demand for high volatility and high beta stocks leads to increases in the prices of such stocks, and explains their subsequent poor performance. In fact low beta and low volatility are positively related attributes, and as shown by Scherer (2011), optimal weights of stocks in the minimum variance portfolio are negatively proportional to their betas. A related explanation is linked to the existence of institutional limitations on using leverage. In particular, Frazzini and Pederson (2013) show that investors facing such restrictions bid up high-beta assets and more constrained investors hold riskier assets. A different behavioral explanation is offered by Wang, Yan and Yu (2012), who rely on prospect theory to demonstrate that there is a negative risk/return relation among stocks where the average investor faces prior losses, and vice versa.

Among the skeptics of the low-volatility anomaly, there have been efforts to show that these empirical findings are not robust and hence simply disappear when slightly different methodological choices are made during empirical investigations. For example Fama and French (1992) find that, while the relationship between a stock beta with respect to the market portfolio and the stock average return is not significant, stocks with high exposure to the size and/or B/M factor do earn a higher expected return, in accordance with the predictions of standard asset-pricing models. Thus the "low beta anomaly" is not an anomaly, but instead a mere indication that the cross-section of stock returns is poorly explained by a single-factor model such as the standard CAPM. Similarly, some recent papers have questioned the robustness of the Ang, et al. (2006, 2009) results. Among other concerns, they claim that the low-volatility anomaly does not stand up to changes to data frequency, portfolio formation, portfolio weighting scheme (cap-weighted vs. equally weighted), to the screening out of illiquid stocks (Bali and Cakici, 2008) or to stocks that display extreme past positive returns (Bali, Cakici and Whitelaw, 2011). The use of geometric vs. arithmetic averaging for portfolio return has also been found to have a strong impact on the result (diBartolomeo, 2013).

Other authors find that replacing the short-term measure of volatility in Ang, (2006), with measures obtained over longer periods leads to a positive relationship between risk and return (Fu, 2009, Spiegel and Wang, 2005, Brockman et al. 2009). On a related note, Cao and Xu (2010) break down idiosyncratic volatility into its short- and long-term components and find a positive relationship between the long-term component and expected stock returns. The most popular argument against the low-volatility anomaly stems from the existence of short-term reversals. Indeed, the empirical underperformance of high-volatility stocks tends to be concentrated in the highest decile and a careful analysis reveals that most of the stocks in that decile for one particular sample period have enjoyed a strong rally in the previous (calibration) period. A short-term correction usually occurs after the rally has taken place, hence the one-month performance of the highest volatility stocks tends to be poor subsequent to the determination that these stocks have enjoyed a substantial increase in prices. Indeed, Huang, *et al.*(2010), find that the low-volatility anomaly disappears after adjusting for such short-term return reversals. The risk-return relationship generally becomes increasingly positive as the holding period increases. (See Huang, *et al.*, 2011, for holding periods of 12 months or Goltz and Martellini, 2013 for holding periods up to 60 months.) When liquidity and turnover issues are factored in (see Li, Sullivan and Feijoo, 2013), this argument becomes even more compelling.

The unsettled status of this debate poses a dilemma for investors who may be considering an allocation to a low-volatility strategy. By definition, the long-term consequences of betting on any anomaly are unknown. Recognizing this, the decision to invest in low-volatility stocks should be rooted in risk preferences rather than in the quest for return. As the name suggests, the rational reason to invest in low-volatility stocks should be to gain exposure to the equity market at a relatively low level of risk. Hence the correct question for investors to ask themselves is how a low-volatility strategy fits into their risk budget, so that they can design the investment strategy that best serves their risk objectives. Low-volatility portfolios can be constructed in many different ways and the risks and performance of a particular strategy will depend on the choices that are made about the critical components of the strategy. These decisions include, among many others, the selection of an appropriate stock universe, the weighting scheme applied to the chosen universe, and, with regard to the optimization procedure, the choice of constraints, the risk model and even the optimization algorithm. It is important to bear in mind that these are independent, separable choices, so their expected individual impact can and should be evaluated before making a commitment in order to reach an optimal combination. In the end, low-volatility portfolios tend to be better diversified than their cap-weighted counterparts, and this comparative diversification benefit will generate attractive relative performance, regardless of the existence and persistence of the low-volatility anomaly. On the other hand, investors sensitive to the risk of deviations from a benchmark must understand that investment in a low-volatility strategy is likely to induce sector and factor exposures that differ from their designated benchmark. This may lead to significant variation in the relative performance of their low-volatility portfolio for prolonged periods. Although such risks can be managed in a variety of ways, investors must conduct a comprehensive assessment of expected portfolio behavior in different market environments, as that will enable them to manage the risks in an efficient manner. ~

References

Due to space constraints, the references to this article have been omitted. The full list of references can be obtained on request from research@edhec-risk.com

INDEXES

Measuring and Controlling the Risks of Smart-beta Investing: The Smart Beta 2.0 Approach

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The advent of smart-beta investing came with a plethora of commercial indexes with embedded portfolio construction methods that represent alternatives to cap weighting. In addition, as smart-beta strategies gain importance in the investment process, the question of the impact of smart-beta strategies on the risk of the investor's allocation arises, both in terms of absolute risk and of relative risk, since smart beta is often perceived as a less costly and better-performing substitute for benchmarked active managers, who are typically subjected to risk constraints.

Amenc, Goltz and Martellini (2013) argue that the traditional Smart Beta "1.0" indexes present systematic risks (common factor exposures) and specific risks (risks linked to the underlying model and input variables) that are neither documented nor explicitly controlled by their promoters. Indeed, the focus of Smart Beta 1.0 offers, providing investors with pre-packaged bundles of methodological choices, is on generating performance over cap-weighted indexes without explicit attention to risk transparency and risk choice. Such inadequate information and risk management leads to doubt about the performance presented and implies considerable risk-taking that is not controlled by investors when they choose new equity benchmarks. Thus, the authors advocate a new approach, dubbed "Smart Beta 2.0," in which investors, not index promoters, can make an explicit choice of the risk exposures for their smart-beta benchmarks. Indeed, the results show that this choice, and therefore the associated risk controls, is not inconsistent with smart-beta benchmark performance.

In this article, we will illustrate that the Smart Beta 2.0 approach allows investors not only to measure but also to customize their risk exposures throughout the portfolio construction process. In particular, we discuss the following types of risk: i) exposure to systematic risk factors (which can be tailored through stock selection decisions or factor constraints); ii) exposure to strategy specific risk (which can be diversified away across strategies); and iii) relative performance risk with respect to traditional market cap-weighted benchmarks (which can be managed through tracking error control).

Risks of smart-beta solutions

First, these new forms of indexes are exposed to **systematic risk** factors, depending on the methodological choices guiding their construction and on the universe of stocks to which their construction schemes are applied. For example, given that a cap-weighted index is typically concentrated in the most highly capitalized stocks, any departure from it through deconcentration will necessarily lead to an increase in the exposure to smaller-cap stocks, which also are less liquid stocks. Exhibit 1 shows that compared to cap-weighting, alternative weighting schemes (exemplified by Scientific Beta USA indexes) induce a small-cap tilt.

Second, smart-beta solutions are exposed to **specific risks,** which are related to the characteristics of a given portfolio construction methodology and encompass two sources of risk: parameter estimation risk, and optimality risk. There exists a trade-off between, on the one hand, the desire to limit the number of parameters that need to be estimated for the index construction in order to gain robustness out of sample and, on the other hand, the desire to use all the available information to target a portfolio that is in principle closer to the true optimal maximum Sharpe ratio (MSR) benchmark (less optimality risk) but that will involve more estimation risk (more parameters and harder parameters to estimate).

Finally, the choice of a given weighting scheme leads to a different set of risk and return properties relative to cap-weighted indexes. In Exhibit 2 we show that all strategies have positive returns relative to the cap-weighted benchmark.

On the other hand, all smart-beta strategies need to deviate from the cap-weighted index in terms of factor exposures and portfolio construction methodology, to generate outperformance. The risk choices of smart-beta strategies may be less rewarded than those of the cap-weighted index in certain periods, which makes them susceptible to periods of serious underperformance, i.e., significant and lasting relative drawdowns (Amenc, *et al.*, 2012).

EXHIBIT 1

Risk factor exposures and risk-adjusted performance of US Scientific Beta indexes.
The analysis is based on daily total returns from 6/21/2002 to 12/31/2012. All statistics are annualized; yield on secondary market US Treasury Bill (3M) is used as a proxy for the risk-free rate, and the Scientific Beta USA cap-weighted index is used as the cap-weighted reference. The coefficients statistically significant at the 95% confidence level are in bold.

	Scientific Maximum Deconcentration	Scientific Beta USA Diversified Risk Parity	Scientific Beta USA Maximum Decorrelation	Scientific Beta USA Efficient Minimum Volatility	Scientific Beta USA Maximum Sharpe Ratio	Scientific Beta USA Cap-Weighted
Alpha	*0.40%*	*0.75%*	*0.37%*	*1.90%*	*0.77%*	*0.00%*
Market	1.01	0.96	0.96	0.84	0.93	1.00
Size (SMB)	0.44	0.37	0.40	0.22	0.34	0.00
Value (HML)	-0.01	-0.01	-0.06	-0.06	-0.05	0.00
Adj. R-Square	>99%	>99%	99%	98%	99%	100%
Sharpe Ratio	0.28	0.30	0.28	0.36	0.30	0.21

EXHIBIT 2

Relative performance of US Scientific Beta indexes with regard to cap-weighted reference index.
The analysis is based on daily total returns from 6/21/2002 to 12/31/2012. All statistics are annualized; yield on secondary market US Treasury Bill (3M) is used as a proxy for the risk-free rate, and the Scientific Beta USA cap-weighted index is used as the cap-weighted reference. Extreme Relative Return (5%) measures the maximum amount of the relative loss that the strategy can suffer at a 95% confidence level, based on the distribution of one-year rolling window relative returns.

	Scientific Maxmium Deconcentration	Scientific Beta USA Diversified Risk Parity	Scientific Beta USA Maximum Decorrelation	Scientific Beta USA Efficient Minimum Volatility	Scientific Beta USA Maximum Sharpe Ratio
Relative Return	2.02%	2.05%	1.53%	2.16%	1.72%
Tracking Error	3.62%	3.08%	3.57%	4.60%	3.39%
Extreme Relative Return (5%)	-3.78%	-3.03%	-4.24%	-3.92%	-2.72%
Maximum Relative Drawdown	13.76%	10.39%	12.29%	7.12%	9.15%
Maximum Time Under Water	453	453	729	168	740

[2] A distinction between stock universe selection and the selection of a diversification-based weighting scheme recognizes that, in principle, methodological choices can be made independently in these two steps that are used in the construction of advanced beta equity strategies. Flexibly combining different possible choices in the two steps allows us to test the performance and risk of the possible methodologies and to assess commercially available advanced beta strategy indexes by constructing strategies with similar objectives and constraints.

Managing the exposure to systematic risk factors through stock selection or sector constraints

Today, paying attention to the systematic risks of smart beta is not only a genuine opportunity to create added value, but also a condition for its sustainability. While smart beta can play an important role in institutional investors' allocations, this can only be at the price of implementing an effective risk management process.

The first approach to accounting for the systematic risks of investing in smart beta is the disentangling of the two ingredients that form the basis of any smart-beta index construction: the stock selection and weighting phases. Indeed, a clear distinction between the stock selection phase and the weighting phase allows correction of implicit factor tilts that may arise from the weighting scheme through an explicit choice of the universe in which the strategy invests.[2] Amenc, Goltz and Lodh (2012) show that stock selection is able to correct the risk factor exposures of diversificationweighting schemes by excluding stocks with the undesired characteristics prior to applying a diversification scheme.

For example, the size selection scheme[3] allows the exposure to the size factor to be modified. As shown in Exhibit 3 for the Maximum Deconcentration strategy, the small-cap tilt is reduced from 0.44 in the standard version to +0.19 in the large-cap stock selection and, conversely, to 0.67 when choosing mid-cap stocks only. Interestingly, this shift in size exposure does not affect other factors' initial exposures by a significant amount. The Sharpe ratios of the large-cap and mid-cap indexes are still superior to that of the cap-weighted benchmark (0.21), showing that systematic risk control does not come at the cost of performance.[4]

Naturally, in addition to stock selection, implicit constraints in the optimization can be used to take into account other systematic risks such as relative sector and country exposures with respect to a cap-weighted reference index. As displayed in Exhibit 4, constraining for sector neutrality reduces sector deviations from the cap-weighted reference (e.g. the +11.1% utility sector overweight is reduced to +0.20%). The overall effect across all sectors is equally remarkable: the sum of absolute sector deviations drops from 46% for the standard version of the USA Efficient Minimum Volatility index to 10% for the sector-neutral version.

Exhibit 5 shows a comparison of performance statistics for the Scientific Beta cap-weighted reference index with Scientific Beta USA minimum volatility portfolios with and without sector-neutrality constraints to examine the effect of additional constraints on the strategy's ability to achieve its portfolio-level objective.

The results show that imposing sector-neutrality constraints does not affect the performance of the Scientific Beta USA minimum volatility strategy by a large amount. The unconstrained portfolio achieves 16.0% volatility reduction over the cap-weighted benchmark while the sector-constrained portfolio still achieves an 11.8% reduction.

Managing the exposure to strategy-specific risks through diversification across strategies

The analysis of Martellini, Milhau and Tarelli (2013) can also be used to reduce the specific risks of smart-beta benchmarks. In particular, one may seek to have a strategic exposure to various smart-beta benchmarks so as to diversify away these risks. For example, Exhibit 6, below, shows the average Sharpe ratio of GMV and EW portfolios estimated across 2,757 different sets of reasonable parameter values for the S&P 500 universe.[5] These Sharpe ratios obtained in the presence of realistic levels of estimation errors on the covariance matrix (a problem that only affects the GMV portfolio, not the EW portfolio) can be compared to the value 0.8759, which happens to be the average Sharpe ratio of the long-only maximum Sharpe ratio (MSR) portfolio under the assumption that true covariance and expected returns parameters are known with certainty. The distance between the Sharpe ratio of the GMV and EW portfolios with respect to the Sharpe ratio of the true MSR portfolio underlines the opportunity costs involved in optimality risk and estimation risk for such portfolios. On the other hand, we find that the average Sharpe ratio of the GMV portfolios remains rather stable across errors in covariance parameter estimates.

Moreover, a mixture of GMV and EW portfolios generates an average Sharpe ratio higher than that of the GMV and EW portfolios alone, with very modest dispersion.[6] This result is

EXHIBIT 3

Controlling for the size factor exposure of the Scientific Beta Maximum Deconcentration Indexes

The analysis is based on daily total returns from 6/21/2002 to 12/31/2012. All statistics are annualized; yield on secondary market US Treasury Bill (3M) is used as a proxy for the risk-free rate, and the Scientific Beta USA cap-weighted index is used as the cap-weighted reference. The coefficients statistically significant at the 95% confidence level are in bold. Mid-cap (large-cap) indexes are based on bottom (top) 50% market-cap stocks in the Scientific Beta USA universe.

	Scientific Beta USA Mid-Cap Maximum Deconcentration	Scientific Beta USA Maximum Deconcentration	Scientific Beta USA Large-Cap Maximum Deconcentration	Impact of change in stocks selection on factor exposure
Fama - French Factors	Coefficient	Coefficient	Coefficient	
Annualized Alpha	0.45%	0.40%	0.49%	
Market	1.01	1.01	1.00	Low Impact
Size (SMB)	0.67	0.44	0.19	High Impact
Value (HML)	-0.08	-0.01	0.06	Low Impact
Sharpe Ratio	0.30	0.28	0.26	

EXHIBIT 4

Sector tilts of the efficient minimum volatility standard and sector-neutral indexes

This chart shows sector exposures (in weight %) of the indexes, based on their stock weight profile at the rebalancing date 12/21/2012. We show the relative sector weights with respect to those of the reference cap-weighted index.

EXHIBIT 5

Controlling sector exposures of minimum volatility indexes

The table shows the risk and return statistics of the standard and sector-neutral Scientific Beta USA minimum volatility index and the Scientific Beta USA cap-weighted index. The analysis is based on daily total return data for the period June 21, 2002 to December31, 2012 downloaded from www.scientificbeta.com. All statistics are annualized

	Scientific Beta USA Minimum Volatility Index	Sector-Neutral Scientific Beta USA Efficient Minimum Volatility Index	Scientific Beta USA Cap-Weighted Index
Annual Returns	8.23%	8.02%	6.07%
Annual Volatility	18.32%	19.24%	21.31%
% Reduction in volatility compared to the reference cap-weighted index	16.0%	11.8%	
Sharpe Ratio	0.36	0.33	0.21

[3]*Scientific Beta's large-cap selection picks the top 50% of stocks by free-float-adjusted market capitalization; conversely the mid-cap selection picks the bottom 50% of stocks by free-float-adjusted market capitalization from the full Scientific Beta USA universe.*

[4]*Amenc, Goltz and Lodh (2012) show that other risk factor tilts like dividend yield or low and high volatility can be reduced or cancelled by using appropriate stock selection while respecting the risk/[solidus]return objectives of the diversification schemes.*

consistent with the findings in Tu and Zhou (2011) that a combination of these two portfolio strategies allows the specific risks of each strategy to be diversified away by exploiting the imperfect correlation between the different strategies' parameter estimation errors and the differences in their underlying optimality assumptions. Moreover, as the single strategies' performance will show different profiles of dependence on market conditions, a multistrategy approach leads to a smoother conditional performance and higher probability of outperforming the cap-weighted index (Amenc, *et al.*, 2012, Badaoui and Lodh, 2013).

Managing relative performance risk with respect to traditional market-cap—weighted benchmarks through tracking-error control

If the goal of investment in smart beta is to outperform cap-weighted market indexes, we should note that this goal is exactly the same as that of a benchmarked active manager. Therefore, the control of tracking error to hedge relative risk, including risk of extreme underperformance, becomes important.

One solution to that issue is proposed by Jorion (2003) and applied by Amenc, Goltz, Lodh and Martellini (2012): align the factor exposures of the strategy with those of its benchmark within the optimization to ensure that the *ex-post* tracking error does not differ significantly from *ex-ante* tracking-error objectives. One can also use a core-satellite approach and mix that (satellite) index with the (core) cap-weighted reference index to adapt the level of tracking error of the final investment to the investor's tracking error budget. This approach allows reliable control of both the average tracking error and the extreme tracking error of smart-beta strategies and maintains the significant potential for outperformance of the smart-beta investment overall.

Exhibit 7 shows the effect of imposing tracking-error constraints on Scientific Beta USA strategy indexes. The 3% tracking-error target is not exceeded substantially by any of the strategies even for the extreme observations of tracking-error, as can be seen from the extreme (95%) tracking-error observations. Despite the risk control, the relative-risk—controlled versions are still able to outperform their cap-weighted benchmarks, albeit by a smaller margin. Investors thus face a clear trade-off between taking on relative risk and generating outperformance. However, in the relative-risk—controlled indexes both overall tracking-error and extreme tracking-error figures are brought down substantially without eroding all of the potential for outperformance. ~

CONCLUSION

Investment in smart beta presupposes measurement of the systematic risk factors and integration of the factors, not only in absolute terms, to evaluate the real risk-adjusted performance created by better diversification of the benchmark, but also in relative terms, to limit the trackingerror risk and therefore the risk of underperformance in comparison with the cap-weighted index. These statistical analyses should also be completed by thorough due diligence on the specific risk represented by the diversification model and the implementation rules and methods. The Smart Beta 2.0 approach allows investors to invest in these advanced forms of benchmarks with full knowledge and control of the risks of their choice.

EXHIBIT 6

Ex-ante Sharpe ratios for selected long-only weighting schemes in the presence of estimation errors in expected excess returns and covariances.

Results taken from Martellini, Milhau and Tarelli (2013). These results have been obtained by simulating ("true") population parameters and estimation errors. The 2nd and 3rd columns contain results with simulated estimation errors for risk parameters. The statistics are averaged across different sets of "true" parameters.

Portfolio Strategy	*Average Sharpe ratio with estimation risk*	*Standard deviation of Sharpe ratio with estimation risk*
GMV (Global Minimum Variance)	0.5228	0.0381
EW (Equal-Weighted)	0.5455	0.0000
50% GMV + 50% EW	0.5638	0.0147

EXHIBIT 7

Relative risk of Scientific Beta USA strategy indexes.

The table displays the relative risk and return statistics of four Scientific Beta USA strategy indexes without any relative risk control (panel 1), and of the same strategies with 3% tracking-error control (panel 2). The benchmark used is the Scientific Beta USA Cap-Weighted index and the analysis is based on daily total return data for the period June 21, 2002 to December 31, 2012 downloaded from www.scientificbeta.com. 95% TE is computed using a rolling window of one-year length and one-week step size over the analysis period. All statistics are annualized.

Panel 1	Smart Beta *without* Relative Risk Control: Scientific Beta USA Indexes			
	Efficient Maximum Sharpe	Efficient Minimum Volatility	Maximum Decorrelation	Maximum Deconcentration
Excess Returns over CW	1.72%	2.16%	1.53%	2.02%
Tracking Error	3.39%	4.60%	3.57%	3.62%
Information Ratio	0.51	0.47	0.43	0.56
95% Tracking Error	5.28%	8.01%	5.58%	6.36%

Panel 2	Smart Beta *with* Relative Risk Control: Scientific Beta USA Indexes (3% Tracking Error)			
	Efficient Maximum Sharpe	Efficient Minimum Volatility	Maximum Decorrelation	Maximum Deconcentration
Excess Returns over CW	0.68%	0.71%	0.99%	0.90%
Tracking Error	1.83%	2.10%	2.03%	1.86%
Information Ratio	0.37	0.34	0.49	0.48
95% Tracking Error	3.01%	4.30%	3.55%	2.83%

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[5]To generate realistic parameter values, they use the following approach. They perform the analysis on weekly data over a 2-year rolling window, with a rolling step equal to one week. For each position of the time-window considered, they take into account all stocks belonging to the universe, without introducing a survivorship bias. They then use a robust estimator of the covariance matrix, based on the Fama-French three-factor model, which is also used for the estimation of expected excess returns.

[6]These results, which can be extended to other possible combinations of smart-beta indexes, are consistent with theoretical results by Kan and Zhou (2007), who show that a portfolio that combines the sample-based MSR and GMV portfolios dominates the sample-based MSR alone in the presence of parameter uncertainty. In the same spirit, the Scientific Beta Diversified Multi[set solid] strategy weighting scheme combines five diversification weighting schemes in equal proportions.

INDEXES

From Single to Multistrategy Indexes

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In contrast to cap-weighted indexes, alternative equity-diversification strategies aim at delivering a fair risk-adjusted return over the long run by combining stocks in a portfolio in accordance with modern portfolio theory (Markowitz, 1952). In fact, such strategies are designed to benefit from the fact that stock returns are imperfectly correlated, and represent proxies for the tangency portfolio. They are also deemed to address one notable criticism of the cap-weighted paradigm: concentration in fewer stocks or sources of risks than is possible in their underlying investment universe.

There are two distinct approaches to diversification: either searching for an implementable proxy for the tangency portfolio (i.e. the portfolio that truly offers the maximum Sharpe ratio), or instead adopting heuristic approaches to diversification by using an *ad-hoc* diversification objective that matches the investor's views and preferences rather than explicitly aiming to obtain a portfolio with an optimal risk/reward ratio.

As discussed in Amenc, *et al.* (2013), each smartbeta strategy features two very different kinds of risks: systematic risks and specific risks. Systematic risks come from the fact that strategy indexes can be exposed to some common systematic risk factors (e.g. value), while specific risks (encompassing estimation and optimality risks) are related to the portfolio construction methodology and the set of assumptions that are made upstream.

In this article, we first explain the conceptual groundings of the five alternative weighting schemes identified by ERI Scientific Beta as being different credible vehicles for diversification. We emphasize their intrinsic specific and systematic risks, as well as the impacts of these risks on their individual conditional and unconditional performance and risk profiles. We finally introduce the Scientific Beta Multistrategy, which aims to diversify away these risks and stands as a very first solution to gaining robust exposure to the smart-beta paradigm.

Diversification strategies

Equal weighting (also known as the 1/N weighting scheme) is a simple way of deconcentrating a portfolio in terms of stock weights or maximizing the effective number of stocks.[7] This strategy has been shown to deliver attractive performance even in comparison to sophisticated portfolio optimization strategies (De Miguel *et al.*, 2009b). Depending on the size of the stock universe, equal-weighting can lead to relatively high turnover and liquidity problems.[8] **Maximum deconcentration** addresses this drawback and minimizes the distance of weights from the equal weights subject to constraints on turnover and liquidity.

Extending the notion of weight deconcentration to risk deconcentration, the general risk parity approach aims to equalize the risk contributions of constituent stocks to the total portfolio risk:

$$w_i \frac{\partial \sigma_p}{\partial w_i} = w_j \frac{\partial \sigma_p}{\partial w_j}$$

where w_i is the (positive) portfolio weight of stock *i* and σ_p the portfolio volatility (see Maillard, Roncalli and Teïletche, 2010, for a detailed discussion). In the general case no analytical solution is available to this problem; it therefore needs to be solved numerically. **Diversified risk parity,** which is based on a specific case of the general risk parity problem, is a weighting scheme that attempts to equalize the risk contributions of individual stocks to the total risk of the index, assuming uniform correlations across stocks. This assumption has the advantage that the optimal weights can be derived analytically, without relying on any numerical resolution. Indeed, in the absence of any constraints, such as tracking error or sector neutrality constraints, diversified risk parity boils down to inverse volatility weighting. Furthermore, the use of identical correlations allows a high level of robustness to be achieved (see Elton and Gruber, 1973).

In fact, maximum deconcentration and diversified risk parity overlook the fact that exploiting the imperfect interactions between the underlying assets is at the heart of diversification. A large body of literature has assessed diversification benefits, notably in the area of international equity portfolio management, by focusing on a measure of how well the portfolio exploits correlation effects.[9] Disentangling the effect of correlations from the effect of individual asset volatilities has been widely discussed (see, e.g., Amenc, Goltz and Stoyanov, 2011) and the **maximum decorrelation** weighting scheme tries to exploit the effect of risk reduction through decorrelation. It is inspired by Christoffersen, *et al.* (2010) who focus on exploiting solely the correlation structure when measuring the benefits of diversification. They assess the diversification potential within a given global equity universe by minimizing the total portfolio variance under the assumption that all individual assets' volatilities are identical, thus relying only on the information about their correlations.

In contrast with the three *ad-hoc* diversification strategies discussed above, the true minimum-volatility portfolio lies on the efficient frontier.

First, the minimum-volatility portfolio corresponds to the portfolio on the efficient frontier that exhibits the lowest level of volatility among all feasible portfolios. The minimum-volatility strategy can be seen as an attempt to exploit information on risk parameters, including stock volatility and correlations across stocks. The minimum-volatility strategy has been shown to be concentrated in low-risk (low volatility or low beta) stocks, which in turn leads to pronounced sector biases toward defensive sectors such as utilities (see Chan, *et al.*, 1999). Introducing weight constraints,[10] Jagannathan and Ma (2003), and in turn De Miguel, *et al.* (2009a]) who implement flexible constraints on overall portfolio concentration (so-called norm constraints), not only reduce the concentration of the portfolio, but also achieve a better out-of-sample risk and return profile.[11] Drawing on *De Miguel*, (2009), the Scientific Beta efficient **minimum-volatility** weighting scheme provides an investible proxy for the minimum-volatility portfolio.

The **Efficient Maximum Sharpe Ratio** strategy is an implementable proxy for the tangency portfolio from modern portfolio theory. As in any mean-variance optimization, the estimation of input parameters is a central ingredient in the implementation of the methodology. In contrast to minimum volatility strategies, the maximum Sharpe ratio strategy relies on estimates of both risk parameters (volatilities and correlations) and expected returns. As direct estimation of expected returns is known to lead to large estimation errors (Merton, 1980), ERI Scientific Beta's efficient maximum Sharpe ratio strategy estimates expected returns indirectly by assuming that they are positively related to a stock's semi-deviation (see Amenc, *et al.*, 2011).[12] More specifically, an extra step is added to the estimation process to provide more robustness: stocks are sorted by their semi-deviation into deciles and all stocks in a decile are then assigned the median value of the decile.

The efficient maximum Sharpe ratio strategy can be an alternative to the minimum-volatility approach, especially for investors who do not wish to hold, for long periods, a portfolio concentrated in low-volatility stocks, since the efficient

EXHIBIT 1

Overview of popular equity diversification strategies

The table indicates, for the diversification strategies, the optimization objective (without taking into account any constraints, turnover control or liquidity rules), its unconstrained solution and the required parameters. The "Optimality conditions" column indicates the conditions under which each diversification strategy would result in the maximum Sharpe ratio portfolio from modern portfolio theory. *N* is the number of stocks, μ_i is the expected return on stock *i*, σi is the volatility for stock i, ρ_{ij} is the correlation between stocks *i* and j, μ is the (Nx1) vector of expected return, $\mathbb{1}$ is the *(Nx1)* vector of ones, σ is the *(Nx1)* vector of volatilities, Ω is the *(NxN)* correlation matrix and $\sum$ is the *(NxN)* covariance matrix.

Strategy	Objective	Unconstrained closed-form solution	Required parameter(s)	Optimality conditions
Maximum Deconcentration	Maximize effective number of stocks	$w^* = \frac{1}{N}\mathbb{1}$	None	$\mu_i = \mu\ \forall i$ $\sigma i = \sigma\ \forall i$ $\rho_{ij} = \rho\ \forall i$
Diversified Risk Parity	Equalize risk contributions under under "Constant Correlation" assumption	$w^* = \frac{diag(\sigma^{-1})}{\mathbb{1}'\, diag(\sigma^{-1})}$	σ_i	$\lambda i = \lambda\ \forall i$ $\rho_{ij} = \rho\ \forall i$
Maximum Decorrelation	Minimize the portfolio volatility under the assumption of identical volatility across all stocks	$w^* = \frac{\Omega^{-1}\mathbb{1}}{\mathbb{1}'\Omega^{-1}\mathbb{1}}$	ρ_{ij}	$\mu_i = \mu\ \forall i$ $\sigma i = \sigma\ \forall i$
Efficient Minimum Volatility	Minimize portfolio volatility	$w^* = \frac{\sum^{-1}\mathbb{1}}{\mathbb{1}'\sum^{-1}\mathbb{1}}$	σ_i, ρ_{ij}	$\mu_i = \mu\ \forall i$
Efficient Maximum Sharpe Ratio	Maximize portfolio Sharpe Ratio	$w^* = \frac{\sum^{-1}\mu}{\mathbb{1}'\sum^{-1}\mu}$	$\mu_i, \sigma_i, \rho_{ij}$	Optimal by construction

[7] *The effective number of stocks is defined as the reciprocal of the Herfindahl index, which is a commonly used measure of portfolio concentration: Effective Number of Stocks =* $\frac{1}{\sum_{i=1}^{n} w_i^2}$ *where* N *is the number of constituent stocks in the index and* w_i *is the weight of stock* i *in the index. In brief, the effective number of stocks in a portfolio indicates how many stocks would be needed in an equal-weighted portfolio to obtain the same level of concentration (as measured by the Herfindahl index). Equal-weighting stocks in a portfolio will lead to the maximum effective number of stocks.*

maximum Sharpe ratio strategy generally features more homogeneous weight distribution across volatility quintiles.

Table 1 summarizes the description of the five diversification strategies. Interestingly, since the diversification strategies differ from each other in the assumptions they make and the objectives they aim to achieve, the combination of these different strategies allows the risks that are specific to each strategy to be diversified away by exploiting the imperfect correlation between the different strategies' parameter-estimation errors and the differences in their underlying optimality assumptions. Moreover, as the single strategies' performance will show different profiles of dependence on market conditions, a multistrategy approach can help investors smooth the overall performance across market conditions.[13] For instance[14], Amenc, *et al.* (2012), form a combination of two diversification approaches[15] that leads to smoother conditional performance and a higher probability of outperforming the cap-weighted index. In the same spirit, the Scientific Beta diversified multistrategy weighting scheme combines in equal proportions the efficient maximum Sharpe ratio, the efficient minimum volatility, the maximum decorrelation, the diversified risk parity and the maximum deconcentration weighting schemes. We will illustrate the properties of such a combination of strategies in the next section.

Performances and risks of smart beta: a call to diversify strategy risks

In this section, we analyze the performance and risks of individual Scientific Beta USA diversification strategy indexes and those of a diversified multistrategy combination.[16] Table 2 shows that all the diversification strategies tend to deliver higher returns than the cap-weighted reference index with annualized outperformance ranging from +1.53% (maximum decorrelation) to +2.16% (efficient minimum volatility). Moreover all of the diversification strategy indexes exhibit lower volatility than the cap-weighted reference index, leading to better risk-adjusted performance, with Sharpe ratios ranging from 0.28 to 0.36 (against 0.21 for the cap-weighted reference index).

The diversified multistrategy index, being an equal-weighted average of the above five indexes, provides close to average values of statistics such as returns, volatility and Sharpe ratio. More interestingly, we show that these indexes are exposed differently to a set of systematic risk factors (i.e. market, SMB and HML factors). In general, all strategies have small-cap exposure brought about by deconcentration caused by departure from cap weighting. Additionally, Amenc, *et al.* (2012), and Badaoui and Lodh (2013) show considerable variation in the performance of some popular smart-beta strategies in different subperiods, revealing the pitfalls of aggregate performance analysis based on long periods. Table 3 shows that certain market conditions favor some smart-beta strategies while proving detrimental to others. The reason for such behavior is that the set of risk factors to which each smart-beta strategy is exposed has been shown to carry time-varying risk premia (Asness, *et al.* 1992, Cohen, Polk, and Vuolteenaho, 2003). Depending on the risk premia earned by these factors under particular economic conditions, the performance of the strategies can vary significantly.

EXHIBIT 2

Absolute and Relative Performance of Diversification Strategy Indexes.
The statistics are based on daily total returns (with dividend reinvested) over the analysis period from inception date (6/21/2002) to 12/31/2012. All statistics are annualized and performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding period returns for investors. The total number of stocks in the Scientific Beta USA universe is 500. We also display the weighted average float-adjusted market capitalization (in millions USD) as an indication of liquidity (or investment capacity).

Scientific Beta USA Indexes	Maximum Deconcentration	Diversified Risk Parity	Maximum Decorrelation	Efficient Minmum	Efficient Maximum	Diversified Multistrategy	Cap-Weighted
Annual Returns	8.09%	8.11%	7.60%	8.23%	7.79%	7.98%	6.07%
Annual Volatility	22.71%	21.47%	21.39%	18.32%	20.49%	20.82%	21.31%
Sharpe Ratio	0.28	0.30	0.28	0.36	0.30	0.30	0.21
Excess Returns	2.02%	2.05%	1.53%	2.16%	1.72%	1.92%	-
Tracking Error	3.62%	3.08%	3.57%	4.60%	3.39%	3.10%	-
Information Ratio	0.56	0.66	0.43	0.47	0.51	0.62	-
Ann 1-Way Turnover	26.6%	25.7%	31.2%	31.4%	31.0%	23.80%	2.8%
Wgt Avg Market Cap	20691	22241	21243	25504	22832	22502	87507
Alpha	0.40%	0.75%	0.37%	1.90%	0.77%	0.84%	0.00%
Market Beta	1.01	0.96	0.96	0.84	0.93	0.94	1.00
Small Cap (SMB) Beta	0.44	0.37	0.40	0.22	0.34	0.35	0.00
Value (HML) Beta	-0.01	-0.01	-0.06	-0.06	-0.05	-0.04	0.00

Separating bull and bear market periods to evaluate performance has been proposed by various authors such as Levy (1974), Turner, Starz and Nelson (1989) and, more recently, Faber (2007). As discussed before, combining strategies allows outperformance to be smoothed out across different market conditions (Amenc, 2012). Indeed, Table 4 shows how the diversified multistrategy approach averages out the excess returns over different market regimes. Unlike some of the diversification strategies, its performance in bull/bear markets and high/low volatility markets is not extreme and its tracking error is quite low compared to its constituent strategies.

As displayed in Table 2, one-way annual turnover of all diversification strategies is close to 30%, showing the effectiveness of turnover rules. The strategies are adequately liquid as their weighted average market capitalization[17] is about one fourth of that of the cap-weighted index, which is highly liquid by construction. An investor wishing to use a better diversified benchmark than a cap-weighted index but disinclined to take on liquidity risk can decide to apply this scheme solely to a very liquid selection of stocks. We construct five smart-beta strategies on the top 50% of stocks in the USA universe in terms of liquidity[18] combine them in equal proportion to form the high-liquidity diversified multistrategy index and compare its performance with the diversified multistrategy index on the full U.S. universe.

The results in Table 4 show that high-liquidity stock selection does not have a big effect on the performance of the strategies. The weighted average market capitalization, an indicator of the investment capacity of the smart- beta strategies, shows a significant increase from around 22.5bn USD to the levels of 37.5bn USD in comparison with 87.5bn USD for the reference cap-weighted index. These results suggest that by selecting the most liquid stocks, one can limit the liquidity problems of smart-beta diversification strategies while maintaining most of the potential for improved risk/reward properties.

[8] *In particular, in very broad universes that contain stocks with little liquidity, the rebalancing back to equal weights may be difficult to implement (see Blitz, 2013). Plyakha, Uppal and Vilkov (2012), Demey, Maillard and Roncalli (2010) and Leote de Carvalho, Lu and Moulin (2012) show equal-weighted strategies have moderately higher levels of turnover compared to market-capitalization weighted portfolios. Dash and Loggie (2008) point out that transaction costs can become important for equal-weighting when the universe includes less liquid stocks. However, the intensity of the liquidity [this is the rest of fn 10 on previous page] problems depends on the universe chosen. The liquidity risk will be lower if one were to apply the equal-weighting scheme to a universe consisting of the largest stocks rather than to a universe including both large-and small-cap stocks.*

[9] *See, for instance, Longin and Solnik (1995) and Goetzmann, Li and Rouwenhorst (2001).*

[10] *In addition to norm constraints, one could use a sector neutrality constraint, which is a more direct tool to control the sector exposure of indexes. See the article "Measuring and Controlling the Risks of Smart Beta Investing: The Smart Beta 2.0 Approach" in this P&I supplement for a detailed example.*

[11] *The authors show that using such flexible concentration constraints instead of rigid upper and lower bounds on individual stock weights (as in Jagannathan and Ma, 2003) allows for a better use of the correlation structure. The quadratic norm constraints used for the strategy can be written in terms of portfolio weights as* $\|w\|_2 = \sum_{i=1}^{N} w_i^2 \le \frac{3}{N}$

[12] *A number of studies show a positive relation between expected return and different measures of downside risk. Bali and Cakici (2004) and Huang, et al. (2012) find that a stock's expected return has a strong positive relation with its VaR and its extreme downside risk, respectively. Chen, et al. (2009) and Estrada (2007) show a positive relation between a stock's expected return and its semi-deviation. Ang, et al. (2006a) document a positive relation between a stock's downside beta (stocks that are strongly correlated with the market when it goes down) and its expected return.*

[13] *This topic is discussed at more length in Badaoui and Lodh (2013).*

[14] *Tu and Zhou (2010), Kan and Zhou (2007) and Martellini, Milhau and Tarelli (2013) among others also study whether a portfolio of strategies can improve the performance of individual strategies.*

[15] *Robust proxies for the minimum volatility portfolio provide defensive exposure to equity markets that does well in adverse market conditions, while robust proxies for maximum Sharpe ratio portfolios provide greater access to the upside of equity markets..*

[16] *Gonzalez and Thabault (2013) present a more detailed performance and risk analysis of these diversification strategies. Amenc, et al. (2013), argue that investors should not only measure but also be allowed to control their risks at each step of the portfolio construction process: factor risks at the stock selection stage, and sector/country relative risks as well as tracking- error risk against the cap-weighted reference index at the optimization stage. Goltz and Gonzalez (2013) show how these risk control choices can be used by investors to tailor smart-beta strategies to their needs (see also "Measuring and Controlling the Risks of Smart Beta Investing: The Smart Beta 2.0 Approach" in the present supplement).*

[17] *Weighted average market cap of index i=* $\sum_{k=1}^{N} W_{k,i}$ *Market Cap*$_k$ *where* $W_{k,i}$ *is the weight of stock k in index i, N is the total number of stocks in the index, and Market Cap*$_k$ *is the float-adjusted market cap of stock k.*

[18] *The stocks are partitioned into high-liquidity and mid-liquidity groups by their liquidity scores, which depend on average traded daily dollar volume and trading ratio (number of days that the stock is exchanged over the total number of business days) of a stock.*

Alternatively, stock selections can also be used to gain exposure to a desired risk factor. Indeed, the value factor is known to be a well rewarded factor over the long term (Fama and French, 1993). For instance, applying the diversified multistrategy approach to the top 50% of stocks in the Scientific Beta USA universe sorted by their book to market (value selection) provides access to a substantial outperformance of 2.71% over the reference cap-weighted index (which is also an improvement over the multistrategy index on the full USA universe) and of about 1.88% over a cap-weighted index constituted on the Scientific Beta Value selection. ~

CONCLUSION

In brief, the diversification strategy indexes address the limitations of cap-weighted indexes such as their high concentration levels (in weight or risk contributions) or inefficient return-to-risk profiles. Although each strategy has its own benefits, it also has certain limitations that stem from its specific risks. The investor can diversify such risks away by allocating across strategies in the form of a diversified multistrategy index, sparing the steps of identifying a model with superior assumptions and forging assumptions on upcoming market conditions. The Scientific Beta diversified multi[solid]strategy index presents itself as an interesting candidate. Indeed, the Scientific Beta diversified multistrategy index presents a good trade-off between return and relative risk as it is a strategy that can achieve substantial outperformance compared to its index constituents while maintaining a low tracking error with respect to the reference cap-weighted index. In fact, the strategy has a return that corresponds to the average return of its five components and a tracking-error level that is lower than the average tracking error of its constituents.

Finally, the Scientific Beta diversified multi strategy index represents only one particular and simple way of diversifying the specific risks of the alternative weighting schemes. Investors could go beyond the simple equal combination of all weighting schemes by i) selecting a set of smart- beta strategies that provide the best diversification potential and ii) developing optimal combinations of these alternative weighting schemes that could potentially outperform the equal-proportion approach.

EXHIBIT 3

Conditional Performance.

The table shows excess returns and tracking error of the Scientific Beta USA indexes in bull/bear and high/low volatility markets. The statistics are based on daily total returns (with dividend reinvested) over the analysis period from inception date (6/21/2002) to 12/31/2012. All statistics are on a quarterly basis and performance ratios that involve the average returns are based on the geometric average. The total number of stocks in the Scientific Beta USA universe is 500. Calendar quarters with positive market index returns comprise bull markets and the rest constitute bear markets. The high- volatility market comprises the top 50% of quarters sorted on the quarterly cap-weighted benchmark's volatility and the low- volatility market comprises the rest.

Panel A: Excess Returns over Scientific Beta USA Cap-Weighted Index

	Maximum Deconcentration	Diversified Risk Parity	Maximum Decorrelation	Efficient Minimum Volatility	Efficient Maximum Sharpe Ratio	Diversified Multistrategy
Bull Market	1.05%	0.59%	0.56%	-0.44%	0.29%	0.41%
Bear Market	-0.28%	0.38%	0.14%	1.91%	0.63%	0.56%
High- Volatility Market	0.13%	0.28%	0.15%	0.72%	0.31%	0.33%
Low- Volatility Market	0.89%	0.73%	0.63%	0.33%	0.56%	0.63%

Panel B: Tracking Error with Respect to Scientific Beta USA Cap-Weighted Index

	Maximum Deconcentration	Diversified Risk Parity	Maximum Decorrelation	Efficient Efficient Minimum Volatility	Maximum Sharpe Ratio	Diversified Multistrategy
Bull Market	1.05%	0.59%	0.56%	-0.44%	0.29%	0.41%
Bear Market	-0.28%	0.38%	0.14%	1.91%	0.63%	0.56%
High- Volatility Market	0.13%	0.28%	0.15%	0.72%	0.31%	0.33%
Low- Volatility Market	0.89%	0.73%	0.63%	0.33%	0.56%	0.63%

EXHIBIT 4

Absolute and Relative Performance of Diversified MultiStrategy Indexes

The statistics are based on daily total returns (with dividend reinvested) over the analysis period from inception date (6/21/2002) to 12/31/2012. All statistics are annualized and performance ratios that involve the average returns are based on the geometric average, which reliably reflects multiple holding- period returns for investors. We also display the weighted average float-adjusted market capitalization (in millions USD) as an indication of liquidity (or investment capacity).

Scientific Beta USA Indexes	Diversified Multistrategy	Diversified Multistrategy	High- Liquidity Diversified Multistrategy	Cap-Weighted
Annual Returns	7.98%	8.78%	7.78%	6.07%
Annual Volatility	20.82%	21.43%	21.92%	21.31%
Sharpe Ratio	0.30	0.33	0.28	0.21
Excess Returns	1.92%	2.71%	1.72%	-
Tracking Error	3.10%	4.21%	2.93%	-
Information Ratio	0.62	0.64	0.59	-
Annual 1-Way Turnover	23.80%	25.7%	23.3%	2.8%
Weighted Average Market Cap	22505	18797	37581	87507

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INDEXES

A Critical Analysis of Alternative Bond Indexes

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Existing bond benchmarks as ill-diversified bundles of unstable factor exposures

Over recent years, a number of concerns have been expressed about the relevance of the corporate and sovereign bond indexes currently offered by existing index providers.

One of the major problems with bond indexes, which simply weight the debt issues by their market value, is the so-called bums' problem (Siegel, 2003). Given the large share of the total debt market accounted for by issuers with large amounts of outstanding debt, market-value–weighted corporate bond indexes will have a tendency to overweight bonds with large amounts of outstanding debt. It is often argued that such indexes will thus give too much weight to riskier assets. This problem occurs not just for individual issues but also for market sectors or industries in corporate indexes. While it is debatable whether debt-weighting really leads to an overweight in the most risky securities[19], it is clear that market-value-debt–weighted indexes lead to concentrated portfolios that are in opposition to investors' standard objective of diversification. In other words, a good case can be made that existing bond indexes tend to be heavily concentrated, poorly diversified, portfolios, regardless of whether or not it is the wrong constituents that are overweighted.[20]

In addition to the problem of concentration, fluctuations in risks' exposure, such as duration or credit risk in existing indexes, are another source of concern. Such uncontrolled time variation in risk exposures is incompatible with investors' requirements that these risk exposures be relatively stable so that allocation decisions are not compromised by implicit choices made by an unstable index. For example, an asset-liability mismatch would be generated by changes in the duration of the bond index if the bond index is used as a benchmark for a pension fund bond portfolio.

One last issue with existing bond benchmarks is liquidity risk. Bond indexes generally consist of a very large number of bonds, which makes them difficult to replicate. Furthermore, bonds with special features or smaller amounts outstanding usually suffer from illiquid trading, resulting in relatively large bid-offer spreads (Bias, *et al.*, 2006), especially during stress times. For instance, Nielsen *et al.*, 2012) show that the spread contribution from illiquidity on corporate bonds increased dramatically with the onset of the subprime crisis. Old issues are also illiquid as investors focus on newly issued bonds. Some corporate bond indexes intend to address these deficiencies by limiting the number of bonds per index and excluding special bond types and old bonds, thus increasing liquidity.

More generally, it appears that existing bond indexes can be regarded as more issuer friendly than investor friendly, in the sense that these bond indexes passively reflect the collective decisions of issuers regarding the maturity and size of bond issues, with no control over risk factor exposures associated with such choices, nor over the reward that investors should deserve from holding a well-diversified portfolio of such factor exposures.

Alternative bond benchmarks as partial and *ad-hoc* answers to otherwise well-identified questions

Recently, a number of index providers have launched new forms of alternative indexes to try to address some of the challenges with traditional weighting schemes based on the market value of debt. A number of *ad-hoc* alternative weighting schemes have been proposed, but these initiatives are based on no academic grounds, and it is unclear whether the portfolios thereby constructed would be optimal benchmarks under any reasonable assumptions.

In what follows, we provide a general overview of these initiatives, which can be broadly classified into three different categories: fundamental approaches, diversification approaches and liability-driven approaches (see Reilly, Kao and Wright, 1992 or Campani and Goltz, 2011 for more details). Our main conclusion is that none of these approaches successfully address all the key concerns and challenges involved in designing a truly investor-friendly bond benchmark, which suggests that further work is needed in this area.

Fundamental approaches to bond indexes address neither concentration risks nor factor exposure risks.

Fundamental indexing in the bond market is a direct transfer of methodologies originally developed for equities. Promoters of fundamentally weighted corporate bond indexes include Research Affiliates in partnership with Citi, as well as Barclays with the Issuer Corporate Bond index launched in September 2010. Research Affiliates has also developed a fundamentally weighted sovereign bond index in partnership with Ryan Lab, and fundamentally weighted sovereign bond indexes have been launched by Lombard Odier Investment Managers (the LOIM sovereign bond index, launched in December 2010) and BlackRock (the BlackRock Sovereign Risk index, launched in June 2012).

The methodology used by Research Affiliates is explained in Arnott, *et al.*, (2010). For corporate bond indexes, the authors use the following five factors in order to give a score to each corporate bond (investment grade and high yield): book value of assets, total dividends, total cash flow, sales, and face value of the debt issue. First, weights are computed for each corporation, and with respect to each factor, by using the trailing five-year average of each of the above metrics over the aggregate five-year average across all corporations. While it might seem unclear why it would be desirable to use a five-year trailing value as opposed to the current value for the fundamentals, the main practical motivation for this *ad-hoc* procedure is that the implied smoothing will lead to a reduction in turnover. Then, the composite measure is obtained by equally weighting four of the measures: assets, dividends, cash flow, and sales. For emerging market sovereign bonds, the approach developed in Arnott (2010) is based on the following five factors: total population, square root of land area (as a crude approximation for resources), total gross domestic product, energy consumption, and face value of the debt issue. Similarly to the case of corporate bonds, weights are first computed for each country and with respect to each factor by using smoothed five-year averages of the above metrics over the aggregate metric across all countries. Then, a country's aggregate weight is the equally weighted average of its score on the five individual factors.

The fundamental approach for constructing a bond index raises several concerns. First of all, the methodology used does not address the concern over stability of factor exposure. Besides, the problem of concentration is approached with a purely *ad-hoc* methodology, and better diversified portfolios could be constructed on the basis of the use of standard risk models. More important, it is unclear why some backward-looking trailing average of some arbitrarily selected variables (e.g., square-root of land area!) should contain more useful information than, say, bond ratings, which for all their flaws are based on a much richer information set.

In the same spirit, a number of institutions (investment banks, index providers and asset managers), including Barclays Capital, Euro MTS and PIMCO, have launched sovereign bond indexes using GDP measures exclusively to adjust the weights allocated to various regions, with methodological details varying across different providers. All these indexes based on the GDP metric (and sometimes adjusted with other macroeconomic factors) give a relatively lower weight to countries that are heavily indebted. For example, Japan is the largest debt issuer in the world, representing 28% in the constitution of the Barclays Capital Global Treasury and Emerging index in 2011, but has a low weight (9%) in the GDP-based version of the same Barclays Capital indexes. Therefore, using GDP-based indexes contributes to dealing with only one drawback of cap-weighted indexes, which is the concentration. On the other hand, relying exclusively on GDP statistics may introduce a significant backward-looking bias, since such data are updated on a quarterly basis. Moreover, the sensitivity to interest rate is not controlled, and the diversification of the issuers is not properly taken into consideration through suitable risk models, which may lead such portfolios to be heavily loaded on the same risk.

More recently, an *ad-hoc* adjustment to cap-weighted schemes was observed in sovereign bond indexes based on some fundamental measures of fiscal sustainability. Indeed, following the European debt crisis and its possible contagion to countries outside the eurozone such as the U.S., which represents the biggest issuer of sovereign debt, Barclays Capital introduced fiscal strength-weighted indexes in August 2011. Inside such indexes, countries are weighted by market capitalization. Then, factors representing financial solvency, external financing and institutional strength are converted into factor scores ranging from 0 to 10. A weighted average of the factor scores is then calculated to give a single score for each country in the index. Finally the market value weight of each country is multiplied by the country's fiscal score. The adjusted market value weights are then normalized to arrive at the fiscal strength country index weights. Adjusting the cap-weighted scheme based on some fundamental measures of fiscal sustainability is meant to be a remedy to the bums problem, since the biggest debt issuers tend to have growing fiscal difficulties. On the other hand, the procedure does not address the factor exposure risks.

Diversification approaches to bond indexes address concentration risk, albeit in an *ad-hoc* manner, but do not address factor exposure risks.

The first *ad-hoc* approach to cope with the problem of concentration risk consists of imposing maximum limits to weight assigned to any particular constituent or issuer. Barcap, for instance, is trying to limit such concentration by capping issuers' weights to a fixed percentage of the index and then redistributing the excess weight across the other issuers (for instance the Barclays Capital U.S. Corporate Aaa Acapped index is market-cap weighted with a 3% cap).

Extending the concept, equally weighted indexes, offered, for example, by Dow Jones, assign the same weight to each bond. The index contains only 96 bonds, which makes it easier to replicate than popular cap-weighted indexes such as Barclays or Bank of America Merrill Lynch, and therefore avoids illiquidity issues. Investment grade bonds that qualify for inclusion (issued in the U.S., and with an outstanding value of at least $500 million) are classified into one of three sectors: financials (48 bonds), industrials (36 bonds), or utilities (12 bonds), and into one of four maturity cells. While this index offers the advantage of dealing with the bums/concentration

[19]*A higher weight for an issuer with a high market value of debt does not necessarily mean that the index is overweighting issuers with a high face value of debt. An issuer with a high amount of par value debt outstanding will get a high weight only if the market value is relatively close to par value, which implies that the issuer is not perceived to be very risky. It is therefore not clear why the market-value—weighted index should become riskier. In addition, loading onto riskier issuers should not be a problem if this risk is rewarded by higher expected returns.*

[20]*A similar problem has been documented for equity indexes (see for example Amenc, Goltz and Le Sourd, 2006).*

problem, since the amount of debt issued does not affect the weighting scheme (even though the number of issues may have an impact), it does not address the lack of duration control, and may exhibit higher turnover levels than cap-weighted indexes, since maintaining the precise sector and maturity repartitions of the index induces more frequent rebalancing than in the case of buy-and-hold market-cap weighted indexes, which require trading only when the index constituency changes over time. Somewhat related versions exist, such as the equally weighted ladder indexes (see for example the Ryan/NASDAQ US Treasury Ladder index which is based on 30 equally weighted U.S. Treasury issues with fixed coupons but different maturities—from 1 to 30 years).

Instead of imposing an identical dollar contribution from various constituents of the bond index universe, one may seek to impose an identical risk contribution from all constituents. Such is the focus of inverse-duration–weighted bond indexes, for which the weight assigned to each bond is equal to the inverse of the (modified) duration of the bond taken as a proxy for the risk level of the bond.

Since duration weighting implies that the overall duration of the index is equal to the number of constituents, this index achieves both stability of factor exposure as well as some form of portfolio diversification. Nothing guarantees, however, that this *ad-hoc* portfolio construction methodology should lead to a benchmark with attractive risk-adjusted characteristics.

Liability-driven approaches to bond indexes address factor exposure risks (at least interest rate risk exposure) but they do not address concentration risks.

Some bond indexes have been created specifically to deal with the need to generate relevant benchmarks for liability-driven investment managers. For instance, the Markit iBoxx US Pension Liability indexes, launched in October 2006, are composed of three indexes: active, retired, and aggregate, which are considered to be benchmarks that reflect pension liability performance. Each index mimics the overall performance of a typical traditional defined-benefit plan (for either an active or retired person) in the U.S., taking into consideration the passage of time and changes in the term structure of interest rates (see Ryan's liability-driven bond indexes for a somewhat similar approach).

One drawback of these indexes is that they are very dependent on the liability data used to compute the discounted cash flows. Indeed the data, provided by Hewitt Associates for the Markit iBoxx US Pension Liability indexes, may not exactly match the specific cash flows of any particular plan sponsor.

Other forms of indexes also aim at controlling duration or maturity. For instance, the Ryan STRIPS index family is a family of 30 single Treasury strips securities that best represent each annual maturity between 1 and 30 years. STRIPS, which is the acronym for Separate trading of registered interest and principal securities, are created from traditional, coupon-paying bonds by decomposing the bond into two distinct cash flows: a zero-coupon bond and a collection of coupons. The zero-coupon part can then be seen as a building block for the design of custom liability indexes.

In a similar spirit, targeted- exposure indexes have been created to allow for a target stable interest rate exposure. For example, the Barclays U.S. Treasury Targeted Exposure index series are designed to reflect the returns of futures positions in U.S. Treasury bonds, where each weight is chosen so that the exposure to changes in yields is kept constant. More precisely, the index uses a fixed level of sensitivity to changes in the yield of the current cheapest-to-deliver bond i.e., each index targets a one- index—point change in index level per one- basis—point change in the yield of the underlying cheapest-to-deliver bond. This makes the indexes inversely linked to the performance of the underlying, in the sense that, when the two-year Treasury note yield increases, then the index targeting a two-year exposure decreases. Like liability-driven indexes, target exposure indexes address the problem posed by uncontrolled changes in index duration over time. These indexes do not, however, address the concentration problem; after all, there are an infinite number of bond portfolios that achieve a given target duration, and the question remains to assess which one of these portfolios, equally desirable for the liability-hedging properties, will enjoy the highest level of risk-adjusted performance.

Designing welfare-enhancing bond benchmarks requires that a number of key challenges be addressed

Given that none of the existing standard or alternative bond benchmarks offer satisfactory solutions to investors' needs, the question arises as to whether one could use risk models to construct improved bond benchmarks with a focus on enhancing diversification and controlling risk exposure, and subject to implementable levels of turnover and liquidity constraints. The performance of portfolio optimization techniques has been the subject of extensive research in the equity universe. One particularly critical research question that has been analyzed under many different angles since the introduction of modern portfolio theory in the seminal paper by Markowitz (1952) is whether the presence of estimation error in input parameters may entirely invalidate the relevance of formal optimization models from an out-of-sample perspective. In particular, given that the number of risk parameters grows faster than linearly in the number of constituents, sample-based estimates have been found to be unreliable for relatively large portfolios and small sample sizes, and several improved estimators for the covariance matrix have been proposed in the literature in an attempt to alleviate the concern over the so-called curse of dimensionality (see for example Ledoit and Wolf, 2003, 2004). In addition, Jagannathan and Ma (2003) find that imposing constraints on the weights in the optimization program improves the risk-adjusted out-of-sample performance in a manner that is similar to some of the aforementioned improved covariance matrix estimators.

The abundance of theoretical and empirical research on the performance of portfolio optimization techniques in the equity universe stands in sharp contrast to the relative scarcity of research about how to form bond portfolios with attractive risk/reward performance from an out-of-sample basis. For example, there is no readily available answer in the academic literature to fundamental questions such as whether an investor in sovereign or corporate bonds would be better off investing in an equally-weighted combination of available bonds vs. an optimally-chosen combination on the basis of careful parameter estimates.

That relatively little is known about the out-of-sample performance of bond portfolio optimization models is perhaps surprising given the importance of fixed-income investments in institutional and private investors' portfolios, as evidenced by the larger size of the bond markets both in terms of number of securities and total wealth invested. One possible explanation is that bonds are often held as part of investors' hedging portfolios, where the focus is on matching interest- rate risk factor exposures on the asset side to interest-rate risk factor exposures on the liability side, as opposed to risk/reward ratio maximization. This is, however, not a sufficient reason for ignoring the need to generate attractive risk-adjusted performance. After all, as indicated above, there are infinitely many bond portfolios with a given target duration, and selecting the one with the highest risk/reward ratio would presumably improve investor welfare. Besides, Treasury and corporate bonds are also natural ingredients within investors' performance-seeking portfolios, where the focus lies precisely on maximizing the risk/reward ratio.

Another possible explanation is that one might be able to transpose the methodologies and related empirical results from the equity universe to the fixed-income universe, thus relaxing the need for more specific analysis on bond portfolios. This explanation is also not entirely satisfying since individual bonds, unlike stocks or constant maturity bond indexes, have a finite maturity, which imposes a very specific structure on the covariance matrix and expected return vector. Moreover, no-arbitrage relationships exist between bonds of various maturities that impose a set of constraints on risk/return parameter estimators, for which no equivalents exist in equity portfolios. Finally, because of the aforementioned liability-hedging motives, bond portfolio optimization is often performed in the presence of duration constraints, which is yet another challenge that is specific to bond portfolio optimization problems.

In a recent paper (Deguest, *et al*., 2013), we extend the existing literature, which has mostly focused on the equity universe, by providing the first formal out-of-sample comparative analysis of the performance of various bond portfolio optimization models in the presence of duration constraints. To do so, we introduce a suitable three-step procedure that can be used to generate risk-return parameter estimates for bond portfolios that are consistent with the absence of arbitrage. At each rebalancing date, we first impose a no-arbitrage restriction that allows us to decompose all bonds available in a given universe into a sum of fictitious pure discount bonds matching coupon or principal payment dates and amounts. In a second step, we use the transition matrix from pure discount bond prices to coupon-paying bond prices obtained in step 1. This is used to extract a consistent covariance matrix for non-stationary coupon-paying bond returns from the covariance matrix for stationary, constant-maturity, pure discount bond returns. This procedure ensures the respect of no arbitrage conditions, as well as the respect of the structure inherent in bond prices, e.g., the convergence of bond return volatility to zero when approaching maturity. In a third step, we strengthen the covariance matrix for coupon-paying bonds obtained in step 2 using a factor model for the term structure. In the empirical analysis, the factors are extracted from a principal component analysis of the return on coupon-paying bonds, with the first two factors (interpreted as the level and slope of the yield curves) typically explaining an exceedingly large fraction of the bond return variance.

Using risk- parameter estimates obtained from the aforementioned three-step procedure, as well as expected return estimates based on the parsimonious prior of a constant reward for the few selected risk factors, we find that the use of Sharpe ratio maximization techniques generates an improvement in investors' welfare compared to the use of *ad-hoc* bond benchmarks such as equally weighted (EW) or cap-weighted (CW) portfolios. In addition to using the maximum Sharpe ratio (MSR), we also test different heuristic portfolio optimization models, including minimum concentration (MC) portfolios (which correspond to the closest approximation to an equally weighted strategy subject to constraints such as duration or weight constraints), global minimum variance (GMV) portfolios and diversified risk parity (DRP) portfolios, also known as factor risk parity portfolios (see Deguest, Martellini and Meucci, 2013). This improvement is relatively small among single issuers, where correlation levels between portfolio constituents are exceedingly high, but becomes substantial in the multiissuer eurozone, at least before the decoupling of the sovereign bond markets that took place over the summer of 2010.

While the encouraging results we have obtained for sovereign bonds suggest that enhanced bond benchmarks can be constructed with improved characteristics in terms of concentration risks and factor exposure risks, a number of additional challenges remain to be addressed before the methodology can be applied in practice, including, among other things, the lack of liquidity of some bond issues, as well as the need to account for the presence of interest- rate risks and credit-risk-related sources of uncertainty and heterogeneity in bond returns. In this context, it is expected that addressing such challenges carefully will pave the way for the emergence of improved bond benchmarks that will provide adequate answers to investors' needs. ~

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DEFINED BENEFIT PLAN

Dynamic Investment Strategies for Corporate Pension Funds in the Presence of Sponsor Risk

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Optimizing the investment policy of a corporate pension fund is typically a challenging problem, owing to the presence of various agents endowed with different preferences with respect to the risk/return profile of the fund strategy. Among them are the beneficiaries of the pension plan, trustees of the pension plan, managers of the pension fund, as well as equity holders, bondholders and managers of the sponsor firm.

To analyze the conflicts of interest that may arise from these different perspectives, one can measure for a given allocation strategy the impact on the fair value of pension claims, corporate bond value, equity value and total value of the firm and pension fund. A structural model extending Merton (1974) is particularly well suited for this analysis, since the payoffs accruing to each group of agents are directly modeled as (non-linear) functions of the firm and pension fund assets. Martellini and Milhau (2011) introduce such a model and find evidence for an asset substitution effect between shareholders and pensioners (similar to the agency conflicts documented by Jensen and Meckling, 1976): if shareholders are entitled to receive pension fund surpluses, then they hold a call on the aggregate assets of the firm and the pension fund. Hence they prefer in general a more risky investment strategy, because this policy tends to increase the volatility of assets. On the other hand, pensioners are hurt by the increased likelihood of a deficit.

Martellini and Milhau (2011) also find that an effective way to align the interests of the two classes of stakeholders that does not require any modification of surplus sharing rules is for the pension fund to adopt some form of risk-controlled investing strategy. These strategies extend to an asset-liability—management (ALM) context the Constant Proportion Portfolio Insurance (CPPI) strategies (Black and Jones, 1987, Black and Perold, 1992), and can be shown to maximize the expected utility of a partial surplus (Detemple and Rindisbacher, 2008). They differ from the traditional CPPI through the building blocks: the "safe" asset is the liability-hedging portfolio (LHP), as opposed to cash, and the "risky" asset is itself a dynamic asset allocation strategy, namely the one that would be optimal in an asset-only setting.

Another example of a portfolio insurance strategy transposed to the ALM context is given in Martellini and Milhau (2012), who derive utility-maximizing policies under minimum funding constraints. The solution is an extension of Option-Based Portfolio Insurance (OBPI) strategies, where the payoff is given by the minimum acceptable wealth plus the payoff of a call written on the optimal unconstrained policy.

One limitation with these otherwise attractive strategies is that they have been designed by considering the pension fund in isolation, without explicitly accounting for the presence of a sponsor, which may or may not be able to cover a large pension deficit. More complex forms of dynamic ALM strategies can be designed to account for the possibility of partial insurance of funding risk by the sponsor, and the impact of these strategies on various stakeholders' wealth levels can be analyzed in the context of a capital structure model similar to that of Martellini and Milhau (2011), where equities, corporate bonds and pension claims are valued as collateralized and defaultable claims issued by the sponsor company.

CPPI strategies in ALM

Traditional CPPI strategies aim at guaranteeing a minimum level of wealth at the investment horizon. In ALM, the quantity of interest is not the value of assets but the funding ratio, which is the ratio of assets to liabilities. In view of regulatory constraints, which often impose a minimum funding ratio, it makes sense for the pension fund to adopt a strategy that keeps the funding ratio above the minimum acceptable level at all times. Mathematically, the floor is kL_T, where k is the minimum funding ratio level and L_T is the liability value at the horizon. The following dynamic investment policy ensures the respect of the minimum funding constraint:

$$w_t = m\left(1-\frac{k}{R_t}\right)w_t^u + \frac{k}{R_t}w_t^{LHP} \qquad \boxed{1}$$

Here, w_t denotes the weight vector at date t, w_t^u is some "unconstrained" portfolio strategy, w_t^{LHP} is the portfolio strategy that best replicates the liabilities (also known as Liability Hedging Portfolio, or LHP), m is the multiplier, and is $R_t = \frac{A_t}{L_t}$ the current funding ratio, where A_t denotes the pension asset value at date t. For simplicity, we take the unconstrained strategy to be invested in a stock index, since this building block is intended to deliver performance. The composition of the LHP is determined by the denomination and the discount rate of liabilities: for instance, if they are unconditionally indexed on inflation, the LHP is an indexed bond with the same duration; if they are fixed in nominal terms or conditionally indexed, the LHP is a duration-matching nominal bond.[21] If liabilities are discounted at the risk-free rate plus a spread

EXHIBIT 1

This figure shows the distribution of the value of the funding ratio after ten years, its conditional mean between 100% and 120%, the fair values of pension claims (P_0), equities (E_0) and corporate bonds (D_0), and the total value of the sponsor firm and the pension fund (v_0). The minimum and the maximum funding are set to 100% and 120%, respectively, the initial asset value of the firm's assets is normalized to 100 and the multiplier is set to 3.

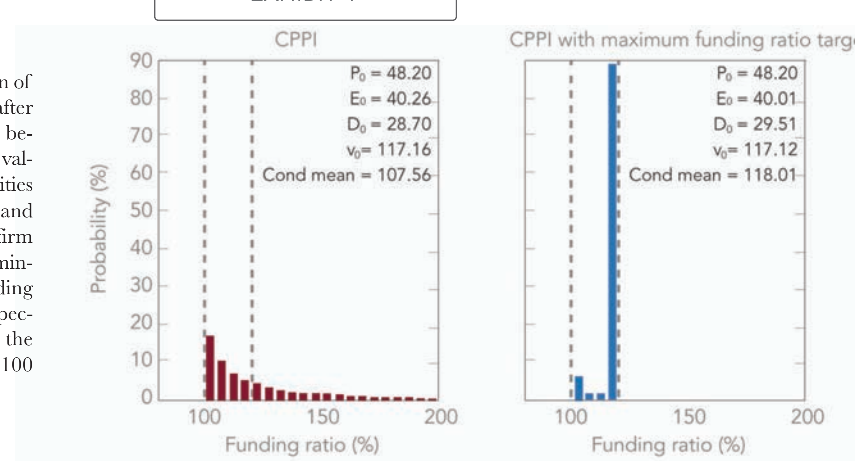
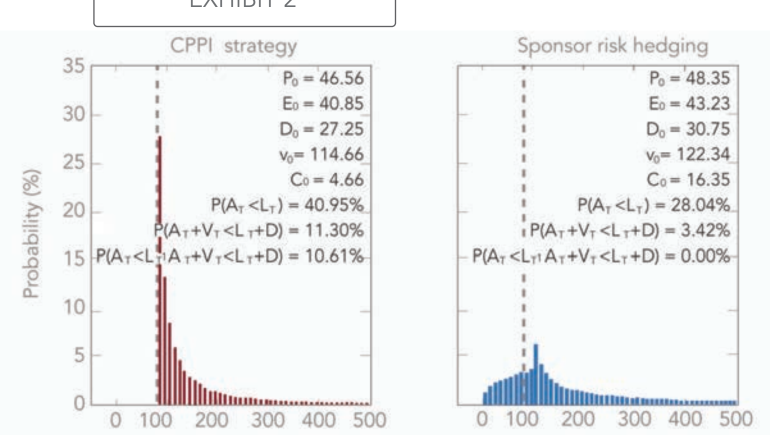


EXHIBIT 2

This figure shows the distribution of the value of the funding ratio after ten years, the fair values of pension claims (P_0), equities (E_0) and corporate bonds (D_0), the total value of the sponsor firm and the pension fund (v_0), and the fair value of the contribution from the sponsor to the pension plan (C_0). The initial asset value of firm's assets is normalized to 100, and the multiplier is set to 3 in the left panel and to 1 in the right one. Here the minimum funding ratio level is taken to be k=80%.



[21] Conditional indexation means that indexation is granted only if the funding ratio is above a certain threshold.

[22] Such risky bonds are not necessarily a perfect hedge for liability risk, because they are subject to the risk of default of their issuer.

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reflecting a given credit rating (it is recommended under accounting standards SFAS 87.44 and IAS 19.78 that the credit rating is taken from AA or AAA bonds), then the LHP would in principle have be invested in risky bonds having the same rating.[22]

In order to analyze the impact of the multiplier on pensioners, equity holders and bondholders, we compute the fair values of pension claims, equities and corporate bonds. To do this, we use the same pricing model as in Martellini and Milhau (2011), which extends Merton (1974) to account for the presence of the pension plan: each claim is valued as the expected value of the payoff, discounted at the risk-free interest rate. The main assumptions made in the valuation exercise are the following: pension assets serve to pay pension claims first; if they do not cover promised benefits, a contribution is required from the sponsor company; if promised payments to pensioners and bondholders exceed available assets, then the firm defaults and equity holders receive nothing. Equity holders receive the fraction of assets that is in excess of the promised payments only if these payments are made. Another key assumption is the pension surplus– sharing rule. If they are entitled to pension surpluses, equity holders hold a call on pension fund assets, and thus benefit from a higher volatility in asset value, hence from a higher multiplier. On the other hand, pensioners have a short position in a put written on the same underlying, thus they prefer a safe strategy, unless they receive guarantee that pension payments will be delivered in all market conditions. Precisely, the strategy described in equation (1) ensures that assets cover liability commitments, but also opens access to the possibility of surpluses: as a result, a higher multiplier leaves pensioners unaffected, but has a positive impact for equity holders.

One drawback of this strategy is that it carries an opportunity cost, which is essentially the price to pay for insuring against the risk of large shortfalls. As shown by Deguest, Martellini and Milhau (2011) in an asset-only context, this cost can be reduced by giving up wealth levels that are in excess of a maximum threshold or cap. In order to impose a cap on the funding ratio, one can modify the strategy in equation (1) to shift progressively to the LHP as the funding ratio approaches either the minimum or the maximum, as opposed to shifting to the LHP near the minimum only. The reader is referred to Martellini, Milhau and Tarelli (2012) for the formal expression of the related allocation strategy. Figure 1 shows the distribution of the value of the funding ratio after ten years, together with the values of the claims, assuming that surpluses are returned to equity holders. Pensioners are not affected in any way by the imposition of a cap because they received exactly the promised payment, but equity value slightly decreases and corporate debt value increases significantly. Overall, the impact on the total value, which is the sum of the three previous prices, is positive.

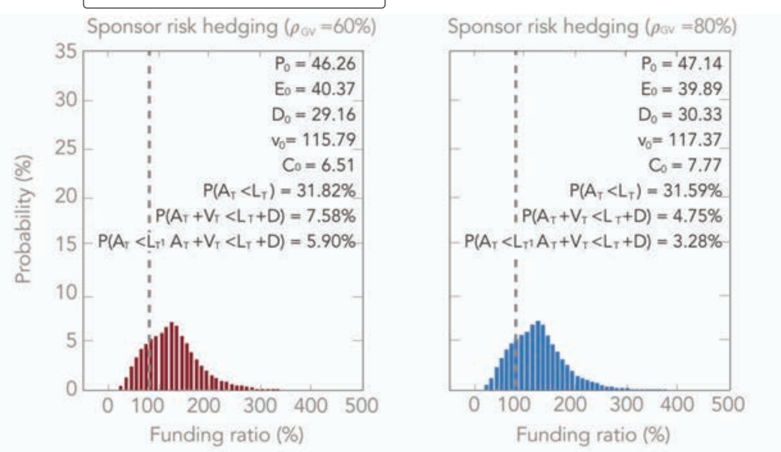
From ALM to integrated ALM: insuring against sponsor risk

Another way to reduce the opportunity cost associated with CPPI strategies is to recognize that insurance against shortfall risk is only required in those situations where the sponsor firm is not in a position to make up for the funding gap; if a pension deficit occurs when the sponsor company is healthy, then the sponsor company will provide insurance without the need to incur the opportunity cost of introducing downside protection at the asset allocation level. Formally, if D denotes the face value of debt, and V_T is the value of the firm's assets, the objective is not to ensure that $A_T \geq L_T$ in all states of the world (which would ignore the possibility for the sponsor to contribute), but rather that $A_T \geq L_T + D - V_T$ whenever $A_T \geq L_T$: this condition means that in the event of a deficit, the sum of the pension fund's and firm's assets must cover commitments to pensioners and bondholders. This suggests the use of a floor F_T^1 equal to the minimum of L_T and $L_T + D - V_T$. But because the quantity $L_T + D - V_T$ can be negative, the preferred option is to impose the following floor:

$$F_T^2 = \max[\min(L_T, L_T + D - V_T), 0] \quad \boxed{2}$$

EXHIBIT 3

Notations and assumptions are the same as in Figure 2, except that the pension fund now has access to an imperfect firm-hedging security, whose value process, G, has correlation 60% (left panel) or 80% (right panel) with the actual firm value process, V. The expected return and volatility of G are set to the same values as those of V.



In order to construct a portfolio strategy that maintains wealth above this floor at all times, one needs a floor-replicating portfolio (denoted as w_t^{F2}): then, this portfolio replaces the LHP in equation (1), and the floor F_T^2 replaces the floor kL_T so that the allocation strategy takes the form:

$$w_t = m\left(1 - \frac{F_t^2}{A_t}\right) w_t^{u} + \frac{F_t}{A_t} w_t^{F2} \quad \boxed{3}$$

The replication of F_T^2 requires a firm-hedging security, that is, an asset perfectly correlated with V, the firm's asset value. As suggested by Merton's model (1974), a natural proxy would be the firm's own stocks. In fact, the firm's equity is also affected by other sources of risk (e.g., interest rate risk in the discount rate of future cash-flows), so that the firm-hedging security would, in principle, instead be a dynamic strategy suitably designed to cancel out the exposure to other risks.

Figure 2 reports, among other indicators and for reasonable parameter values, the probability of the "bad" states of the world, where the pension fund is in deficit and the sponsor cannot cover the deficit through additional contributions: it reaches 10.61% with the extended CPPI strategy from equation (1), and falls to zero, as it should, with a control of sponsor risk. Pension claims are then more valuable because pensioners are more likely to receive the promised payments. Bondholders also benefit from the strategy incorporating an explicit control for sponsor risk, and this for the same reason: either the pension fund is solvent, in which case the sponsor's assets are available to redeem debt, or it is insolvent, but the sponsor's assets are sufficient to deliver the promised payments to pensioners and bondholders. Equity value also increases, because equity holders are more likely to receive a nonzero payment. This increase can be interpreted as an effect of reducing the cost of insurance: with our parameter values, the fair price of the payoff F_T^2, which is the cost of insurance, is lower than the fair price of the floor kL_T.

In spite of these advantages, the respect of the floor F_T^2 does not guarantee the respect of a minimum funding ratio k: to comply with the regulation, the manager can adopt a hybrid floor, which is the maximum of the integrated ALM floor and the regulatory one:

$$F_T^3 = \max[\min(L_T, L_T + D - V_T), kL_T]$$

Since this floor is above kL_T insurance against shortfall risk is more expensive than under the strategy in equation (1). As a matter of fact, it can be shown (Martellini, Milhau and Tarelli, 2012) that equity value is lower than with the basic CPPI strategy, but this loss is more than offset by the positive impacts on pension claims and corporate bonds, so that the total firm value increases.

A practical issue with the previous strategies is the requirement for a perfect firm-hedging security, which is in fact a dynamic strategy involving the sponsor firm's equities. It is more realistic to expect that only an imperfect hedge can be achieved: hence, the floors that are actually replicable are not those introduced in equations (2) and (3) above, but rather those obtained by substituting in the same equations some payoff G_T for V_T, where the processes G and V have a correlation less than 1. An immediate consequence is that the probability for the pension fund to be in deficit and the sponsor not to be able to make the needed contribution cannot be taken down to zero. This is illustrated by Figure 3, in which the correlation was set to 60% (respectively, 80%), and the probability of the "bad" states of the world is 5.90% (respectively, 3.28%). These numbers are not zero, but are still lower than the probability that would be attained without the control of sponsor risk (10.61%).

Comparing the values of the claims with the left panel of Figure 2 (which refers to the basic CPPI), it can be seen that pension claims are more valuable only if the correlation is 80%. Indeed, with a correlation of 60% only, large pension deficits can still occur. With such low correlations, it proves in fact preferable, from the pensioners' perspective, to impose the hybrid floor F_T^3 obtained by replacing ,V_T by G_T in the expression of F_T^3 in Equation (3) (see MMT12 for an illustration). A comparison of Figure 3 with the left panel of Figure 2 also shows that bondholders still benefit from insurance against sponsor risk, even if it is partial. More surprisingly, equity value turns out to be higher when the worse firm-hedging security is employed (left panel). But it is still lower than with the basic CPPI. In summary, the integrated ALM strategies presented above have a positive impact on pension claims and corporate bonds when compared to a CPPI policy that does not control for the presence of sponsor risk. They tend to decrease equity value (except in the rather hypothetical case where a perfect firm-hedging asset is available), but the loss of value is weak, and from a collective perspective, the impact on total value is positive. ~

CONCLUSION

In conclusion, a CPPI strategy adapted to the ALM context reduces the conflicts of interest between pensioners and shareholders by allowing the pension fund to invest more in risky assets, which generates surpluses from which equity holders will benefit, while protecting the funding ratio, which is in the pensioners' interest. By introducing a cap in addition to the floor on the funding ratio, one can also decrease the cost of insurance against shortfall risk, which has a positive effect from bondholders' and pensioners' perspectives. Another way to reduce this cost is to implement policies that aim at insuring, at least partially, against sponsor risk: these strategies avoid, or decrease the likelihood of, large pension plan deficits in situations where the sponsor cannot make the required contributions. Overall, our findings suggest that suitably designed dynamic portfolio strategies can prove to be a very effective answer to some key challenges currently faced by corporate pension plans.

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DEFINED BENEFIT PLAN

Toward a Fair(er) Valuation of Pension Liabilities

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One of the main risks for plan participants, actually the only source of uncertainty for a defined-benefit plan with unconditional liability payments, is that of sponsor bankruptcy when the pension plan is underfunded. In an attempt to address this concern, a number of dramatic changes have been made over the past few decades in the legal, regulatory, accounting and fiscal environments of state as well as corporate pension funds. These have collectively led to a significantly heightened scrutiny over pension liabilities valuation, with a focus on greater transparency with respect to the impact of both market and credit risk components on pension obligation values. The recent and spectacular bankruptcy of the city of Detroit has resulted not only in a heightened concern among workers and retirees that their pension claims may never be honored; it has also revived the longstanding debate over the proper approach to the valuation of pension liabilities. The official value for the pension liabilities was found to have been severely underestimated, with a $3.5 billion hole that suddenly appeared in Detroit's pension system.[23] A similar debate is currently raging in Europe, in particular in the Netherlands, where the government seeks to keep the retirement system viable through the use of an appropriately adjusted discount rate that would lead to a lower reported value for Dutch pension fund liabilities.

Overall, it is fair to say that there is no universally recognized approach for setting a proper discount rate for liabilities, as compared with the standard valuations formula coming from asset valuation procedures. There are at least three different types of discount rates that can be used to value pension liabilities: arbitrary discount rates, market discount rates and endogenous discount rates. In what follows we review some of the key academic insights about the use of various discount rates for liabilities, and the pros and cons associated with such choices. Also, we discuss the importance of the perspective of the key stakeholders in settling these issues.

Fixed arbitrary discount rates vs. allocation-dependent arbitrary discount rates

A distinction here should be made between the use of a fixed arbitrary discount rate and the use of a discount rate that is given by the expected return on the investment portfolio. The latter allocation-dependent discount rate only moves in time if the pension fund allocation moves in time, and stays constant otherwise, regardless of market conditions in general and changes in interest levels in particular.

Fixed arbitrary discount rate

For simplicity and to reduce short-term risks to the sponsoring organization, regulators have employed fixed discount rates in the past. For example, pension plans in the Netherlands employed fixed discount rates for many years. Consequently, the only risks involve changes in the market value of the plan assets and possible changes in future cash liabilities, such as changes in workforce or changes in pension rules.

In effect, regulators following this procedure are allowing the sponsoring organization to smooth the surplus and funding ratio risks. However, there are several difficulties with this. First, there is no sound theoretical approach for setting the fixed discount rate. What should the rate be, and how should this rate be affected by economic conditions, if at all? Second, suppose that the sponsoring organization wishes to minimize its surplus risks in this context. The sole approach would be to minimize asset return volatility by employing short-term, risk-free assets, which could be relatively expensive compared with the true immunization strategies based on long-term bonds. Last, the use of fixed rates can cause considerable confusion regarding unfunded liabilities when the health of the sponsoring organization is threatened. The actual market value of the surplus can be larger or smaller than the regulatory surplus. For these reasons, we do not advocate the use of fixed arbitrary discount rates, a procedure based on no sound academic grounds.

Allocation-dependent discount rate

According to the return on asset assumption (ROA), the discount rate used for valuation of liabilities is given by the expected return on the asset portfolio. This approach poses a number of severe problems. On the one hand, it is not possible to estimate the expected return on risky assets with a sufficient degree of accuracy and therefore the methodology also leads to setting discount rates in a rather arbitrary manner. On the other hand, even if it were possible to obtain reliable estimates for expected returns, the use of an expected return as a discount rate violates almost all important principles of fair valuation methodologies. In particular, it leads to the somewhat perverse conclusion that the most efficient way to address an underfunding situation is to invest more aggressively in risky assets such as equities, which have, arguably, a higher expected return. This procedure appears to be the most misleading possible choice in terms of a valuation methodology for pension liabilities. Another consequence of using the expected return as a discount rate in the case of pension liabilities with conditional indexation (as is, for example, the case in the Netherlands) is a value transfer from younger to older beneficiaries (say workers vs. retirees); indeed the induced decrease in liability value will lead to an increase in the funding ratio, which, in turn, will generate additional indexation or lower pension reduction for the retirees at the expense of current workers.

Market discount rates with or without a credit-risk adjustment

The starting point in any attempt to move away from the use of arbitrary discount rates consists of recognizing that pension liabilities can be interpreted as a yield curve of monthly benefit payments. The no-arbitrage pricing principle, then, stipulates that the monthly promised pension payments should therefore be discounted at a market rate, with a key distinction between Treasury rates, which are presumed to be default-free rates, and corporate rates, which are default-sensitive rates.

Market default-free rate

Support for this approach for valuation is provided by the Financial Accounting Standards Board (FASB). According to Financial Accounting Statement (FAS) 87, in valuing pension liabilities of plan sponsors, the liability is required to be priced as a high-quality, zero-coupon bond whose par value matches the liability payment amount, and whose maturity matches the liability payment date. More precisely, the selection of discount rates is explained in paragraph 186 of FAS 106 (December 15, 1990) as follows: "The objective of selecting assumed discount rates is to measure the single amount that, if invested at the measurement date in a portfolio of high-quality debt instruments, would provide the necessary future cash flows to pay the accumulated benefits when due. Notionally, that single amount, the accumulated postretirement benefit obligation, would equal the current market value of a portfolio of high-quality zero coupon bonds whose maturity dates and amounts would be the same as the timing and amount of the expected future benefit payments."

The argument in favor of using Treasury spot rates is that the present value of the liabilities represents how much in Treasury securities the U.S. Department of the Treasury would have to issue to pay off the projected liabilities (see for example Ryan, 1993, for an early reference). While the use of a market rate unarguably represents progress compared to using a constant arbitrary rate independently of market conditions, a possible question still remains regarding the presence of a credit risk adjustment.

Market defaultable rate

In an attempt to account for the presence of credit risk in pension claims, international accounting standards SFAS 87.44 and IAS19.78 recommend that corporate pension obligations be valued on the basis of a discount rate equal to the market yield on AA corporate bonds, the same rate for all firms. Obviously, default risk on pension liability is a concern not only for corporate pension plans, but for state and local pension plans as well, as was painfully evident in the recent default of the municipality of Detroit, leaving beneficiaries with partially unfunded pension claims.

However, there are at least four problems with using AA- or AAArated corporate bonds (see example Ryan and Fabozzi, 2003, for more details). First, the credit risk premium is not only a reward for additional credit risk in corporate bonds vs. Treasury bonds, but also for additional liquidity risk. In this context, it is unclear why any persistent or temporary liquidity effects in the corporate bond market should be reflected in pension liability values. Second, there are major problems with corporate bond indexes used to represent the corporate AA and AAA markets (see for example Campani and Goltz, 2011). Third, there are not many corporate bonds available with a duration in excess of 15 years, despite the fact that liabilities are far longer than 15 years. Finally, one of the reasons for using a term structure is to create a portfolio of high-quality fixed-income instruments to defease the projected liabilities. Yet, the size and liquidity of the AA- and AAA- rated corporate bond market is such that a defeasance is not possible, which undermines the very foundation of the noarbitrage valuation principle.

More generally, the use of the same market rate to discount all pension liabilities, regardless of the sponsor credit rating, pension funding situations and asset allocation policy, is not likely to lead to a correct assessment by the various stakeholders of the impact of specific default risk on the value of pension obligations. It is hardly justifiable that a widening of credit spreads on corporate bond markets should lead to an improvement of the funding status of defined-benefit pension funds.

In the end, it appears that using an arbitrary credit risk adjustment is not a satisfactory answer to the desire for the presence of a credit risk adjustment.

[23]*See the July 19th, 2013, New York Times article "Detroit Gap Reveals Industry Dispute on Pension Math."*

Endogenous discount rates

Asset pricing theory suggests that the fair value of pension liabilities should be handled by regarding them as privately held, collateralized, defaultable claims issued by the sponsor company, municipality or state to workers and pensioners, and using option pricing methodologies similar to the one introduced by Merton (1974) for the valuation of corporate bonds. Under this framework, the primary perspective is the sponsoring organization.

In a recent paper, Martellini and Milhau (2011) use this approach and argue that valuation principles for liabilities streams should account for differences in financial health and capital structure decisions at the sponsor company level, as well as differences in asset allocation policy at the pension fund level. They find that the existence of a pension plan has a strong impact on capital structure decisions, with the optimal leverage ratio a decreasing function of promised pension payments. They also say that the presence of a pension fund has a substantial impact on debt value and credit ratings. Conversely, these results suggest that capital structure decisions have a substantial impact on the fair value of pension claims, with a pension credit spread that increases approximately proportionally to the leverage ratio.

One important corollary is, therefore, that the regulatory valuation leads to overestimating the fair value of liabilities for highly leveraged firms, while it leads to underestimating the liability value for firms with little debt outstanding. One can, in principle, use an option pricing model to allow for a quantitative estimate of the magnitude of the over/underestimation as a function of parameter values.

These insights have important policy implications in that they provide a first step towards a much needed methodological framework for the design of firm-specific regulatory constraints and liability valuation principles. They also call for the emergence of a scheme-specific pension insurance pricing rule.

It should be emphasized, however, that a move toward an endogenous discount rate rationally taking into account credit risk in liability streams would come with its own pitfalls. First, while the methodology seems to be well-suited for private plans, it would be difficult to apply the option approach to state and local defined benefit plans. Additionally, there are implementation issues with an option approach related in particular to the choice of parameter values, e.g., future volatility of the assets of the firm. Any indeterminacy in the parameter value would translate again into a situation where actuaries could make arbitrary choices that would lead to ambiguities as severe as the ambiguities related to incorrectly estimating future returns. In other words, one could adjust the assumed volatility to get the desired funding ratio, just as one can adjust the expected return assumption to obtain the desired discount rate.

The implementation would require a careful assessment of the inputs to the model. (See Kiska, Lucas and Phaup, 2011, for the use of an option-pricing approach. A model for the joint statistical distribution over time of defined-benefit pension underfunding and sponsor terminations, so as to evaluate the prospective cash flows of the Pension Benefit Guaranty Corporation [PBGC] and appraise its financial position would also be needed.) In addition, using an endogenous discount rate would lead to a counter-cyclical regulation; sponsor companies in bad shape will face a softening of the funding ratio constraints, since a pension fund will report an improvement in the surplus or funding ratio in the event of a decline in the credit quality of pension liabilities. It can be argued that reporting a gain from a decline in credit quality is potentially misleading and can mask a deteriorating situation. On the other hand, it can also be argued that a softening of the funding ratio constraints in difficult situations might help sustain the defined benefit pension system. Of course, in a rational expectation model, employees of the sponsor company would observe the deterioration in present value of their pension benefits and bargain for increases in wages to compensate for the loss, an option that would not be available to retired beneficiaries.

As an alternative, if regulators are able to separate pensions into healthy and underfunded plans, they could allow the healthy plans to take on risky investments, thereby reducing the expected costs of operating the plan for these firms. When plans become underfunded, the regulators would require these plans to become more conservative or to increase their contributions to improve the funding ratio. In this way, the pension plan system would operate in a manner similar to banks, in which the need for capital allocation rules are evident. Of course, such a dynamic regulatory environment requires the regulators to be able to render their rules in a timely fashion and with current information, which may place a burden on organizations such as the Labor Department in the United States.

Conflicts of interest among stakeholders and investment policy implications

The use of various discount rates not only has important regulatory and accounting implications, but it also has substantial investment policy implications. For example, from the standpoint of the pension plan participants and government regulators (rather than insurance organizations such as the PBGC), there should be greater emphasis on fixed-income assets and immunization strategies to protect these stakeholders. It should be noted, however, that the future liabilities can be uncertain because of workforce changes, pension rule changes, and so on. Therefore the immunization strategies can be relatively complicated to implement.

On the other hand, from the perspective of the sponsoring organization, the use of discount rates that depend upon the blended return of the assets will encourage greater risk-taking since higher returns translate into higher discount rates, which means lower contributions—at least in the short run. In a similar vein, the use of endogenous discount rates, which give rise to higher discount rates as the deficit increases, provides advantages for the sponsoring organization during stressful periods but may be at odds with the goals of some of the other stakeholders.

One of the core difficulties with choosing the appropriate discount rate is that it implicitly requires a clear definition of what the perspective is. Indeed, there are various stakeholders involved (shareholders of the sponsor company, pension plan beneficiaries, taxpayers who ultimately provide implicit or explicit insurance against public and private pension deficits, etc.), and there are clear indications of the presence of conflicts of interest among these stakeholders. For example, Martellini and Milhau (2011) find that the fair value of promised payments to pensioners (and bondholders) decreases if the allocation to risky assets by the pension fund increases when the correlation between the value of the firm process and the stock index return process is positive. This is a clear case of asset substitution, since a higher allocation to risky assets leads to an increase in the total riskiness of the total assets held by the firm (financial assets held off the balance sheet through the pension funds and real assets directly held on the balance sheet). This is the underlying state variable on which the value of such claims is based. Similarly, assuming they do not have access to any surplus of the pension fund, risk-taking is detrimental from the pensioners' perspective because it involves increasing the likelihood of partial recovery of pension claims, while risk-taking allows shareholders to reduce the burden on contributions needed to meet expected pension payments caused by exposure to the upside potential of the performance-seeking assets.

These conflicts of interest could be mitigated by granting pensioners some partial access to the surplus (the case with conditional indexation rules in the Netherlands), thereby allowing plan beneficiaries to benefit from the increases in expected performance related to more aggressive investment strategies. Another effective way to align the incentives of shareholders and pensioners without any complex adjustment to the pension plan structure consists of enlarging the set of admissible investment strategies so as to include dynamic risk-controlled strategies, sometimes referred to as contingent immunization strategies or dynamic liability-driven investment (LDI) strategies (Martellini, Milhau, and Tarelli, 2012). In fact, implementing risk-controlled strategies aiming at insuring a minimum funding ratio level above 100% allows shareholders to get some, limited, access to the upside performance of risky assets, while ensuring that pensioners will not be hurt by the induced increase in risk.

Because of the complexity and conflicting goals of the major stakeholders, it is important to evaluate the pros and cons of the alternative discount rates when determining the proper discount rate. Asset pricing theory unambiguously suggests that the only meaningful set of discount rates is given by the term structure of risk-free rates. One might argue that using Treasury bond yields as discount rates does not reflect the impact of default on pension values and would lead to overestimating the value of pension claims from the beneficiaries' perspective. On the other hand, prudent regulation is precisely designed to make the impact of default on pensions as minimal as possible. A prudent regulation would require higher contributions, anyway, in the event of increases in the default probability, which would be inconsistent with decreasing liability values in response to increases in default probability. To our minds, the use of market default-free rates therefore appears to be a reasonable and pragmatic approach to pension valuation problems. Given that it is believed that a government (the PBGC in the U.S.) will bail out the defined benefit system under a systematic failure, it is, after all, only fair and meaningful that pension liabilities should be valued at the government bailout cost. Admittedly, such an approach leads to higher liability values and, therefore, lower funding ratios, compared to the use of arbitrary market rates or corporate discount rates. In this context, it could be possible to accompany the use of a lower discount rate with a more lenient regulatory environment, if needed, particularly in terms of minimum funding ratio requirements.

In this article, we assume that the liability cash flows are certain or can be approximated by expected values. If liabilities are a direct function of a factor such as inflation, the discount rate should be related to this factor: herein, the inflation-adjusted government bond rate (real rate). As always, the duration of the liabilities must be linked to the duration of the discount rates.

Arguably, the proper way to handle the pension crisis is not by hiding the truth about current funding deficits. While an integrated asset and liability management system provides the ideal setting to analyze the subtle questions related to pension liability valuation (see Mulvey, 1994, and Mulvey, *et al.*, 2006, 2008), in our experiences with pension plans over the past 20 to 30 years this type of discussion has rarely occurred in practice. Still, these issues are critical to the future success of the defined benefit pension system, especially as defined contribution plans have become the mainstay of most new organizations and provide much easier and cheaper administration by the various organizations. This trend, of course, also poses a serious challenge, as it is unclear whether individuals will be able to make educated and proper decisions about their own welfare. ~

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